UNIT 6 Linear Equations and Graphing

Suppose you are planning a ski trip to Nakiska, a ski resort 90 km west of Calgary.

What do you need to know to organize this trip?

You want to calculate the total cost for all Grade 8 students. What assumptions do you make? What other things do you need to consider?

What You'll Learn

- Model and solve problems using linear equations.
- Solve equations using models, pictures, and symbols.
- Graph and analyse two-variable linear relations.

Why It's Important

- Solving equations is a useful problem-solving tool.
- You find linear relations in everyday situations, such as rates of pay, car rentals, and dosages of medicine.

Chair Lift Ticket Rates

	Price per day ($)	Price per half day ($)
Child (6 to 12 years)	17	14
Youth (13 to 17 years)	37	29
Adult (18 to 64 years)	53	42
Senior (65 years and older)	42	34

Equipment Rental

Skis and Snowboards	Price per 1 day ($)	Price per day for 3 or more days ($)
Complete Package	32	30
Skis or Boards	23	22
Boots	9	8
Poles	7	7

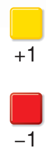
Key Words

- distributive property
- expand
- linear relation
- ordered pair
- discrete data

6.1 Solving Equations Using Models

Focus Use concrete materials to model and solve linear equations.

Recall that one yellow unit tile represents $+1$.
One red unit tile represents -1.
What happens when you combine one red unit tile and one yellow unit tile?



The yellow variable tile  represents any variable, such as n or x.

Investigate

Work with a partner.

Marie received three $100 savings bonds on her first birthday.
Her grandmother promised to give her 2 savings bonds each year after that for her birthday. How old will Marie be when she has 13 savings bonds?

Let n represent Marie's age in years.
Write an equation you can use to solve for n.
Use tiles to represent the equation.
Use the tiles to solve the equation.
Sketch the tiles you used.

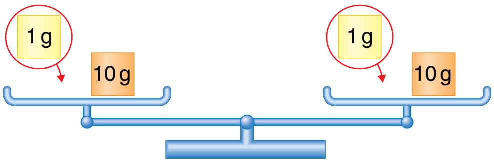
Reflect & Share

Compare your equation and its solution with those of another pair of classmates.
Did you write the same equation?
If your answer is yes, find another way to write the equation.
If your answer is no, are both equations correct?
How do you know?
Share your strategies for solving the equation using tiles.

Connect

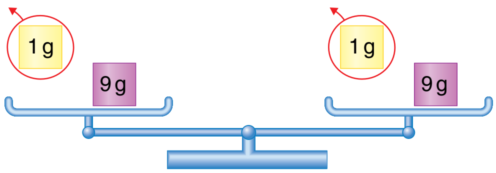
We can use a balance-scales model to solve an equation.

To keep the scales balanced, we must do the same to both sides.
For example,
we add the same mass:



The scales remain balanced.

or remove the same mass:



The scales remain balanced.

Example 1

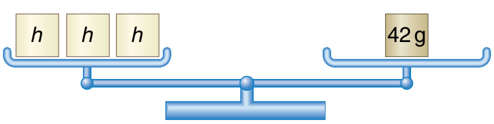
Herman is in the last round of the spell-a-thon in his school.
A contestant receives 3 points for every word spelled correctly.
Herman has 42 points. How many words has he spelled correctly?

A Solution

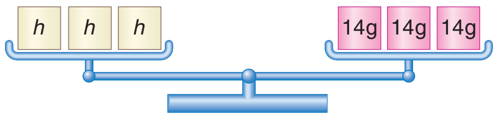
Let h represent the number of words Herman has spelled correctly.
Then, the number of points is 3 times h, or $3h$.

Since Herman has 42 points, the algebraic equation is $3h = 42$.
Use a balance-scales model to represent this equation.

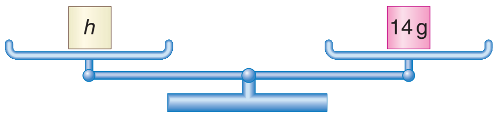
On the left side, show masses to represent $3h$.
On the right side, show a mass to represent 42.



Since there are 3 identical unknown masses in the left pan,
replace 42 g in the right pan with 3 equal masses.
Each mass is 14 g.



So, each unknown mass is 14 g.
$h = 14$



Herman has spelled 14 words correctly.

Check: 14 words worth 3 points each is $14 \times 3 = 42$ points.
The solution is correct.

