

$$\begin{array}{l}
 \text{1. } \frac{1}{3} = a + (2-1)d \quad \frac{13}{3} = a + (8-1)d \\
 \frac{1}{3} = a + d \quad \frac{13}{3} = a + 7d \\
 \\
 \text{(2) } \frac{13}{3} = a + 7d \\
 \text{(1) } \frac{1}{3} = a + d \\
 \hline
 \text{(2) - (1) } \frac{12}{3} = 6d \\
 4 = 6d \\
 \frac{4}{6} = d \\
 \boxed{d = \frac{2}{3}} \\
 \\
 \left. \begin{array}{l} \text{(2) } \frac{13}{3} = a + 7d \\ \text{(1) } \frac{1}{3} = a + d \end{array} \right\} \rightarrow \begin{array}{l} \frac{1}{3} = a + \frac{2}{3} \\ \frac{1}{3} - \frac{2}{3} = a \\ \boxed{a = -\frac{1}{3}} \end{array}
 \end{array}$$

$$\begin{array}{l}
 \text{(b) } t_n = -\frac{1}{3} + (n-1)\frac{2}{3} \\
 = -\frac{1}{3} + \frac{2}{3}n - \frac{2}{3} \\
 t_n = \frac{2}{3}n - \frac{3}{3} \\
 t_n = \frac{2}{3}n - 1
 \end{array}$$

$$\begin{array}{l}
 \text{(c) } t_{12} = \frac{2}{3}(12) - 1 \\
 = 8 - 1 \\
 = 7
 \end{array}$$

$$\begin{array}{l}
 t_{26} = \frac{2}{3}(26) - 1 \\
 = \frac{52}{3} - \frac{3}{3} \\
 = \frac{49}{3}
 \end{array}$$

$$\begin{array}{l}
 \text{(d) } 9 = \frac{2}{3}n - 1 \\
 10 = \frac{2}{3}n \\
 30 = 2n \\
 15 = n
 \end{array}$$

$$\begin{array}{l}
 \frac{97}{3} = \frac{2}{3}n - 1 \\
 \frac{97}{3} + 1 = \frac{2}{3}n \\
 \frac{97+3}{3} = \frac{2}{3}n \\
 \frac{100}{3} = \frac{2}{3}n \\
 100 = 2n \\
 50 = n
 \end{array}$$

$$\begin{array}{l}
 \text{(e) } S_{200} = \frac{200}{2} \left[2\left(-\frac{1}{3}\right) + (200-1)\frac{2}{3} \right] \\
 = 100 \left[-\frac{2}{3} + 199\left(\frac{2}{3}\right) \right] \\
 = 100 \left[-\frac{2}{3} + \frac{398}{3} \right] \\
 = 100 \left[\frac{396}{3} \right] \\
 = 13200
 \end{array}$$

2. $162 = ar^{5-1}$ $13122 = ar^{9-1}$ (b) $t_n = 2(3)^{n-1}$ (d) $9565938 = 2(3)^{n-1}$
 (a) $162 = ar^4$ $13122 = ar^8$ (c) $t_3 = 2(3)^{3-1}$ $4782969 = 3^{n-1}$

$$\frac{13122}{162} = \frac{ar^8}{ar^4}$$

$$81 = r^4$$

$$3 = r$$

$$162 = a(3)^4$$

$$162 = 81a$$

$$2 = a$$

$$t_{10} = 2(3)^{10-1}$$

$$= 2(3)^9$$

$$= 39366$$

$$\log_3 4782969 = n-1$$

$$14 = n-1$$

$$15 = n$$

$$1458 = 2(3)^{n-1}$$

$$729 = 3^{n-1}$$

$$\log_3 729 = n-1$$

$$6 = n-1$$

$$7 = n$$
 (e) $S_{10} = \frac{2[3^{10}-1]}{3-1}$

$$= 59048$$

****n** has to be a whole number**

3. $t_n = 2 - n + 3n^2$
 $t_1 = 2 - 1 + 3(1)^2 = 4$
 $t_2 = 12$
 $t_3 = 26$
 $t_4 = 46$
 $t_5 = 72$

4. Need n

(a) geometric
 $t_n = ar^{n-1}$
 $3 = 12582912 \left(\frac{1}{2}\right)^{n-1}$
 $\frac{3}{12582912} = \left(\frac{1}{2}\right)^{n-1}$
 $\log_{\frac{1}{2}} \left(\frac{3}{12582912}\right) = n-1$
 $22 = n-1$
 $n = 23$
 $S_{23} = \frac{12582912 \left[\left(\frac{1}{2}\right)^{23} - 1\right]}{\left(\frac{1}{2} - 1\right)}$
 $= 25165821$

(b) arithmetic
 $12030 = 24 + (n-1)6$
 $12030 = 24 + 6n - 6$
 $12030 = 18 + 6n$
 $12012 = 6n$
 $2002 = n$
 $S_{2002} = \frac{2002}{2} [2(24) + (2002-1)6]$
 $= 12066054$

5(a) $t_{20} = -231$; $t_{35} = -411$

$$\begin{aligned} -231 &= a + (20-1)d & -411 &= a + (35-1)d \\ -231 &= a + 19d & -411 &= a + 34d \end{aligned}$$

$$\begin{aligned} \textcircled{1} \quad -411 &= a + 34d \\ \textcircled{2} \quad -231 &= a + 19d \end{aligned}$$

$$\textcircled{1} - \textcircled{2} \quad \begin{array}{r} -180 = 15d \\ -12 = d \end{array}$$

$$\begin{aligned} -231 &= a + 19(-12) \\ -3 &= a \end{aligned}$$

$$S_{30} = \frac{30}{2} [2(-3) + (30-1)(-12)] = -539100$$

5(b) $1835008 = ar^{10-1}$ $112 = ar^{3-1}$
 $1835008 = ar^9$ $112 = ar^2$

$$\begin{aligned} \textcircled{1} \quad 1835008 &= ar^9 \\ \textcircled{2} \quad 112 &= ar^2 \end{aligned}$$

$$\textcircled{1} \div \textcircled{2} \quad \frac{16384}{112} = r^7$$

$$\sqrt[7]{16384} = r$$

$$4 = r$$

$$112 = a(4)^2$$

$$7 = a$$

$$S_{14} = \frac{7[4^{14} - 1]}{4 - 1} = 626349395$$

5(c) $-170 = a + (15-1)d$ $-86 = a + (9-1)d$
 $-170 = a + 14d$ $-86 = a + 7d$

$$\begin{aligned} \textcircled{1} \quad -170 &= a + 14d \\ \textcircled{2} \quad -86 &= a + 7d \end{aligned}$$

$$\textcircled{1} - \textcircled{2} \quad \begin{array}{r} -84 = 7d \\ -12 = d \end{array}$$

$$\begin{aligned} -86 &= a + 7(-12) \\ -86 &= a - 84 \\ -2 &= a \end{aligned}$$

$$S_{20} = \frac{20}{2} [2(-2) + (20-1)(-12)] = -239200$$

5(d) $885735 = ar^{12-1}$ $135 = ar^{4-1}$
 $885735 = ar^{11}$ $135 = ar^3$

$$\begin{aligned} \textcircled{1} \quad 885735 &= ar^{11} \\ \textcircled{2} \quad 135 &= ar^3 \end{aligned}$$

$$\textcircled{1} \div \textcircled{2} \quad \frac{6561}{135} = r^8$$

$$\sqrt[8]{6561} = r$$

$$3 = r$$

$$135 = a(3)^3$$

$$5 = a$$

$$S_4 = \frac{5[3^4 - 1]}{3 - 1} = 11957420$$

6(a) $5 + 15 + 45 + 135 + \dots$ keeps getting larger (∞) therefore diverges

6(b) $50 + 25 + 12.5 + \dots$

$$S_n = \frac{50 \left[\left(\frac{1}{2} \right)^n - 1 \right]}{\frac{1}{2} - 1} \rightarrow \frac{50[0 - 1]}{\frac{1}{2} - 1}$$

$$\lim_{n \rightarrow \infty} \frac{50 \left[\left(\frac{1}{2} \right)^n - 1 \right]}{\frac{1}{2} - 1} = 100$$

7.

(a) $V = 5000(1.025)^t$

(b) 5125, 5253.13, 5384.45, ...

