Find S_{99} 82, 73, 64, 55, ... (a)

arithmetic

$S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{99}{2}[2(82)+(99-1)(-9)]$

$\frac{99}{2}[164+98(-9)]$

$= -35541$

Determine the sum of each series:

geometric

a. 7 + 21 + 63 +...+ 1240029

* find n first

* n must be a whole number

$t_n = ar^{n-1}$

$1240029 = 7(3)^{n-1}$

$\frac{1240029}{7} = \frac{\cancel{7}(3)^{n-1}}{\cancel{7}}$

$177147 = 3^{n-1}$

$\log_3 177147 = n-1$

$\frac{\log 177147}{\log 3} = n-1$

$11 = n-1$

$12 = n$

now

$S_{12} = \frac{a[r^n-1]}{r-1}$

$= \frac{7[3^{12}-1]}{[3-1]}$

$= 1860040$

b. 478+475+472+469+466+...-1928

arithmetic

use t_n to find n

$t_n = a+(n-1)d$

$-1928 = 478+(n-1)(-3)$

$-1928 = 478-3n+3$

$-1928 = 481-3n$

$-1928-481 = -3n$

$\frac{-2409}{-3} = \frac{-3n}{-3}$

$803 = n$

$S_{803} = \frac{n}{2}[2a+(n-1)d]$

$= \frac{803}{2}[2(478)+(803-1)(-3)]$

$\frac{803}{2}[956+(802)(-3)]$

$\frac{803}{2}[956-2406]$

$\frac{803}{2}(-1450) \Rightarrow -582175$

Given $t_{15} = -170$ and $t_8 = -86$, if the sequence is arithmetic find S_{175}

$t_n = a + (n-1)d$

$-170 = a + (15-1)d$ $\quad -86 = a + (8-1)d$

$-170 = a + 14d$ $\quad -86 = a + 7d$

① $-170 = a + 14d$

② $-86 = a + 7d$

①-② $-84 = 7d$

$-12 = d$

$-170 = a + 14(-12)$

$-170 = a - 168$

$-2 = a$

$S_{175} =$

$\frac{175}{2}[2(-2) + (175-1)(-12)]$

$\frac{175}{2}[-4 + (174)(-12)]$

$\Rightarrow -183050$

Given $t_{12} = 885735$ and $t_4 = 135$, if the sequence is geometric find S_{12}

$t_n = ar^{n-1}$

$885735 = ar^{12-1}$ $\quad 135 = ar^{4-1}$

$885735 = ar^{11}$ $\quad 135 = ar^{3}$

① $885735 = \cancel{a}r^{11}$

② $135 = \cancel{a}r^{3}$

$1 \div 2$

$6561 = r^8$

$(6561)^{1/8} = r$

$3 = r$

$135 = a(3)^3$

$135 = 27a$

$\frac{135}{27} = a$

$5 = a$

NOW

$S_{12} = \frac{a[r^n - 1]}{[r-1]}$

$= \frac{5[3^{12} - 1]}{[3-1]}$

$= 1328600$

Determine if the following converges or diverges, if it converges state to what value it converges (aka: $\lim_{n\to\infty}$)

$$t_n = \left(\frac{5n^2 - 3}{3n^2 + 2}\right)$$

= converges to $5/3$

$$t_n = \frac{5n^3 - 6n + 1}{12 - 4n - 3n^3}$$

$= \frac{5}{-3}$ converges

$$t_n = \left(\frac{3}{2}\right)^n$$

$\left(\frac{3}{2}\right)^\infty = \infty$

exponential fraction > 1 ∴ diverges

$$t_n = \frac{7n^5 + 2n^2}{3n^3 + 8}$$

$= 7/0$ ∴ diverges

Find the infinite sum:

$$1875+375+75+\ldots\ldots\ldots$$

geometric

$$r=\frac{t_2}{t_1}=\frac{375}{1875}=1/5$$

$$* \quad S_n=\frac{a[r^n-1]}{[r-1]}$$

$$S_\infty=\frac{1875[(\frac{1}{5})^\infty-1]}{[\frac{1}{5}-1]}$$

$$\frac{1875[0-1]}{[1/5-1]}$$

$$* \ \frac{1}{5}-\frac{5}{5}=-\frac{4}{5}$$

$$\frac{1875(-1)}{(-4/5)}$$

$$=2343.75$$

$$\sum_{i=1}^{160} -8i^3+12$$

cubic constant

$$-8\left[\frac{n(n+1)}{2}\right]^2+12n$$

$$-8\left[\frac{160(161)}{2}\right]^2+12(160)$$

$$\Rightarrow -1327153280$$

$$\sum_{i=50}^{200} 6i^2-9i+2$$

$\Rightarrow$

$$\sum_{i=1}^{200} 6i^2-9i+2$$

$$6\frac{n(n+1)(2n+1)}{6}-9\frac{n(n+1)}{2}+2n$$

$$6\frac{(200)(201)(401)}{6}-\frac{9(200)(201)}{2}+2(200)$$

$$15939700$$

$$\sum_{i=1}^{49} 6i^2-9i+2$$

$$6\frac{n(n+1)(2n+1)}{6}-9\frac{n(n+1)}{2}+2n$$

$$231623$$

$$15939700-231623$$
$$=15708077$$

Solve: $\dfrac{20}{3x^2-11x-4} + \dfrac{8}{x-4} = -4$

Factor:
$3x^2-11x-4$
$3x^2-12x+x-4$
$3x(x-4)+1(x-4)$
$(3x+1)(x-4)$

$\cancel{(3x+1)(x-4)}\dfrac{20}{\cancel{(3x+1)(x-4)}} + \dfrac{8}{x-4}(3x+1)(x-4) = -4(3x+1)(x-4)$

$20+8(3x+1)=-4(3x+1)(x-4)$

$20+24x+8=-4(3x^2-11x-4)$

$24x+28=-12x^2+44x+16$

$12x^2-20x+12=0$

Quad formula

$\dfrac{20\pm\sqrt{-176}}{24}$ NO SOL'N

Restrictions: $x\neq -\frac{1}{3}, 4$

Factor: $2x^4+9x^3-37x-30$

$P(-1)=0$

$$\begin{array}{r} 2x^3+7x^2-7x-30 \\ x+1\,\overline{)\,2x^4+9x^3+0x^2-37x-30} \\ \underline{2x^4+2x^3} \\ 7x^3+0x^2 \\ \underline{7x^3+7x^2} \\ -7x^2-37x \\ \underline{-7x^2-7x} \\ -30x-30 \\ \underline{-30x-30} \\ 0 \end{array}$$

$$\begin{array}{r} 2x^2+11x+15 \\ x-2\,\overline{)\,2x^3+7x^2-7x-30} \\ \underline{2x^3-4x^2} \\ 11x^2-7x \\ \underline{11x^2-22x} \\ 15x-30 \\ 15x-30 \end{array}$$

* Synthetic

Pt 1

$$\begin{array}{r|rrrrr} +1 & 2 & 9 & 0 & -37 & -30 \\ & \downarrow & 2 & 7 & -7 & -30 \\ \hline & 2 & 7 & -7 & -30 & \end{array}$$

$(x+1)(x-2)(2x^2+11x+15)$

$2x^2+6x+5x+15$

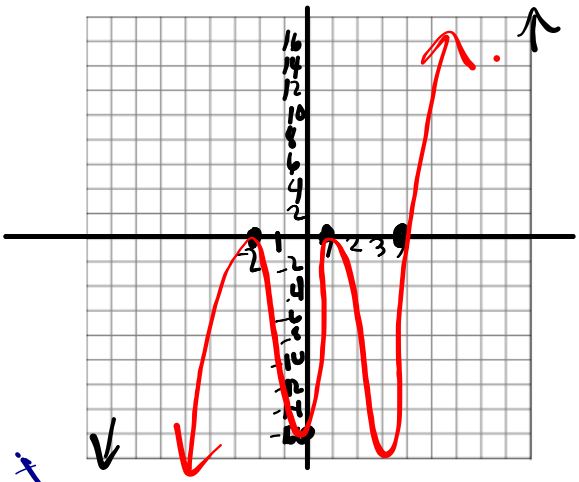
$2x(x+3)+5(x+3)$

$(x+3)(2x+5)$

$\boxed{\text{Ans: } (x+1)(x-2)(x+3)(2x+5)}$

Given: $y = (x+2)^2(x-1)^2(x-4)$

A. Degree → 5
B. Turning Points → 4
C. Intervals where positive and negative
D. End Behaviour
E. x and y intercepts
F. Clearly sketch