Another strategy to solve an equation is to use algebra tiles.
We rearrange the tiles to end up with variable tiles on one side and unit tiles on the other side.

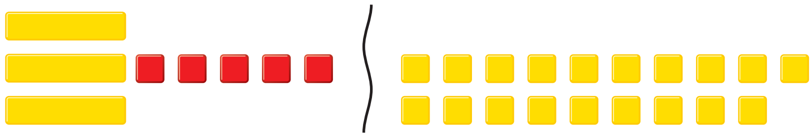
Example 2

Jodee is also a contestant in the spell-a-thon.
A contestant receives 3 points for every word spelled correctly.
Because of a technical penalty, Jodee loses 5 points. She now has 19 points.
How many words has Jodee spelled correctly?

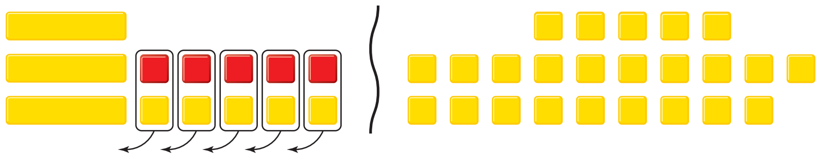
A Solution

Let j represent the number of words Jodee has spelled correctly.
Then, the number of points she receives is $3j$.
When the penalty is considered, the number of points is $3j - 5$.
So, the equation is: $3j - 5 = 19$

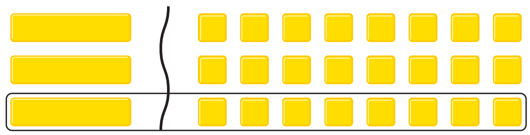
On the left side, place tiles to represent $3j - 5$.
On the right side, place tiles to represent 19.



To isolate the j-tiles on the left side, add 5 positive unit tiles to make zero pairs.
To keep the balance, add 5 positive unit tiles to the right side, too.

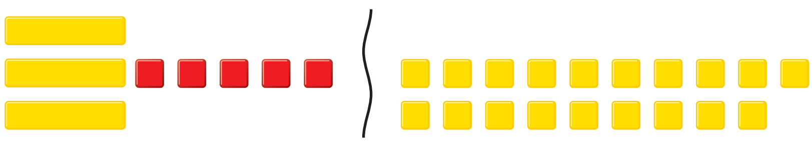


There are 3 j-tiles. So, arrange the unit tiles into 3 equal groups.

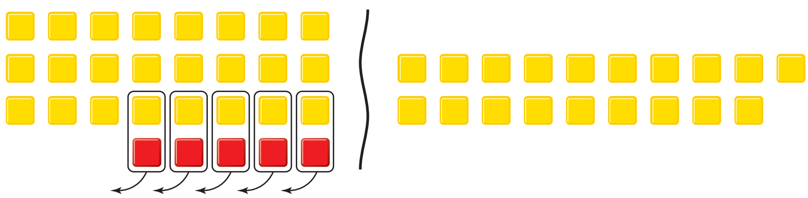


The solution is $j = 8$.
Jodee has spelled 8 words correctly.

We can verify the solution by replacing each positive variable tile with 8 positive unit tiles. Then:



becomes:



Since there are now 19 positive unit tiles on each side, the solution is correct.

Many different types of equations can be modelled and solved with algebra tiles.

The opposite of x is $-x$.
So, the red variable tile  represents $-x$.
As with the unit tiles, a positive variable tile and
a negative variable tile combine to model 0.

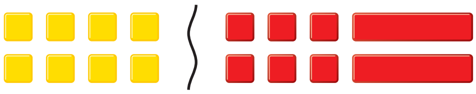
To make some equations easier to solve,
it helps to make the variables positive.

Example 3

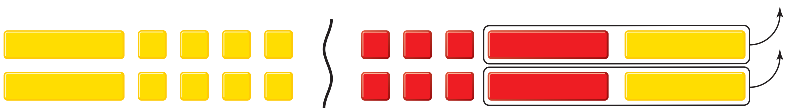
Use algebra tiles to solve: $8 = -6 - 2x$
Verify the solution.

A Solution

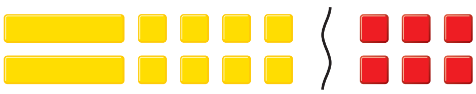
Place tiles to model the equation.