(c) $V = 5000(1.025)^8$
 $= 6092.01$

(d) $11866 = 5000(1.025)^t$
 $2.3732 = 1.025^t$
 $\log_{1.025} 2.3732 = t$
 $t = 34.9999$
 35 yrs

8. $S_n = \frac{a[r^n - 1]}{r - 1}$

$-3280 = \frac{a[(3)^8 - 1]}{3 - 1}$

$-3280 = \frac{a[6560]}{-4}$

$2 = a$

$r = -3$

$a = t_1 = 2$

9. (a) conv to 0

(b) conv to $5/3$

(c) conv to 0

(d) diverge

10. (a) $-3/4$

(b) expand lim first $2 \rightarrow \infty$ $\frac{14x^2 + 29x - 15}{10x^2 - 6x - 4}$
 $= \frac{14}{10} = \frac{7}{5}$

(c) $\frac{0}{10} = 0$

(d) $10(0) = 0$

(e) ∞ OR DNE

(f) 0

11. (a) $\sum_{k=1}^{90} 4k^3$
 $4 \left[\frac{n(n+1)}{2} \right]^2$
 $4 \left(\frac{90(91)}{2} \right)^2$
 $= 67076100$

(b) $\sum_{k=1}^{999} 17$
 $= 17n$
 $= 17(999)$
 16983

(c) $\sum_{k=1}^{75} 8k^2 - 15k + 2$
 $8 \left[\frac{n(n+1)(2n+1)}{6} \right] - 15 \left[\frac{n(n+1)}{2} \right] + 2n$
 $8 \left[\frac{75(76)(151)}{6} \right] - 15 \left(\frac{75(76)}{2} \right) + 2(75)$
 $= 1105000$

(d) $\sum_{k=50}^{150} (k^3 - 3)$
 $\sum_{k=1}^{150} (k^3 - 3) - \sum_{k=1}^{49} (k^3 - 3)$
 $\left(\left[\frac{n(n+1)}{2} \right]^2 - 3n \right) - \left(\left[\frac{n(n+1)}{2} \right]^2 - 3n \right)$
 $\left[\frac{150(151)}{2} \right]^2 - 3(150) - \left[\left(\frac{49(50)}{2} \right)^2 - 3(49) \right]$
 $128255175 - 1500478$
 126754697

$\sum_{k=75}^{100} (2k^2 - 10)$

↑ Quad ↑ const

$\left(3 \left[\frac{n(n+1)(2n+1)}{6} \right] - 10(n) \right) - \left(3 \left[\frac{n(n+1)(2n+1)}{6} \right] - 10(n) \right)$
 $\left(3 \left[\frac{100(101)(201)}{6} \right] - 10(100) \right) - \left(3 \left[\frac{74(75)(149)}{6} \right] - 10(74) \right)$

$\sum_{k=1}^{10} k^2$ $36 + 49 + 64 + 81 + 100$

$\sum_{k=1}^{10} (1 + 4 + 9 + 25) + 36 + 49 + 64 + 81 + 100$

Unit 2.

1. 21 sandwiches

$$21 \times 3 \times 8 \times 7 = 3528$$

2. PERMUTATION

$$\frac{11!}{2!}$$

$$= 19958400$$

more
3unEi is
correct

COMBINATION

$$\frac{11!}{2!2!2!}$$

$$= 4989600$$

3. EEEEEENNNNN

$$\frac{12!}{6!6!} = 924$$

4. 5 burnt out letters

9 Good letters

3 good and 2 burnt out

$${}^9C_3 \times {}^5C_2$$

$$= 84 \times 10$$

$$= 840$$

5. CHARLOTTETOWN

(a) $\frac{13!}{2!3!}$

$$= 518918400$$

(b) CHARLOTTETOWN

$$\frac{12!}{2!3!}$$

$$= 39916800$$

(c) 518918400
 $- 39916800$

 479001600

(d) start end

L
CAARLOTTETOWN

$$\frac{11!}{2!2!}$$

$$= 9979200$$

6. letter, letter, num, num, num, num

(a) $26 \times 26 \times 10 \times 10 \times 10 \times 10 = 6760000$

(b) $26 \times 25 \times 10 \times 9 \times 8 \times 7 = 3276000$

7. (a) ${}_{12}P_3 = 1320$ (e) ${}_{14}C_9 = 2002$

(b) $\frac{10!}{10} = 362880$ (f) ${}_{10}P_2 = 90$

(c) ${}_{15}C_7 = 6435$ (g) ${}_{50}P_3 = 117600$

(d) ${}_{15}C_{12} = 455$

8. (a) 1 10 45 120 210 252 210 120 45 10 1

(b) $(4a+5b)^4$: ${}_{4}C_0(4a)^4(5b)^0 = 1(256a^4)(1) = 256a^4$
 ${}_{4}C_1(4a)^3(5b)^1 = 4(64a^3)5b = 1280a^3b$
 ${}_{4}C_2(4a)^2(5b)^2 = 6(16a^2)25b^2 = 2400a^2b^2$
 ${}_{4}C_3(4a)^1(5b)^3 = 4(4a)125b^3 = 2000ab^3$
 ${}_{4}C_4(4a)^0(5b)^4 = 1(1)625b^4 = 625b^4$

$256a^4 + 1280a^3b + 2400a^2b^2 + 2000ab^3 + 625b^4$

(c) $(\frac{1}{2}a - 8b)^9$ 4th term is ${}_{9}C_3(\frac{1}{2}a)^3(-8b)^6$
 $= 84(\frac{1}{8}a^3)262144b^6$
 $2752512a^3b^6$

$$\begin{array}{cccccccc}
 & & & & & & & 1 \\
 & & & & & & & 1 \\
 & & & & & & 1 & 2 & 1 \\
 & & & & & 1 & 3 & 3 & 1 \\
 & & & 1 & 4 & 6 & 4 & 1 & & \\
 & & 1 & 5 & 10 & 10 & 5 & 1 & & \\
 & 1 & 6 & 15 & 20 & 15 & 6 & 1 & &
 \end{array}$$

what is the fifth term in the expansion

of: $(\frac{1}{9}x + 27)^7$

${}_{7}C_0$
 ${}_{7}C_1$
 ${}_{7}C_2$
 ${}_{7}C_3$
 ${}_{7}C_4(\frac{1}{9}x)^3(27)^4$
 $35(\frac{1}{729}x^3)(531441)$
 $25515x^3$

$$9. a) 15 \times 15 \times 15 \times 15 \times 15 = 759375$$

$$b) 759375 \text{ combinations} \times \frac{12 \text{ sec}}{\text{each}} = 9112500 \text{ seconds}$$

$$151875 \text{ min,}$$

$$2531.25 \text{ hours}$$

$$10. (a) {}_{30}C_6 = 593775$$

$$(b) 3 \text{ females and } 3 \text{ males}$$

$${}_{17}C_3 \times {}_{13}C_3$$

$$= 680 \times 286$$

$$= 194480$$

$$(c) \text{ Ajay, } 2 \text{ males and } 3 \text{ females}$$

$${}_{12}C_2 \times {}_{17}C_3$$

$$= 66 \times 680$$

$$= 44880$$

$$11. 3 \text{ of } 6 \text{ questions and two of three}$$

$${}_{6}C_3 \times {}_{3}C_2$$

$$= 20 \times 3$$

$$= 60$$

12. C, Jim, Jer, Lucia, Lisa, Mel, Zyo

$$(a) \text{ —, Lucia, —, —, —, —, —}$$

$$6! = 720$$

$$b) \text{ Lisa, —, —, —, —, Zyo, —}$$

$$5! = 120$$

$$(c) \text{ —, —, —, —, —, —, —}$$

(3 boys, 4 girls)

All Boys followed by all girls

$$3! \times 4! = 144$$

1. $P(2) = 2^4 - 3(2)^3 - 5(2)^2 + 7(2) + 14$
 $= 0 \therefore x-2$ is a factor.