$y = (x+2)(x+2)(x-1)(x-1)(x-4)$

$y = x^5$

Lead Coeff is 1 positive

Degree is 5 odd

End Behav: ↓ ↑

E. x-int: -2, 1, 4
just touches, just touches, through

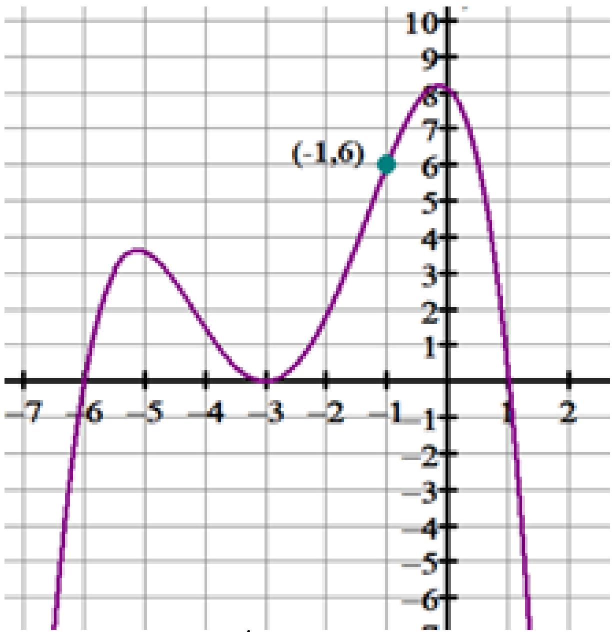
y-int (let x=0) $y = (0+2)^2(0-1)^2(0-4)$
$(4)(1)(-4) = -16$

Do C last:
negative $(-\infty, -2)$
$(-2, 1)$ $(1, 4)$

positive $(4, \infty)$

Get from graph

State the equation of the polynomial



$y = a(x+6)(x+3)^2(x-1)$

put point (-1,6) in and solve for a

$6 = a(-1+6)(-1+3)^2(-1-1)$

$6 = a(5)(2)^2(-2)$

$6 = a(5)(4)(-2)$

$6 = -40a$

$\frac{6}{-40} = a$

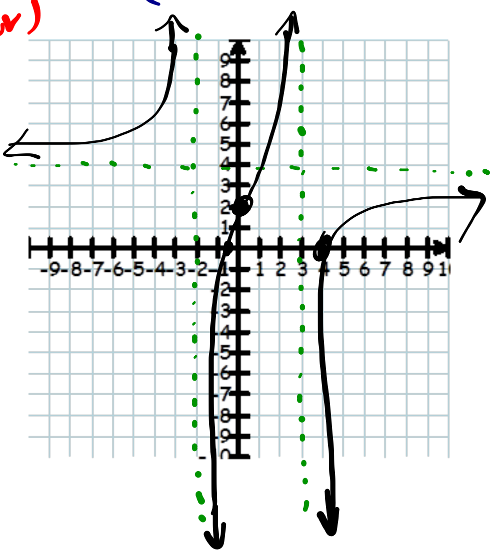
$a = \frac{-3}{20}$ $\Rightarrow$ EQ'N $y = \frac{-3}{20}(x+6)(x+3)^2(x-1)$

Sketch: state intercepts, asymptotes, Point of Discontinuity if they exist

$$f(x)=\frac{4x^2-13x-12}{x^2-x-6}\ ; \quad f(x)=\frac{(x-4)(4x+3)}{(x-3)(x+2)}$$

$4x^2-16x+3x-12$
$4x(x-4)+3(x-4)$
$(x-4)(4x+3)$

x-int: 4, $-3/4$ ⇐ (zeros of numerator)
y-int: 2 ⇐ (let $x=0$)
VA: $x=3$, $x=-2$ ⇐ zeros of denom.
HA: $y=4$ ⇐ (highest power)
POD: ——
Slant Asy: ——



