

Math 10 GMF

Monday Nov 20th

Compound Interest

Compound Interest \rightarrow interest that is paid on the principal plus interest. It is interest that is paid on what you put in, as well as any interest already made on the investment.

Simple Interest \rightarrow Interest is paid on the principal only.

From last week \rightarrow Simple interest
 $I = Prt$

Example: How much will your investment be worth in 10 years if you invest \$ 1000.00 today at 6.25% / a

$$\begin{aligned} I &= Prt \\ &= 1000.00(0.0625)(10) \\ &= \$625.00 \end{aligned}$$

↙ 6.25% / yr
↑ interest made

$$\begin{aligned} A &= P + I \\ &= \$1000.00 + 625.00 \end{aligned}$$

$$A = 1625.00$$

You will have \$1625.00 in 10 yrs.

For Compound Interest

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$P = \frac{A}{\left(1 + \frac{r}{n} \right)^{nt}}$$

$A \rightarrow$ Future Value

\rightarrow Amount that the investment is worth at the end of the term.

$P \rightarrow$ principal (start)

$r \rightarrow$ rate from bank as a decimal.

$n \rightarrow$ number of compounding periods per year.

$t \rightarrow$ term \rightarrow in years.

$n \rightarrow$ Number of compounding periods per year.

	n value
annually \longrightarrow	1
Semi-annually \longrightarrow	2
quarterly \longrightarrow	4
monthly \longrightarrow	12
weekly \longrightarrow	52
daily \longrightarrow	365.25

Example:

You invest \$ 1000.00 today at 10% / a
Compound semi-annually for 5 years.

- a) How much is your investment worth at this time?
- b) How much interest did you make?

$$\begin{aligned}
 A &= P \left(1 + \frac{r}{n} \right)^{nt} \\
 &= 1000 \left(1 + \frac{0.10}{2} \right)^{2(5)} \\
 &= 1000 (1 + 0.05)^{10} \\
 &= 1000 (1.05)^{10} \\
 &= 1000 (1.628894627)
 \end{aligned}$$

$$A = \$1628.89$$

your investment is worth $\$1628.89$ at this time.

$$(b) \underline{I} = A - P$$

$$= 1628.89 - 1000.00 = \$628.89$$

Example #2

You invest \$25 000.00 at 12% / a
Compounded quarterly for 10 years.

- a) What is value of investment at this time?
- b) How much interest did you make?

$$\begin{aligned} a) A &= P \left(1 + \frac{r}{n} \right)^{nt} \\ &= 25000 \left(1 + \frac{0.12}{4} \right)^{4(10)} \\ &= 25000 \left(1 + 0.03 \right)^{40} \\ &= 25000 \left(1.03 \right)^{40} \\ &= 25000 (3.262637792) \\ &= 81550.94 \end{aligned}$$

$$\begin{aligned} b) I &= A - P \\ &= 81550.94 - 25000.00 \\ \underline{I} &= 56550.94 \end{aligned}$$

Example: You would like to have
\$15,000.00 in 2.5 yrs to go to
School. How much should you invest
today at 10% / a compounded monthly
to have this amount?

$$\begin{aligned}
 P &= \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}} \\
 &= \frac{15000.00}{\left(1 + \frac{0.10}{12}\right)^{12(2.5)}} \\
 &= \frac{15000.00}{\left(1 + 0.008333\right)^{30}} \\
 &= \frac{15000.00}{(1.008333)^{30}} \\
 &= \frac{15000.00}{1.282694691}
 \end{aligned}$$

$$P = 11694.13$$

Invest \$11,694.13 so you will have 15,000.00 in 2.5 yrs.

How much should you invest today
at 12%/a compounded semi-annually
so that you will have \$20,000.00
in 5 years.

$$\begin{aligned}P &= \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}} \\&= \frac{20\,000.00}{\left(1 + \frac{0.12}{2}\right)^{2(5)}} \\&= \frac{20\,000.00}{(1.06)^{10}} \\&= \frac{20\,000.00}{1.796847697}\end{aligned}$$

$$P = \$11\,167.90$$

Invest \$11,167.90 today to have \$20,000.00 in 5 years.