2. (a) $x^4 - x^3 - 2x^2$
 $x^2(x^2 - x - 2)$
 $x^2(x-2)(x+1)$

b) $x^3 + 5x^2 - 12$
 $P(-2) = 0$
 $\therefore x+2$ is a factor

$(x+2)(x^2 + 3x - 6)$

$$\begin{array}{r} x^2 + 3x - 6 \\ x+2 \overline{) x^3 + 5x^2 + 0x - 12} \\ \underline{x^3 + 2x^2} \\ 3x^2 + 0x \\ \underline{3x^2 + 6x} \\ -6x - 12 \\ \underline{-6x - 12} \\ 0 \end{array}$$

(c) $15x^4 - 22x^2 + 8$
 $15x^4 - 10x^2 - 12x^2 + 8$
 $5x^2(3x^2 - 2) - 4(3x^2 - 2)$
 $(5x^2 - 4)(3x^2 - 2)$

Treat like a quadratic: add -22
 mult 120
 -10 & -12

(d) $6x^4 + 13x^3 - 8x^2 - 17x + 6$
 $P(1) = 6 + 13 - 8 - 17 + 6 = 0$

$$\begin{array}{r} 6x^3 + 19x^2 + 11x - 6 \\ x-1 \overline{) 6x^4 + 13x^3 - 8x^2 - 17x + 6} \\ \underline{6x^4 - 6x^3} \\ 19x^3 - 8x^2 \\ \underline{19x^3 - 19x^2} \\ 11x^2 - 17x \\ \underline{11x^2 - 11x} \\ -6x + 6 \\ \underline{-6x + 6} \\ 0 \end{array}$$

$(x-1)(6x^3 + 19x^2 + 11x - 6)$

$$\begin{array}{r} 6x^2 + 7x - 3 \\ x+2 \overline{) 6x^3 + 19x^2 + 11x - 6} \\ \underline{6x^3 + 12x^2} \\ 7x^2 + 11x \\ \underline{7x^2 + 14x} \\ -3x - 6 \\ \underline{-3x - 6} \\ 0 \end{array}$$

$(x-1)(x+2)(6x^2 + 7x - 3)$
 $(x-1)(x+2)(2x+3)(3x-1)$

Hard Trinomial
 $6x^2 + 7x - 3$
 $6x^2 + 9x - 2x - 3$
 $3x(2x+3) - 1(2x+3)$
 $(2x+3)(3x-1)$

$$f(x) = x^3 - 17x^2 + 80x - 100$$

$$P(2) = 0$$

$$\begin{array}{r} x^2 - 15x + 50 \\ x-2 \overline{) x^3 - 17x^2 + 80x - 100} \\ \underline{x^3 - 2x^2} \\ -15x^2 + 80x \\ \underline{-15x^2 + 30x} \\ 50x - 100 \\ \underline{50x - 100} \\ 0 \end{array}$$

$$(x-2)(x^2 - 15x + 50)$$

$$(x-2)(x-5)(x-10)$$

$$3. f(x) = 2x^4 + 3x^3 - x^2 - 3x - 1$$

$$P(1) = 0$$

$$\begin{array}{r} 2x^3 + 5x^2 + 4x + 1 \\ x-1 \overline{) 2x^4 + 3x^3 - x^2 - 3x - 1} \\ \underline{2x^4 - 2x^3} \\ 5x^3 - x^2 \\ \underline{5x^3 + 5x^2} \\ -6x^2 - 3x \\ \underline{-6x^2 + 4x} \\ x - 1 \\ \underline{x - 1} \\ 0 \end{array}$$

$$(x-1)(2x^3 + 5x^2 + 4x + 1)$$

$$P(-1) = 0$$

$$\begin{array}{r} 2x^2 + 3x + 1 \\ x+1 \overline{) 2x^3 + 5x^2 + 4x + 1} \\ \underline{2x^3 + 2x^2} \\ 3x^2 + 4x \\ \underline{3x^2 + 3x} \\ x + 1 \\ \underline{x + 1} \\ 0 \end{array}$$

$$(x-1)(x+1)(2x^2 + 3x + 1)$$

$$(x-1)(x+1)(2x+1)(x+1)$$

x-int: 1, -1, -1/2

$$4. y = (x+4)^2(x-1)^2(x+1)$$

- degree = 5 (odd)
- turning pts = 4
- end behaviour (lead coeff = +)
degree = 5

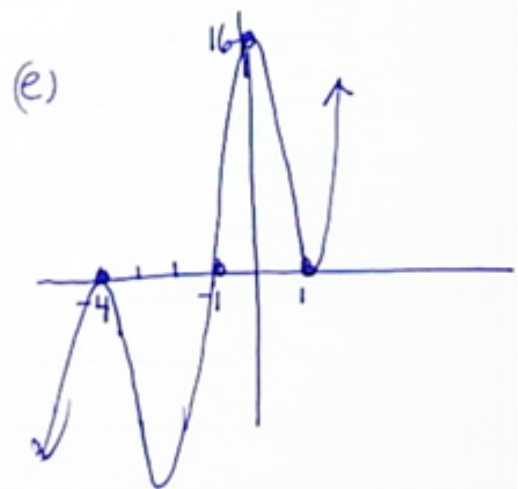
$$\begin{array}{ll} x \rightarrow -\infty & y \rightarrow -\infty \\ x \rightarrow +\infty & y \rightarrow +\infty \end{array}$$

zeros:
-4, 1, -1

(d)	$(x+4)^2(x-1)^2(x+1)$	-4	
$(-\infty, -4)$	+	-	- neg
$(-4, -1)$	+	-	- neg
$(-1, 1)$	+	+	+ pos
$(1, \infty)$	+	+	+ pos

OR

positive
 $(-1, 1)$ and $(1, \infty)$
 neg
 $(-\infty, -4)$ and $(-4, -1)$



$$5. \quad y = -x^4 + 2x^3 + 3x^2 - 4x - 4$$

$$y = -1(x^4 - 2x^3 - 3x^2 - 4x - 4) \quad * \text{factor theorem}$$

$$y = -1(x+1)^2(x-2)^2$$

(a) degree 4

(b) turning points = 3

(c) End behaviour lead coeff = neg
deg = +

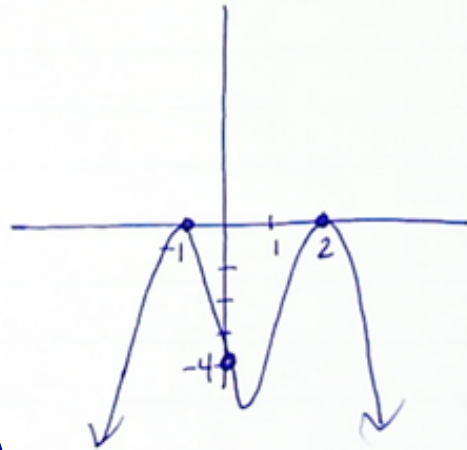
$$x \rightarrow -\infty \quad x \rightarrow \infty$$

$$y \rightarrow -\infty \quad y \rightarrow -\infty$$

zeros
-1, 2

(d)

	$-1(x+1)^2(x-2)^2$			y
$(-\infty, -1)$	-	+	+	- neg
$(-1, 2)$	-	+	+	- neg
$(2, \infty)$	-	+	+	- neg

 x -int -1 and 2 y -int -4or neg $(-\infty, -1)$
 $(-1, 2)$
 $(2, \infty)$ 

6. (a) Intercepts: -3, 1, 7

$$y = a(x+3)(x-1)(x-7)$$

pt
(2,30)

$$30 = a(2+3)(2-1)(2-7)$$

$$30 = a(5)(1)(-5)$$

$$30 = -25a$$

$$\frac{30}{-25} = a$$

$$-\frac{6}{5} = a \quad y = -\frac{6}{5}(x+3)(x-1)(x-7)$$

(b). Intercepts: -5, -1, 2, 4
 ↑
 just touches

$$y = a(x+5)(x+1)^2(x-2)(x-4)$$

$$20 = a(0+5)(0+1)^2(0-2)(0-4)$$

$$20 = a(5)(1)(-2)(-4)$$

$$20 = a(40)$$

$$\frac{20}{40} = a$$

$$a = 0.5$$

$$y = 0.5(x+5)(x+1)^2(x-2)(x-4)$$

Unit 4

$$1.(a) f(x) = \frac{x^2 - 4x - 12}{x - 6}; f(x) = \frac{(x-6)(x+2)}{x-6} \quad \text{POD } x=6 \quad (6, 8)$$

$$(b) f(x) = \frac{x^2 + 7x - 18}{x - 3}; f(x) = \frac{(x+9)(x-2)}{(x-3)} \quad \text{VA } x=3$$

