Chapter 6 Surface Area, Volume, and Capacity, Practise Your New Skills, p378–381
Student Resource p268–271

1. a) Volume to surface area of a cube of side length ℓ :
Surface area = $6\ell^2$
Volume = ℓ^3

$$\frac{\text{Volume}}{\text{Surface area}} = \frac{\ell^3}{6\ell^2}$$

$$\frac{\text{Volume}}{\text{Surface area}} = \frac{\ell}{6}$$

b) Volume to surface area of a sphere of radius r:
Volume = $\frac{4}{3}\pi r^3$
Surface area = $4\pi r^2$

$$\frac{\text{Volume}}{\text{Surface area}} = \frac{(\frac{4}{3}\pi r^3)}{4\pi r2}$$

$$\frac{\text{Volume}}{\text{Surface area}} = \frac{r}{3}$$

c) Relationship between the length of the cube and the radius of the sphere:
surface area of cube = surface area of sphere
$6\ell^2 = 4\pi r^2$
$\ell^2 = \frac{2}{3}\pi r^2$
$\ell = r \times \sqrt{\frac{2}{3}\pi}$
$\ell \approx 1.447r$
d) Volume of cube = volume of sphere

$$\ell^3 = \frac{4}{3}\pi r^3$$

$$\ell = r(\frac{4}{3}\pi)^{\frac{1}{3}}$$

$\ell \approx 1.612r$

2. a) Convert the diameter of the pipe to metres.

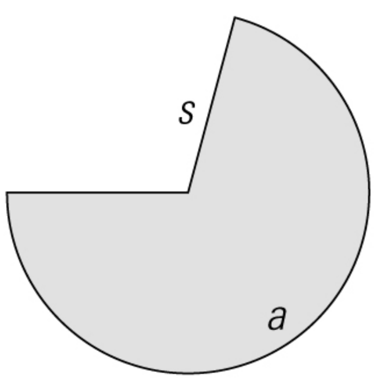
$$762 \text{ mm } \times \frac{1 \text{ m}}{1000 \text{ mm}} = 0.762 \text{ m}$$

Divide by 2 to calculate the radius.
$0.762 \div 2 = 0.381$ m
Calculate the volume.
$V = \pi r^2 \ell$
$V = \pi(0.381)^2(1000)$
$V \approx 456.0 \text{ m}^3$
A 1-km section of pipeline in Canada will hold 456 cubic metres of water.
b) The diameter is 24 inches, which is equal to 2 feet. The radius is 1 ft.
Calculate the volume.
$V = \pi r^2 \ell$
$V = \pi(1)^2(5280)$

$V \approx$ 16 588 ft^3
A 1-mile section of pipeline in the U.S. will hold 16 588 cubic feet of water.
c) Calculate how much water was needed in the Canadian section.
$V = \pi r^2 \ell$
$V = \pi(0.381)^2(567\ 000)$
$V \approx$ 258 573 m^3
Convert to litres.
$$258\ 573 \text{ m}^3 \times \frac{1000 \text{ L}}{1 \text{ m}^3} = 258\ 573\ 000 \text{ L}$$
Calculate the volume of the U.S. section.
r = 0.610 m ÷ 2
r = 0.305 m
$V = \pi r^2 \ell$
$V = \pi(0.305)^2(535\ 000)$
$V \approx$ 156 352 m^3
Convert to litres.
$$156\ 352 \text{ m}^3 \times \frac{1000 \text{ L}}{1 \text{ m}^3} = 156\ 352\ 000 \text{ L}$$
Add to find the total volume.
156 352 000 + 258 573 000 = 414 925 000 L
The pipeline held 414 925 000 L, or about 415 million litres, of water from end to end.

3. a) Volume of large pyramid = $\frac{1}{3} \times 25^2 \times 26.8$
$V \approx 5583$ cm^3
Volume of small pyramid = $\frac{1}{3} \times 7.5^2 \times 7.5$
$V = 140.625$ cm^3
Volume of rectangular prism = 25 × 25 × 40
V = 25 000 cm^3
Total volume = 25 000 + 5583 − 140.625
V = 30 442.375
The volume of the bin is 30 442 cm^3.
b) $\frac{30\ 442}{2250} \approx 13.5$ kg
The bin holds 13.5 kg of coffee.
Students should realize they can subtract the small pyramid from the large pyramid to find the volume of the bottom of the bin.

4. a)



b) Edge *a* is the length of the circumference of the base of the cone, or $2\pi r$.

5. a) Surface area of top = πr^2
$SA = \pi(900)^2$
$SA \approx 2\ 544\ 690$ mm^2
Surface area of cylinder = $2\pi rh$
$SA = 2\pi(900)(1570)$
$SA \approx 8\ 878\ 141$ mm^2
The surface area of the lateral face of a cone equals πrs where s equals the slant height.
Use the Pythagorean theorem to find s.
$s^2 = r^2 + h^2$
$s^2 = 900^2 + 1930^2$
$s \approx 2130$ mm
Surface area of cone = $\pi(900)(2130)$
$SA \approx 6\ 022\ 433$ mm^2
Total surface area:
2 544 690 + 8 878 141 + 6 022 433 = 17 445 264 mm^2
The amount of sheet metal needed is 17 445 264 mm^2.
b) 1 m^2 = (1000 × 1000) mm^2
1 m^2 = 1 000 000 mm^2
$\frac{17\ 445\ 264}{1\ 000\ 000} \approx 17.445$
The amount of sheet metal needed, in metres, is 17.445 m^2.