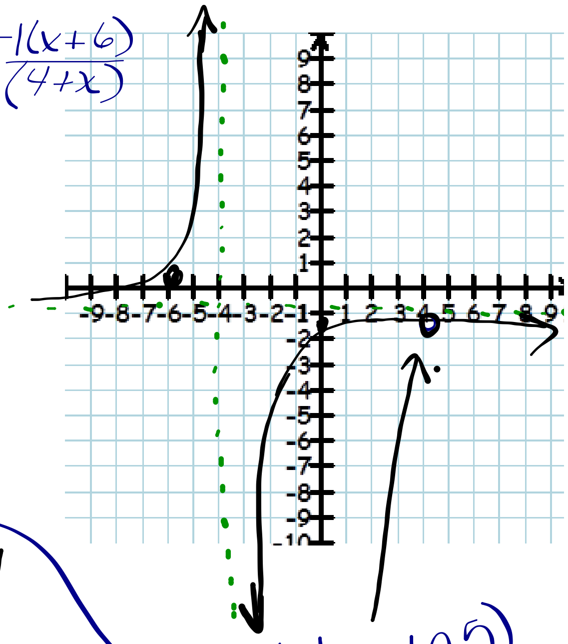
Behaviour at VA

$x=-2$: -2.1 | -1.9; $-\ -$ | $-\ -$; $-\ -$ | $-\ +$; $(+\infty)$ $(-\infty)$

$x=3$: 2.9 | 3.1; $-\ +$ | $-\ +$; $-\ +$ | $+\ +$; $(+\infty)$ $(-\infty)$

Sketch: state intercepts, asymptotes, Point of Discontinuity if they exist

$$f(x)=\frac{x^2+2x-24}{16-x^2}\;;\; f(x)=\frac{(x+6)\cancel{(x-4)}}{\cancel{(4-x)}(4+x)}\;,\; f(x)=\frac{-1(x+6)}{(4+x)}$$



x-int: -6

y-int: $-24/16 = -1.5$

VA: $x=-4$

HA: $y=\frac{1}{-1}=-1$

POD: $x=4$ Plug $x=4$

$y=\frac{-1(4+6)}{4+4}=\frac{-10}{8}$

$(4, -10/8)$

slant —

Bch. at VA: $x=-4$

$y=\frac{-(x+6)}{(4+x)}$

-4.1	-3.9
$\frac{-+}{-}$	$\frac{-+}{+}$
$+\infty$	$-\infty$

POD $(4, -1.25)$

Sketch: state intercepts, asymptotes, Point of Discontinuity if they exist

$$f(x) = \frac{x^2 - 5x + 4}{x - 2}; \quad f(x) = x - 3 + \frac{-2}{x-2}$$

$$f(x) = \frac{(x-4)(x-1)}{(x-2)}$$

$$\begin{array}{r} x - 3 \\ x-2 \overline{)\, x^2 - 5x + 4} \\ \underline{x^2 - 2x} \\ -3x + 4 \\ \underline{-3x + 6} \\ -2 \end{array}$$