$$(c) f(x) = \frac{x^3 - 6x^2 + 11x - 6}{x^2 + 2x - 15}; f(x) = \frac{(x-1)(x-2)(x-3)}{(x+5)(x-3)} \quad \text{VA } x=-5 \\ \text{POD } x=3 \quad (3, \frac{7}{8})$$

$$(d) f(x) = \frac{6x^2 - x - 12}{x^2 - 36}; f(x) = \frac{(3x+4)(2x-3)}{(x-6)(x+6)} \quad \text{VA } x=6, x=-6$$

$$2. (a) 2 \quad (d) \frac{9}{2} = 0 \\ (b) \frac{5}{3} \quad (e) \frac{9}{3} = 3 \\ (c) \frac{1}{6} \therefore \text{DNE}$$

$$3.(a) f(x) = \frac{x^2 + 8x - 20}{x^2 + x - 6}; f(x) = \frac{(x+10)(x-2)}{(x+3)(x-2)} \\ \text{VA } x=-3 \\ \text{HA } y=1 \\ \text{POD } x=2 \quad (2, \frac{12}{5})$$

$$3.(b) f(x) = \frac{x^2 - 3x - 40}{2x^2 + 7x - 4}$$

$$f(x) = \frac{(x-8)(x+5)}{(2x-1)(x+4)}$$

$$\text{VA } x = \frac{1}{2}, x = -4 \\ \text{HA } y = \frac{1}{2}$$

$$(c) f(x) = \frac{x^4 - 2x^3 - 63x^2}{x^2 - 10x + 16}$$

$$f(x) = \frac{x^2(x-9)(x+7)}{(x-8)(x-2)}$$

$$\text{VA } x=8, x=2 \\ \text{HA } -$$

$$(d) f(x) = \frac{2x^3 - 18x}{x^3 - x^2 - 2x}$$

$$f(x) = \frac{2x(x-3)(x+3)}{x(x-2)(x+1)}$$

$$\text{VA } x=2, x=-1 \\ \text{HA } y=2 \\ \text{POD } x=0 \quad (0, -\frac{9}{2}) \\ (0, \frac{9}{2})$$

$$4.(a) f(x) = \frac{x^2 + 8x + 12}{x+5}; f(x) = x+3 + \frac{-3}{x+5}$$

$$\begin{array}{r} x+5 \overline{) x^2 + 8x + 12} \\ \underline{x^2 + 5x} \\ 3x + 12 \\ \underline{3x + 15} \\ -3 \end{array}$$

$$\text{Slant} \\ \text{asy } y = x+3$$

$$4. (b) f(x) = \frac{x^2+4}{x-6}; f(x) = x+6 + \frac{40}{x-6} \quad (c) f(x) = \frac{2x^2+4x-5}{x+2} \quad f(x) = 2x + \frac{-5}{x+2}$$

$$\begin{array}{r} x+6 \\ x-6 \overline{) x^2+0x+4} \\ \underline{x^2-6x} \\ 6x+4 \\ \underline{6x-36} \\ 40 \end{array}$$

slant
asy $y = x+6$

$$\begin{array}{r} 2x \\ x+2 \overline{) 2x^2+4x-5} \\ \underline{2x^2+4x} \\ -5 \end{array}$$

slant
asy $y = 2x$

$$5. (a) f(x) = \frac{x^2-2x+6}{x+1}; f(x) = x-3 + \frac{9}{x+1} \quad (b) f(x) = \frac{x^2+3x-28}{x+7}; f(x) = \frac{(x+7)(x-4)}{(x+7)}$$

slant asy $y = x-3$
VA $x = -1$
HA —

VA —
HA —
POD: $x = -7$ $(-7, -11)$

$$(c) f(x) = \frac{x^2+9x+18}{x+4}; f(x) = \frac{(x+6)(x+3)}{x+4}; f(x) = x+5 + \frac{-2}{x+4}$$

VA $x = -4$
HA —
slant = $y = x+5$

$$(d) f(x) = \frac{x^2-10x+21}{x-3}; f(x) = \frac{(x-7)(x-3)}{(x-3)} \quad (e) f(x) = \frac{2x^2+1}{z^2-4z-45}; f(x) = \frac{2x^2+1}{(z-9)(z+5)}$$

POD $x = 3$ $(3, -4)$

VA $x = 9, x = -5$
HA $y = 2$

$$f) f(x) = \frac{4x+8}{3x^2+8x+4}; f(x) = \frac{4x+8}{(3x+2)(x+2)}$$

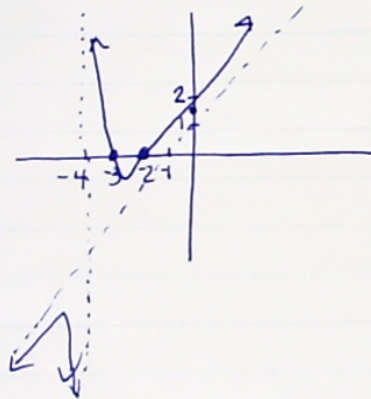
VA $x = -\frac{2}{3}$ $x = -2$
HA —

6. (a) $f(x) = \frac{x^2 + 5x + 6}{x + 4}$; $f(x) = \frac{(x+3)(x+2)}{x+4}$; $f(x) = x+1 + \frac{2}{x+4}$

x-int: -3, -2
 y-int: ~~4~~ 4
 VA $x = -4$
 HA —
 POD —
 slant $y = x+1$

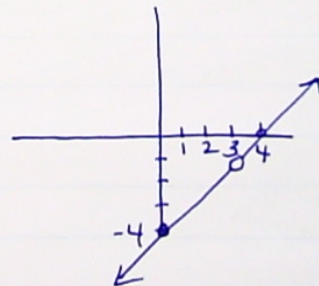
Behaviour

-4.1	-3.9
-	+
$(-\infty)$	$(+\infty)$



(b) $f(x) = \frac{x^2 - 7x + 12}{x - 3}$; $f(x) = \frac{(x-3)(x-4)}{(x-3)}$

x-int: 4
 y-int: -4
 VA —
 HA —
 POD $x = 3$ (3, -1)

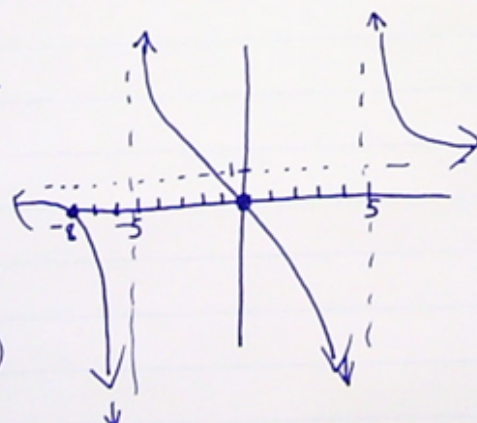


(c) $f(x) = \frac{x^2 + 8x}{x^2 - 25}$; $f(x) = \frac{x(x+8)}{(x-5)(x+5)}$

x-int: 0, -8
 y-int: 0
 VA $x = 5, x = -5$
 HA $y = 1$

Behaviour

-5.1	-4.9	4.9	5.1
-+	-+	++	++
-	-	-+	+
$(-\infty)$	$(+\infty)$	$(-\infty)$	$(+\infty)$



$$7. (a) \frac{(a-3)(a+3)}{a-3} - 2 = \frac{(a-3)(a+3)}{a+3}$$

$$(2a-3)(a+3) - 2(a-3)(a+3) = 12(a-3)$$

$$2a^2 + 3a - 9 - 2(a^2 - 9) = 12a - 36$$

$$2a^2 + 3a - 9 - 2a^2 + 18 = 12a - 36$$

$$3a - 12a = -36 - 9$$

$$-9a = -45$$

$$a = 5$$

$$(c) x - \frac{2}{x-3} = \frac{x-1}{3-x}$$

$$(x-3)x - \frac{2(x-3)}{x-3} = \frac{x-1}{-1(x-3)}$$

$$x(x-3) - 2 = -1(x-1)$$

$$x^2 - 3x - 2 = -x + 1$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3, -1$$