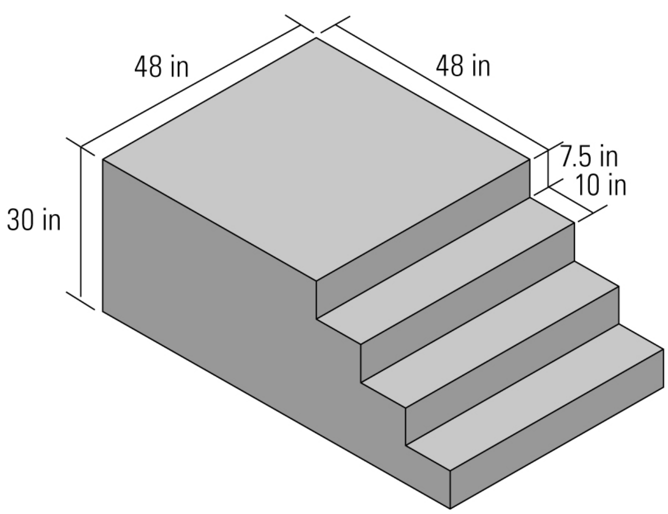
6. a) Volume of one basket = $\frac{1}{2} \times \frac{4}{3}\pi r^3$
$$V = \frac{1}{4} \times \frac{4}{3}\pi\left(\frac{50}{2}\right)^3$$
$V \approx 32\ 725$ cm^3
Convert to litres.
1 litre = (10 × 10 × 10) cm^3
Capacity of one basket:
$\frac{32\ 725}{1000} = 32.725$ L
Soil needed for 48 baskets:
48 × 32.725 ≈ 1571 L
Ara will need 1571 L of soil.
b) Bags:
1571 ÷ 60 ≈ 26.2, rounded up to 27 bags
27 × 13.50 = 364.50
Bags of soil would cost $364.50.
Bulk soil:
Use the given conversion factor.
one cubic yard = 27 × 28.23
one cubic yard = 762.21 L
1571 ÷ 762.21 ≈ 2.06 cubic yards
2.06 × 41.50 = 85.49
Bulk soil costs $85.49.
Ara should buy the bulk potting soil.

7. The awning has a top, sloped face, and two ends. The back and bottom are open.
Total surface area = sum of areas of faces

Area of top = 20×1.5
$A = 30 \text{ ft}^2$
Area of sides (both):
$$A = 2\left((2 \times 1.5) + \frac{1}{2}(2)(2.5)\right)$$
$A = 11 \text{ ft}^2$
Area of sloped face:
First calculate the width using the Pythagorean theorem.
$w^2 = 2^2 + 2.5^2$
$w = \sqrt{2^2 + 2.5^2}$
$w \approx 3.2$ ft
$A = 3.2 \times 20$
$A = 64 \text{ ft}^2$
Total area = 64 + 11 + 30
$A = 105 \text{ ft}^2$
The total area of fabric Seth needs is 105 ft^2.

8. Surface area = $\frac{1}{2}(4\pi r^2)$
$$SA = \frac{1}{2} \times 4\pi \left(\frac{7}{2}\right)^2$$
$SA \approx 77.0 \text{ ft}^2$
The surface area is 77.0 ft^2.

9. a)



Calculate the cross-sectional area and multiply by the width of the stairs.
Surface area:
$(10 \times 7.5) + (10 \times 15) + (10 \times 22.5) + (48 \times 30)$
$SA = 1890 \text{ in}^2$
Volume:
Volume = 1890×48
Volume = $90\,720 \text{ in}^3$
Volume = $\frac{\left(\frac{90\,720}{1728}\right)}{27}$

Volume ≈ 1.94 cubic yards
The volume of concrete needed is 1.94 cubic yards.
b) Surface area:
SA of ends + SA of treads + SA of risers + SA of landing
Total surface area = 2(1890) + 3(10 × 48) + 4(7.5 × 48) + 48 × 48
$SA = 8964\ \text{in}^2$
$SA = 62.25\ \text{ft}^2$
The surface area that needs to be covered is 62.25 ft^2.

10. a) Volume = $\frac{1}{3}$(area of base) × (height)
$V = \frac{1}{3}(230.56)^2 \times 138.75$
$V \approx 2\ 458\ 553\ \text{m}^3$
The volume of the stone used to build the pyramid is 2 458 553 m^3.
b) 2.56 × 2 458 553 ≈ 6 293 896
The weight of the pyramid is about 6 293 896 tonnes or 6.3 million metric tonnes.

Extension

You may wish to give students further practise in calculating the volume of complex objects using Blackline Master 6.10 (p. 410).

Extension Solutions

$V_{\text{rectangle}} = \text{length} \times \text{width} \times \text{height}$
$V_{\text{rectangle}} = 20 \times 13 \times 13$
$V_{\text{rectangle}} = 3380\ \text{m}^3$
$V_{\text{cylinder}} = \pi r^2 h$
$V_{\text{cylinder}} = \pi \times (6)^2 \times (7)$
$V_{\text{cylinder}} \approx 792\ \text{m}^3$
$V_{\text{rectangle}} = \text{length} \times \text{width} \times \text{height}$
$V_{\text{rectangle}} = 8 \times 8 \times 16$
$V_{\text{rectangle}} = 1024\ \text{m}^3$
$V_{\text{pyramid}} = \frac{1}{3}(\text{length of base} \times \text{width of base} \times \text{height})$
$V_{\text{pyramid}} = \frac{1}{3}(8 \times 8 \times 12)$
$V_{\text{pyramid}} = 256\ \text{m}^3$
Total volume = 3380 + 792 + 1024 + 256
Total volume = 5452 m^3