x-int 4, 1

y-int $4/_{-2} = -2$

VA: $x = 2$

HA: $y = 1/0$ ∴ NO HA

POD —

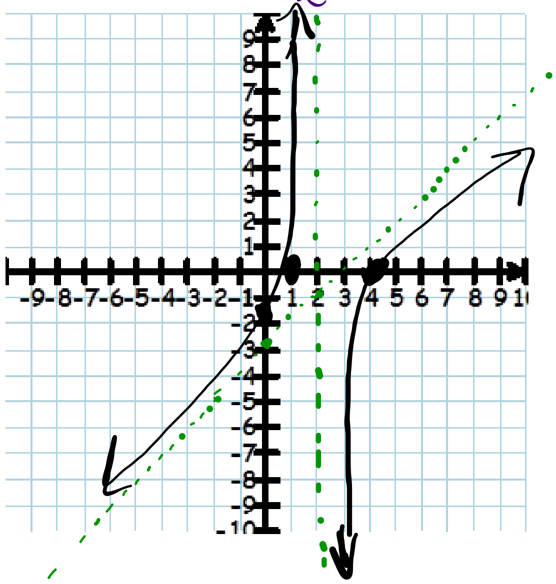
Slant $y = x - 3$

Beh. at $x=2$

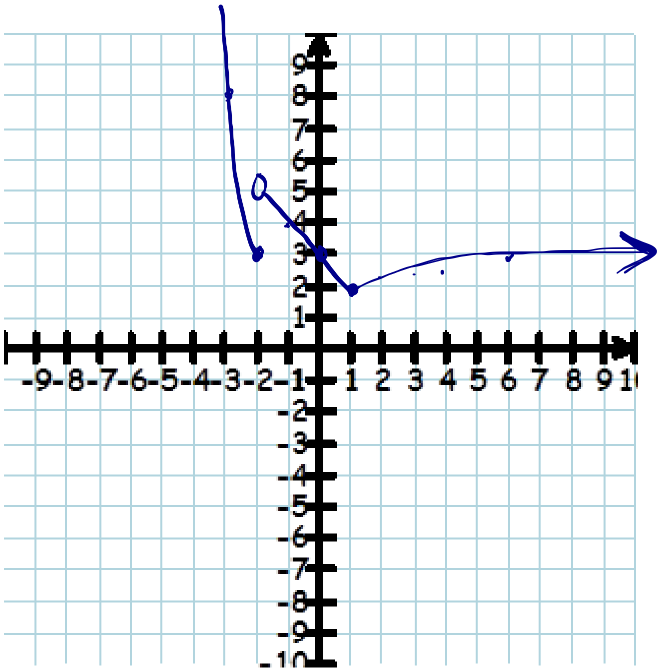
1.9	2.1
$\frac{-+}{-}$	$\frac{-+}{+}$
$+\infty$	$-\infty$

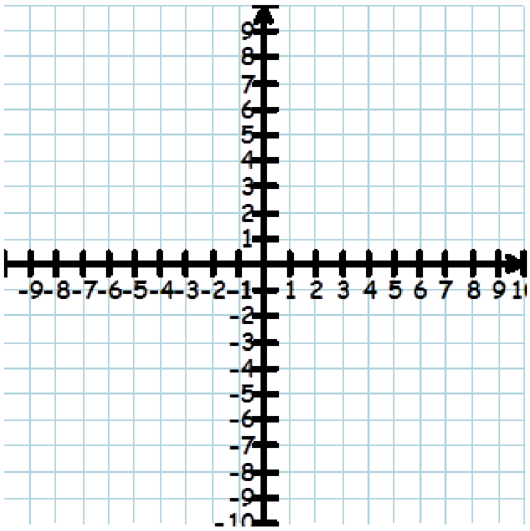
* slant $y = x - 3$

x	y
-2	-5
0	-3
2	-1



$$g(x)=\begin{cases} x^2-1, & x\le -2 \\ 3-x, & -2<x<1 \\ \sqrt{x+3}, & x\ge 1 \end{cases}$$