inadmissible

$$b. \frac{k}{k^2 - 8k + 12} = \frac{k}{k-2} + \frac{1}{k-6}$$

$$\frac{k}{(k-6)(k-2)} = \frac{k}{k-2} + \frac{1}{k-6}$$

$$k = k(k-6) + 1(k-2)$$

$$k = k^2 - 6k + k - 2$$

$$k = k^2 - 5k - 2$$

$$0 = k^2 - 6k - 2$$

$$\frac{6 \pm \sqrt{44}}{2} = \begin{cases} 6.3 \\ -0.3 \end{cases}$$

$$(d) \frac{(m-4)(m+1)}{m-4} + \frac{(m-1)(m+1)}{(m-1)(m+1)} = \frac{1}{(m-1)(m+1)}$$

$$2m(m+1) + (m-5) = 1(m-1)(m+1)$$

$$2m^2 + 2m + m - 5 = m^2 - 1$$

$$2m^2 + 3m - 5 = m^2 - 1$$

$$m^2 + 3m - 4 = 0$$

$$(m+4)(m-1) = 0$$

$m = -4, 1$
inadmissible

$$8. \sqrt{-3x+4} = (2-x)^2$$

$$(a) -3x+4 = 4-4x+x^2$$

$$0 = x^2 - x$$

$$0 = x(x-1)$$

$$x=0; x=1$$

$$(b) \sqrt{x-5} = 11 - \sqrt{x+6}$$

$$\sqrt{x-5} = (11 - \sqrt{x+6})^2$$

$$x-5 = 121 - 22\sqrt{x+6} + x+6$$

$$x-5 = 127+x-22\sqrt{x+6}$$

$$-132 = -22\sqrt{x+6}$$

$$6 = \sqrt{x+6}$$

$$36 = x+6$$

$$30 = x$$

$$\begin{aligned}
 \text{(c)} \quad & \sqrt{x+7} + \sqrt{x+4} = 11 \\
 & \sqrt{x+7} = (11 - \sqrt{x+4})^2 \\
 & \cancel{x+7} = 121 - 22\sqrt{x+4} + \cancel{x+4} \\
 & -118 = -22\sqrt{x+4} \\
 & \left(\frac{118}{22}\right)^2 = (\sqrt{x+4})^2 \\
 & \frac{13924}{484} = x+4 \\
 & \frac{2997}{121} = x
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad & \sqrt{y+3} - \sqrt{2y-8} = 1 \\
 & (\sqrt{y+3})^2 = (1 + \sqrt{2y-8})^2 \\
 & y+3 = 1 + 2\sqrt{2y-8} + 2y-8 \\
 & y+3 = 2y-7 + 2\sqrt{2y-8} \\
 & (-y+10)^2 = (2\sqrt{2y-8})^2 \\
 & y^2 - 20y + 100 = 4(2y-8) \\
 & y^2 - 20y + 100 = 8y - 32 \\
 & y^2 - 28y + 132 = 0 \\
 & (y-22)(y-6) = 0 \\
 & y = 22 ; y = 6
 \end{aligned}$$

$\begin{aligned}
 & * y+3 \geq 0 \quad * 2y-8 \geq 0 \\
 & y \geq -3 \quad \quad 2y \geq 8 \\
 & \quad \quad \quad y \geq 4
 \end{aligned}$

$$9. (a) y = x^2 - 5$$

$$\text{Inv: } x = y^2 - 5$$

$$x + 5 = y$$

$$\sqrt{x+5} = y$$

$$(b) y = 10 + \sqrt{2x-1}$$

$$\text{Inv: } x = 10 + \sqrt{2y-1}$$

$$x - 10 = \sqrt{2y-1}$$

$$(x-10)^2 = 2y-1$$

$$(x-10)^2 + 1 = 2y$$

$$\frac{(x-10)^2 + 1}{2} = y$$

$$\text{OR } \frac{x^2 - 20x + 101}{2} = y$$

$$(c) y = 3(x-4)^2 - 9$$

$$\text{Inv: } x = 3(y-4)^2 - 9$$

$$x + 9 = 3(y-4)^2$$

$$\frac{x+9}{3} = (y-4)^2$$

$$\sqrt{\frac{x+9}{3}} = y-4$$

$$\sqrt{\frac{x+9}{3}} + 4 = y$$

$$(d) y = x^2 - 8x + 13$$

$$x = y^2 - 8y + 13$$

$$x = y^2 - 8y + 16 - 16 + 13$$

$$x = (y^2 - 8y + 16) - 16 + 13$$

$$x = (y-4)^2 - 3$$

$$x + 3 = (y-4)^2$$

$$\sqrt{x+3} = y-4$$

$$\sqrt{x+3} + 4 = y$$

$$(e) y = \frac{3x-5}{x+4}$$

$$\text{Inv: } x = \frac{3y-5}{y+4}$$

$$x(y+4) = 3y-5$$

$$xy + 4x = 3y-5$$

$$xy - 3y = -4x-5$$

$$y(x-3) = -4x-5$$

$$y = \frac{-4x-5}{x-3}$$

$$(f) y = \frac{9+x}{3x-2}$$

$$\text{Inv: } x = \frac{9+y}{3y-2}$$

$$x(3y-2) = 9+y$$

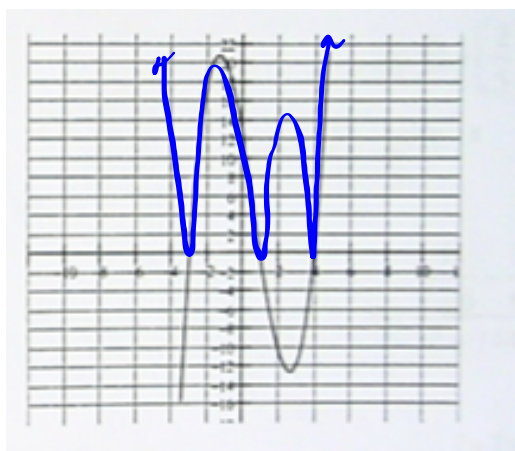
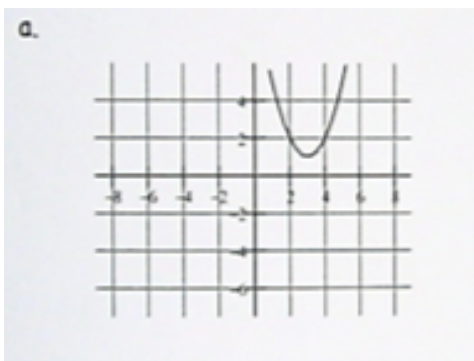
$$3xy - 2x = 9+y$$

$$3xy - y = 2x+9$$

$$y(3x-1) = 2x+9$$

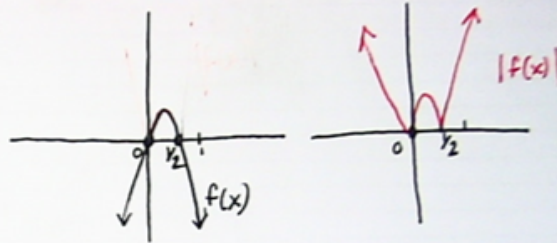
$$y = \frac{2x+9}{3x-1}$$

10.



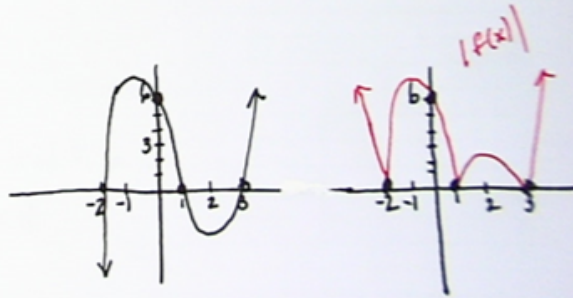
11. a $f(x) = -4x^2 + 2x$
 $f(x) = -2x(2x-1)$
 x-int: $0, \frac{1}{2}$
 y-int: 0

	$-2x(2x-1)$		
$(-\infty, 0)$	+	-	neg
$(0, \frac{1}{2})$	-	-	pos
$(\frac{1}{2}, \infty)$	-	+	neg

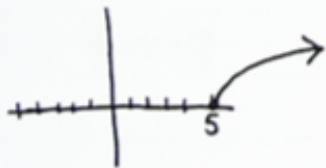


b) $f(x) = x^3 - 2x^2 - 5x + 6$
 $f(x) = (x+2)(x-1)(x-3)$
 x-int: $-2, 1, 3$
 y-int: 6

	$(x+2)(x-1)(x-3)$			
$(-\infty, -2)$	-	-	-	- neg
$(-2, 1)$	+	-	-	+ pos
$(1, 3)$	+	+	-	- neg
$(3, \infty)$	+	+	+	+ pos



c) $f(x) = \sqrt{x-5}$

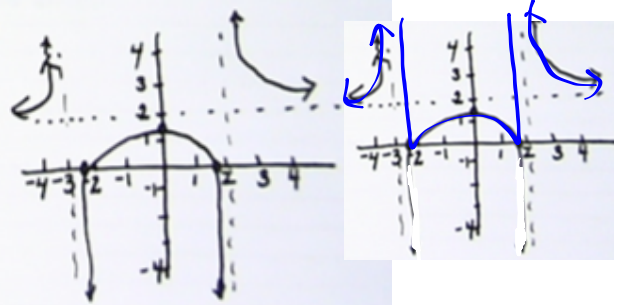


d) $f(x) = \frac{2x^2+x-8}{x^2+x-6}$; $\frac{2x^2+x-8}{(x+3)(x-2)}$ QF $\frac{-1 \pm \sqrt{65}}{4}$
 $1.77 \quad -2.27$

x-int: $1.77, -2.27$
 y-int: $-\frac{4}{3} = 1\frac{2}{3}$
 VA $x = -3, 2$
 HA $y = 2$

Behaviour

-3.1	-2.9	1.9	2.1
$\frac{+}{-}$	$\frac{-}{+}$	$\frac{+}{-}$	$\frac{+}{+}$
(∞)	$(-\infty)$	(∞)	(∞)



(e) $f(x) = \frac{x^2+4x+1}{x+3}$; $f(x) = x+1 + \frac{-2}{x+3}$

$$\begin{array}{r} x+1 \\ x+3 \overline{) x^2+4x+1} \\ \underline{x^2+3x} \\ x+1 \\ \underline{x+3} \\ -2 \end{array}$$

x-int: $\frac{-4 \pm \sqrt{12}}{2} \begin{cases} -0.27 \\ -3.73 \end{cases}$

y-int: y_3

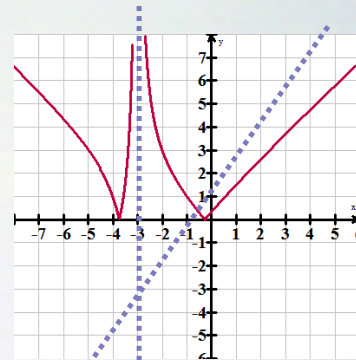
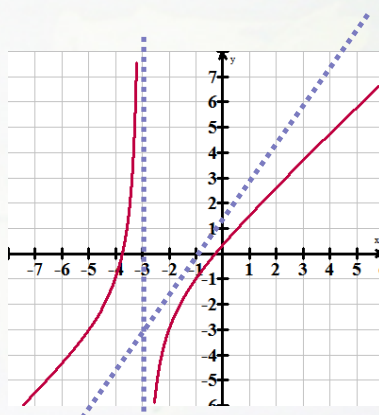
VA: $x = -3$

HA —

slant $y = x+1$

Behaviour

-3.1	2.9
-/-	-/+
$+\infty$	$-\infty$



12. (a) $|2x-5|=7$

$$\begin{array}{ll} 2x-5=7 & 2x-5=-7 \\ 2x=12 & 2x=-2 \\ x=6 & x=-1 \end{array}$$

b. $|x-12| \geq -6$

$x \in \mathbb{R}$

c. $|3x+1| < 15$

$$\begin{array}{ll} 3x+1 < 15 & \text{or } 3x+1 > 15 \\ 3x < 14 & 3x > -16 \\ x < 14/3 & x > -16/3 \\ -16/3 < x < 14/3 \end{array}$$

d. $|x^2-12|=13$

$$\begin{array}{ll} x^2-12=13 & x^2-12=-13 \\ x^2-25=0 & x^2+1=0 \\ (x-5)(x+5)=0 & \text{No sol'n} \\ x=5, -5 & \end{array}$$

$$\begin{array}{l}
 \text{1. a. } \lim_{x \rightarrow 5} \frac{x^3 - 125}{x^2 - 25} \\
 \lim_{x \rightarrow 5} \frac{(x-5)(x^2+5x+25)}{(x-5)(x+5)} \\
 = \frac{5^2 + 5(5) + 25}{5+5} \\
 = \frac{25 + 25 + 25}{10} \\
 = 75/10
 \end{array}$$

$$\begin{array}{l}
 \text{b. } \lim_{x \rightarrow \infty} 7 + \frac{5}{x} - \frac{x^3}{3x^3 - 2} \\
 7 + 0 - \frac{1}{3} \\
 = 20/3
 \end{array}$$

$$\begin{array}{l}
 \text{c. } \lim_{x \rightarrow 3} \frac{x^2 + x - 12}{x^3 + 2x^2 - 11x - 12} \\
 \lim_{x \rightarrow 3} \frac{(x+4)(x-3)}{(x-3)(x+4)(x+1)} \\
 = \frac{7}{7(4)} \\
 = 1/28
 \end{array}$$

$$\begin{array}{l}
 \text{d. } \lim_{x \rightarrow \infty} \left(\frac{4}{3}\right)^x \\
 = \infty
 \end{array}$$

$$\begin{array}{l}
 \text{e. } \lim_{x \rightarrow 5} \frac{\sqrt{3x+1} - 4}{5-x} \times \frac{\sqrt{3x+1} + 4}{\sqrt{3x+1} + 4} \\
 \lim_{x \rightarrow 5} \frac{(3x+1) - 16}{(5-x)(\sqrt{3x+1} + 4)}
 \end{array}$$

$$\begin{array}{l}
 \text{f. } \lim_{x \rightarrow -4} \frac{16-x^2}{3x^2+10x-8} \\
 \lim_{x \rightarrow -4} \frac{(4-x)(4+x)}{(3x-2)(x+4)} \\
 = \frac{8}{-14}
 \end{array}$$

$$\lim_{x \rightarrow 5} \frac{3x-15}{(5-x)(\sqrt{3x+1} + 4)}$$

$$\lim_{x \rightarrow 5} \frac{3(x-5)}{(5-x)(\sqrt{3x+1} + 4)}$$

$$\begin{array}{l}
 \lim_{x \rightarrow 5} \frac{-3}{\sqrt{3x+1} + 4} \\
 \frac{-3}{\sqrt{16} + 4} = -3/8
 \end{array}$$

$$\begin{array}{l}
 \text{g. } \lim_{x \rightarrow \infty} \frac{(x-7)(2x-3)}{10-3x^2} \\
 \lim_{x \rightarrow \infty} \frac{2x^2 - 17x + 21}{10 - 3x^2} \\
 = -2/3
 \end{array}$$

$$\begin{array}{l}
 \text{h. } \lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x+12} - \sqrt{4x}} \times \frac{\sqrt{x+12} + \sqrt{4x}}{\sqrt{x+12} + \sqrt{4x}}
 \end{array}$$

$$\lim_{x \rightarrow 4} \frac{(x-4)(\sqrt{x+12} + \sqrt{4x})}{(x+12) - (4x)}$$

$$\lim_{x \rightarrow 4} \frac{(x-4)(\sqrt{x+12} + \sqrt{4x})}{-3x+12}$$

$$\lim_{x \rightarrow 4} \frac{(x-4)(\sqrt{x+12} + \sqrt{4x})}{-3(x-4)}$$

$$= \frac{\sqrt{16} + \sqrt{16}}{-3} = 8/-3$$

i. $\lim_{x \rightarrow -6} \frac{6x^2 - 12x}{6-x}$
 $= \frac{216 + 72}{12}$
 $= \frac{288}{12}$

j. $\lim_{x \rightarrow -\infty} 12 - \left(\frac{2}{5}\right)^x$
 $12 - \left(\frac{2}{5}\right)^{-\infty}$
 $12 - \left(\frac{5}{2}\right)^{\infty}$
 $-\infty$