a. $\lim_{x \to -3^-} f(x)$ 3

b. $\lim_{x \to -3^+} f(x)$ −3

c. $\lim_{x \to -3} f(x)$ DNE

d. $\lim_{x \to 2^+} f(x)$ −3

e. $\lim_{x \to 2^-} f(x) = 1$

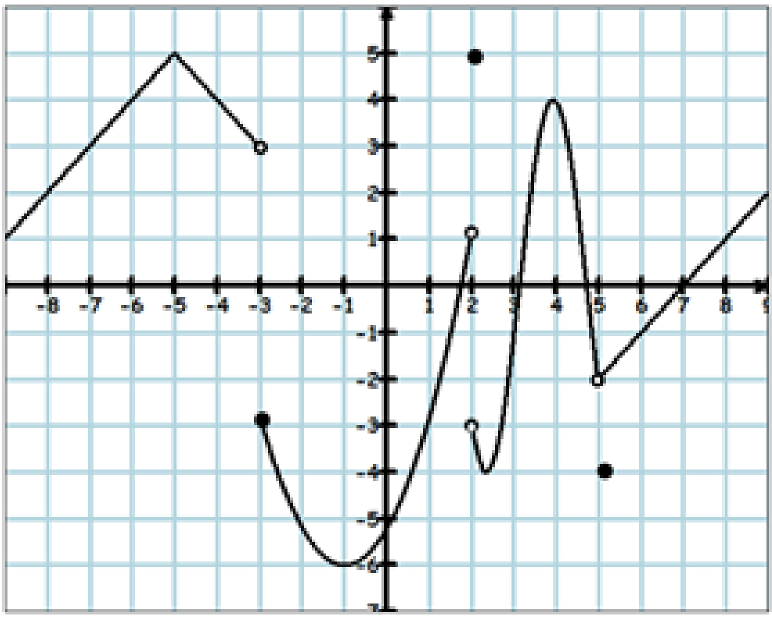
f. $\lim_{x \to 2} f(x)$ DNE

g. $\lim_{x \to 5} f(x)$ −2

h. $f(-3) = -3$

i. $f(2) = 5$

j. $f(5) = -4$



$\lim_{x \to 5^-} f(x) = -2$ $\lim_{x \to 5^+} f(x) = -2$ $\lim_{x \to 5} f(x) = -2$

$$\lim_{x\to\infty}\frac{2x^3-4x}{7x^3+5x^2-2x}$$

$= 2/7$

limits:
$\infty\}$ ÷ by highest power.

$$\lim_{x\to 3}\frac{27-x^3}{3-x}$$

$$\lim_{x\to 3}\frac{\cancel{(3-x)}(9+3x+x^2)}{\cancel{(3-x)}}$$

sub $x=3$ $9+9+9$
$= 27$

evaluate:
factor
rationalize
complex fractions

$$\lim_{x\to 10}\frac{\sqrt{4x+9}-7}{100-x^2}\times\frac{\sqrt{4x+9}+7}{\sqrt{4x+9}+7}$$

$$\lim_{x\to 10}\frac{(4x+9)-49}{(100-x^2)(\sqrt{4x+9}+7)}$$

$$\lim_{x\to 10}\frac{4x+9-49}{(100-x^2)(\sqrt{4x+9}+7)}$$

$$\lim_{x\to 10}\frac{4x-40}{(100-x^2)(\sqrt{4x+9}+7)}$$

$$\lim_{x\to 10}\frac{4\overset{-1}{\cancel{(x-10)}}}{\cancel{(10-x)}(10+x)(\sqrt{4x+9}+7)}$$

$$\lim_{x\to 10}\frac{-4}{(10+x)(\sqrt{4x+9}+7)}$$

$$\frac{-4}{(20)(\sqrt{49}+7)}$$

$$\frac{-4}{20(7+7)}$$

$$\frac{-4}{20(14)}$$

$$\frac{-4}{280}$$

$$\lim_{x\to 12}\frac{\frac{6}{x}-\frac{1}{2}}{x^2-144}$$

$$\lim_{x\to 12}\frac{12-x}{2x(x^2-144)}$$

$$\lim_{x\to 12}\frac{\cancel{12-x}^{\,-1}}{2x\cancel{(x-12)}(x+12)}$$

$$\lim_{x\to 12}\frac{-1}{2x(x+12)}$$

$$\frac{-1}{24(24)}$$

$$\frac{-1}{576}$$

Definition of the derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = 5x^2 + 11x - 9$$

$$f(x+h) = 5(x+h)^2 + 11(x+h) - 9$$

$$\lim_{h \to 0} \frac{5(x+h)^2 + 11(x+h) - 9 - (5x^2 - 11x - 9)}{h}$$

$$\lim_{h \to 0} \frac{5(x^2 + 2xh + h^2) - 11x + 11h - 9 - 5x^2 + 11x + 9}{h}$$

$$\lim_{h \to 0} \frac{\cancel{5x^2} + 10xh + 5h^2 - \cancel{11x} + 11h - \cancel{9} - \cancel{5x^2} + \cancel{11x} + \cancel{9}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(10x + 5h + 11)}{\cancel{h}}$$

sub $h = 0$ $\quad 10x + 5(0) + 11$

$$\boxed{10x + 11} = f'(x)$$