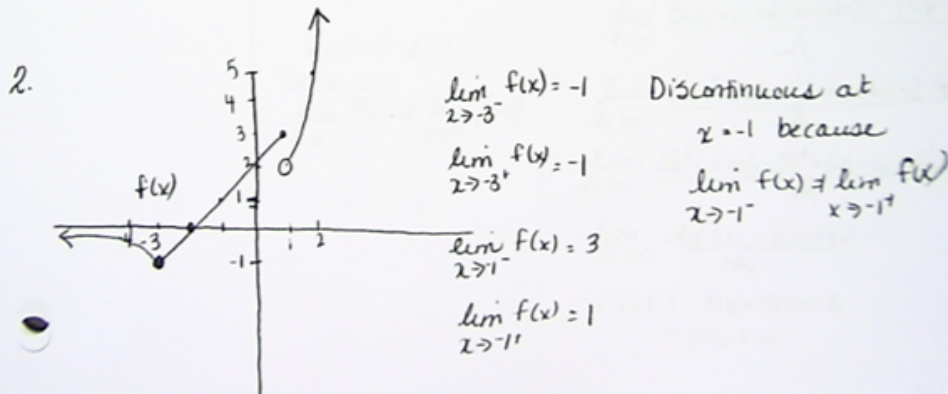
k. $\lim_{x \rightarrow \sqrt[3]{3}} \frac{\sin^2 x}{\sin^{\frac{2}{3}} x}$
 $\frac{\left(\frac{\sqrt{3}}{2}\right)^2}{\frac{3}{4}}$

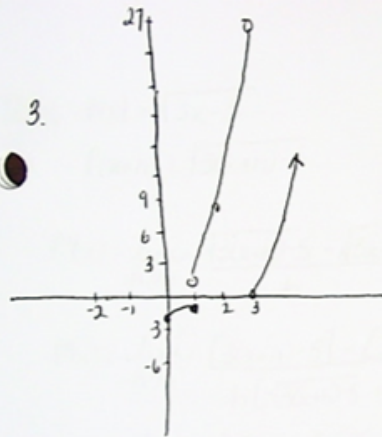
l. $\lim_{x \rightarrow 12} \frac{\frac{6(x-6)}{x-6} - \frac{1}{6}}{x^2 - 144}$
 $\lim_{x \rightarrow 12} \frac{6 - (x-6)}{6(x-6)(x^2 - 144)}$
 $\lim_{x \rightarrow 12} \frac{6 - x + 6}{6(x-6)(x-12)(x+12)}$
 $\lim_{x \rightarrow 12} \frac{12 - x}{6(x-6)(x-12)(x+12)}$
 $\lim_{x \rightarrow 12} \frac{-1}{6(x-6)(x+12)}$
 $\frac{-1}{6(6)(24)}$
 $= \frac{1}{864}$

m. $\lim_{x \rightarrow 0} \frac{\sin x \cos x - \sin^2 x}{\sin x}$
 $\lim_{x \rightarrow 0} \frac{\sin x (\cos x - \sin x)}{\sin x}$
 $= \cos 0 - \sin 0$
 $= 1 - 0$
 $= 1$

n. $\lim_{x \rightarrow -\infty} \frac{5x^6 - 9x^4}{x^2 - 4x^6}$
 $= \frac{5}{-4}$

p. $\lim_{x \rightarrow 7^-} \frac{3+2x}{x-7}$
 $\frac{69}{-0}$
 $-\infty$ (7 from left)

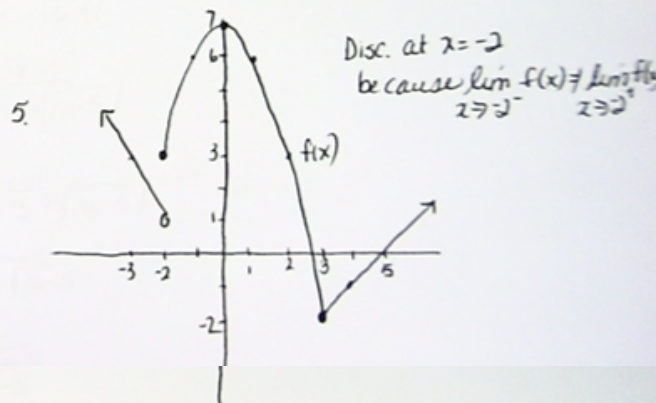




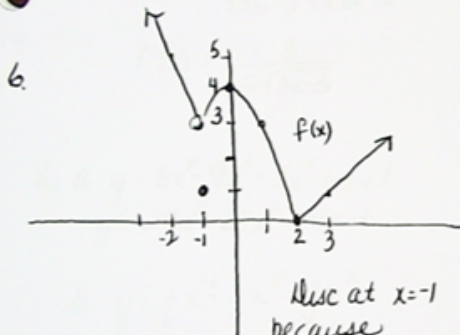
Disc. at $x=1$ because $\lim_{x \rightarrow 1^-} g(x) \neq \lim_{x \rightarrow 1^+} g(x)$

at $x=3$ because $\lim_{x \rightarrow 3^-} g(x) = \lim_{x \rightarrow 3^+} g(x) \neq g(3)$

4. a. 1 f. 0
 b. 2 g. 1
 c. DNE h. 1
 d. 0 i. 1
 e. 0 j. -3



Disc. at $x=-2$ because $\lim_{x \rightarrow -2^-} f(x) \neq \lim_{x \rightarrow -2^+} f(x)$



Disc at $x=-1$ because $\lim_{x \rightarrow -1^-} f(x) \neq \lim_{x \rightarrow -1^+} f(x)$

7. (a) $f(x) = 5x^2 + 6x - 2$
 $f(x+h) = 5(x+h)^2 + 6(x+h) - 2$
 $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
 $\lim_{h \rightarrow 0} \frac{5(x+h)^2 + 6(x+h) - 2 - (5x^2 + 6x - 2)}{h}$
 $\lim_{h \rightarrow 0} \frac{5(x^2 + 2xh + h^2) + 6x + 6h - 2 - 5x^2 - 6x + 2}{h}$
 $\lim_{h \rightarrow 0} \frac{5x^2 + 10xh + 5h^2 + 6x + 6h - 2 - 5x^2 - 6x + 2}{h}$
 $\lim_{h \rightarrow 0} \frac{h(10x + 5h + 6)}{h}$
 $f'(x) = 10x + 5(0) + 6 = 10x + 6$

$$7.(b) f(x) = \sqrt{3x-5}$$

$$f(x+h) = \sqrt{3(x+h)-5}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{3(x+h)-5} - \sqrt{3x-5}}{h} \cdot \frac{\sqrt{3(x+h)-5} + \sqrt{3x-5}}{\sqrt{3(x+h)-5} + \sqrt{3x-5}}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{[3(x+h)-5] - [3x-5]}{h[\sqrt{3(x+h)-5} + \sqrt{3x-5}]}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3x+3h-5-3x+5}{h[\sqrt{3(x+h)-5} + \sqrt{3x-5}]}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3h}{h[\sqrt{3(x+h)-5} + \sqrt{3x-5}]}$$

$$\lim_{h \rightarrow 0} \frac{3}{\sqrt{3(x+h)-5} + \sqrt{3x-5}}$$

$$f'(x) = \frac{3}{\sqrt{3x-5} + \sqrt{3x-5}}$$

$$f'(x) = \frac{3}{2\sqrt{3x-5}}$$

$$8(a) \quad y = 6x^4 - 9x^3 + 7x^2 - 6x + 1$$

$$y' = 24x^3 - 27x^2 + 14x - 6$$

$$(b) \quad y = \frac{1}{6}x^{18} - \frac{3}{4}x^{12} + 14x - 6$$

$$y' = 3x^{17} - 9x^{11} + 14$$

$$(c) \quad f(x) = -24x^7 + 12x^5 - 6$$

$$f'(x) = -168x^6 + 60x^4$$

$$(d) \quad f(x) = 2x^{-4} - 9\sqrt{2}x^3 + x^2 - 12x + x^{\frac{1}{2}}$$

$$f'(x) = -8x^{-5} - 27\sqrt{2}x^2 + 2x - 12 + \frac{1}{2}x^{-\frac{1}{2}}$$

$$(e) \quad y = (7x^3 - 9x)(2x^2 + 4)$$

$$y' = (7x^3 - 9x)(4x) + (2x^2 + 4)(21x^2 - 9)$$

$$(f) \quad y = \frac{-x^4 + 8x^2 - x}{(x^3 - 4x)}$$

$$y' = \frac{(x^3 - 4x)(-4x^3 + 16x - 1) - (-x^4 + 8x^2 - x)(3x^2 - 4)}{(x^3 - 4x)^2}$$

$$(g) \quad y = (3x^2 - 6x + 2)^5$$

$$y' = 5(3x^2 - 6x + 2)^4(6x - 6)$$

$$(h) \quad f(x) = (4x^3 + (x^2 - 1)^{-2})^6$$

$$f'(x) = 6(4x^3 + (x^2 - 1)^{-2})^5(12x^2 - 2(x^2 - 1)^{-3} \cdot 2x)$$

$$(i) \quad f(x) = x^{\frac{3}{4}}(x - 2x^2)^{\frac{1}{2}}$$

$$f'(x) = x^{\frac{3}{4}} \cdot \frac{1}{2}(x - 2x^2)^{-\frac{1}{2}}(1 - 4x) + (x - 2x^2)^{\frac{1}{2}} \cdot \frac{3}{4}x^{-\frac{1}{4}}$$

$$(j) \quad f(x) = \frac{2}{(3x^4 + 4x^2)^3}$$

$$f(x) = 2(3x^4 + 4x^2)^{-3}$$

$$f'(x) = -6(3x^4 + 4x^2)^{-4}(12x^3 + 8x)$$

9. $f(x) = 5x^3 - 6x^2 + 7x - 8$ (1, -2)

$f'(x) = 15x^2 - 12x + 7$

$m_{x=1} = 15(1)^2 - 12(1) + 7$
 $m = 10$

$y + 2 = 10(x - 1)$
 $y + 2 = 10x - 10$
 $y = 10x - 12$

10. $f(x) = 12x^{\frac{1}{2}} - 6x$ at $x = 4$

$f'(x) = 6x^{-\frac{1}{2}} - 6$

$m = 6(4)^{-\frac{1}{2}} - 6$
 $= 6 \cdot \frac{1}{4^{\frac{1}{2}}} - 6$
 $= \frac{6}{2} - 6$
 $m = -3$

at $x = 4$
 $y = 12(4)^{\frac{1}{2}} - 6(4)$
 $= 24 - 24$
 $y = 0$
 $(4, 0)$

$y - 0 = -3(x - 4)$
 $y = -3x + 12$

11. $f(x) = 3x^2 - 12x^{-1}$ (2, 6)

$f'(x) = 6x + 12x^{-2}$

$m = 6(2) + 12(2)^{-2}$
 $= 12 + 12 \cdot \frac{1}{2^2}$
 $= 12 + 3$
 $m = 15$

$y - 6 = 15(x - 2)$
 $y - 6 = 15x - 30$
 $y = 15x - 24$

12. $y = 5x^4 - 40x^2 + 75$

$y' = 20x^3 - 80x$

slope of horizontal line is 0

$20x^3 - 80x = 0$
 $20x(x^2 - 4) = 0$
 $20x(x - 2)(x + 2) = 0$
 $x = 0, 2, -2$

To find pts sub $x = 0, 2, -2$
 into original $y = 5x^4 - 40x^2 + 75$

$x = -2$ $y = 80 - 160 + 75 = 5$
 $x = 0$ $y = 0 - 0 + 75 = 75$
 $x = 2$ $y = 80 - 160 + 75 = 5$
 $(-2, 5)$ $(0, 75)$ $(2, 5)$

$$13. y = \frac{1}{4}(x^2 + 5x)^2 \text{ at } x = -2 \rightarrow y = \frac{1}{4}(4 - 10)^2$$

$$y' = \frac{2}{4}(x^2 + 5x)(2x + 5)$$

$$m_{x=-2} = \frac{1}{2}(4 - 10)(-4 + 5)$$

$$= \frac{1}{2}(-6)(1)$$

$$= -3$$

$$y = \frac{1}{4}(-6)^2$$

$$y = 9$$

$$m = -3 \quad (-2, 9)$$

$$y - 9 = -3(x + 2)$$

$$y = -3x - 6 + 9$$

$$y = -3x + 3$$

$$14. f(x) = (x - x^2)(x^3 - 2)^{\frac{1}{2}} \text{ at } x = 3$$

$$f'(x) = (x - x^2) \frac{1}{2}(x^3 - 2)^{-\frac{1}{2}} \cdot 3x^2 + (x^3 - 2)^{\frac{1}{2}}(1 - 2x)$$

$$m_{x=3} = (3 - 9) \frac{1}{2}(25)^{-\frac{1}{2}} \cdot 27 + (25)^{\frac{1}{2}}(-5)$$

$$(-6) \left(\frac{1}{2}\right) \left(\frac{1}{5}\right) (27) + 5(-5)$$

$$= \frac{-162}{10} - 25$$

$$= \frac{-81}{5} - \frac{125}{5}$$

$$= -206$$

$$15. f(x) = x^{\frac{3}{2}} \text{ parallel to } 6x - y = 10$$

$$f'(x) = \frac{3}{2}x^{\frac{1}{2}}$$

$$6x - 10 = y$$

$$m = 6$$

$$\text{let } \frac{3}{2}x^{\frac{1}{2}} = 6$$

$$3\sqrt{x} = 12$$

$$\sqrt{x} = 4$$

$$x = 16$$

sub $x = 16$ into original

$$x = 16 \quad y = 16^{\frac{3}{2}} = 64$$

$$(16, 64)$$

9. $f(x) = 5x^3 - 6x^2 + 7x - 8$ at $(1, -2)$
 $f'(x) = 15x^2 - 12x + 7$
 $m_{x=1} = 15(1)^2 - 12(1) + 7$
 $m = 10$
 $y + 2 = 10(x - 1)$
 $y + 2 = 10x - 10$
 $y = 10x - 8$

10. $f(x) = 12x^{\frac{1}{2}} - 6x$ at $x = 4$
 $f'(x) = 6x^{-\frac{1}{2}} - 6$
 $m = 6(4)^{-\frac{1}{2}} - 6$
 $= 6 \cdot \frac{1}{4^{\frac{1}{2}}} - 6$
 $= \frac{6}{2} - 6$
 $m = -3$

at $x = 4$
 $y = 12(4)^{\frac{1}{2}} - 6(4)$
 $24 - 24$
 $y = 0$
 $(4, 0)$
 $y - 0 = -3(x - 4)$
 $y = -3x + 12$

11. $f(x) = 3x^2 - \frac{12}{x}$ at $(2, 6)$ $m = 15$ at $(2, 6)$
 $f(x) = 3x^2 - 12x^{-1}$
 $f'(x) = 6x + 12x^{-2}$
 $m_{x=2} = 6(2) + 12(2)^{-2}$
 $12 + 12\left(\frac{1}{2^2}\right)$
 $12 + 12 \cdot \frac{1}{4}$
 $= 12 + 3$
 $= 15$

$y - 6 = 15(x - 2)$
 $y - 6 = 15x - 30$
 $y = 15x - 24$

12. $f(x) = 5x^4 - 40x^2 + 75$ tangent is horizontal
 \therefore slope = 0
 $f'(x) = 20x^3 - 80x$
 set $m = 0$
 $20x^3 - 80x = 0$
 $20x(x^2 - 4) = 0$
 $20x(x - 2)(x + 2) = 0$
 $x = 0, 2, -2$

sub x 's into original to get points

$f(x) = 5x^4 - 40x^2 + 75$
 $x = -2$ $y = 5(-2)^4 - 40(-2)^2 + 75 = -5$
 $x = 0$ $y = 5(0)^4 - 40(0)^2 + 75 = 75$
 $x = 2$ $y = 5(2)^4 - 40(2)^2 + 75 = -5$

$(-2, -5)$ $(0, 75)$ $(2, -5)$

13 $f(x) = x^{\frac{3}{2}}$ parallel to $6x - y = 10$
 $f'(x) = \frac{3}{2}x^{\frac{1}{2}}$ $6x - 10 = y$
 $m = 6$

set $\frac{3}{2}x^{\frac{1}{2}} = 6$
 $3x^{\frac{1}{2}} = 12$
 $x^{\frac{1}{2}} = 4$
 $x = 16$

sub $x = 16$ into original
 $y = 16^{\frac{3}{2}}$
 $y = 64$

Extra Inverse of log and exponential functions