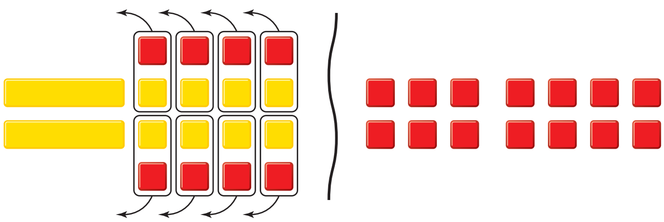
To make the variable tiles positive,
add 2 positive variable tiles to each side.



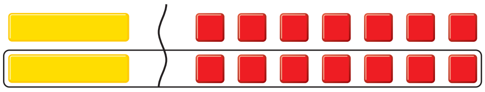
These tiles remain.



To isolate the variable tiles, add 8 negative unit tiles to each side.
Remove zero pairs.



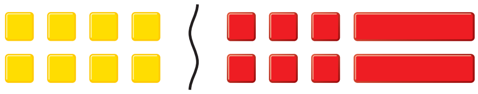
Arrange the tiles remaining on each side into 2 equal groups.



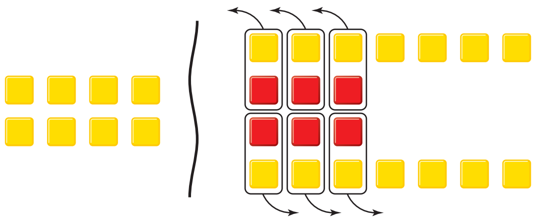
The solution is $x = -7$.

We can verify the solution.
The original equation has negative variable tiles.
If $x = -7$, then $-x = -(-7) = 7$
So, replace each variable tile in the original equation with 7 positive unit tiles.

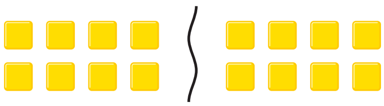
Then:



becomes:



Remove zero pairs.



Since there are now 8 positive unit tiles on each side, the solution is correct.

Discuss the ideas

1. In *Example 1*, we used h to represent the number of words Herman spelled correctly. Why do you think we used h? Could we have used a different letter? Explain.
2. In *Example 2*, we solved the equation with tiles to represent $3j - 5$ on the left side and 19 on the right side. Could we have solved the equation with 19 on the left side and $3j - 5$ on the right side? Would the solution have been different? Justify your answer.
3. In *Example 3*, how would you solve the equation by leaving the variable tiles on the right side of the equation?
4. Could we have solved the equation in *Example 3* using a balance-scales model? Why or why not?

Practice

Check

Use a model to solve each equation.

5. Draw pictures to represent the steps you took to solve each equation.
   a) $4s = 16$ b) $5t = -15$
   c) $18 = 6a$ d) $-18 = 3b$

6. Draw pictures to represent the steps you took to solve each equation.
   a) $3x + 2 = 8$ b) $4s - 3 = 9$
   c) $10 = 6c + 4$ d) $-4 = 5m + 6$

7. Three more than six times a number is 21. Let n represent the number.
   a) Write an equation you can use to solve for n.
   b) Represent the equation with tiles. Use the tiles to solve the equation. Sketch the tiles you used.
   c) Verify the solution.

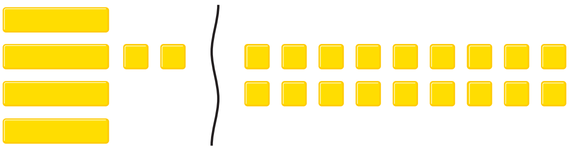
Apply

8. Three less than six times a number is 21. Let n represent the number.
   a) Write an equation you can use to solve for n.
   b) Represent the equation with tiles. Use the tiles to solve the equation. Sketch the tiles you used.
   c) Verify the solution.

9. Maeve wants her friend to guess how many cards she has in her hand. She says that if the number of cards in her hand is tripled, and 4 are added, then she has 22 cards.
   a) Choose a variable. Write an equation for this situation.
   b) Use a balance-scales model to solve the equation.
   c) Verify the solution. Show how you did this.

10. Curtis is practising modelling equations. He is trying to model the equation: $4x - 2 = 18$
Curtis begins by using algebra tiles.

a) Check Curtis' work. Is this the correct model? Explain.



b) If your answer to part a is yes, use the tiles to try to solve the equation. If your answer to part a is no, describe the error, correct it, then use algebra tiles to solve the equation.

11. Use a model to solve each equation. Verify the solution.

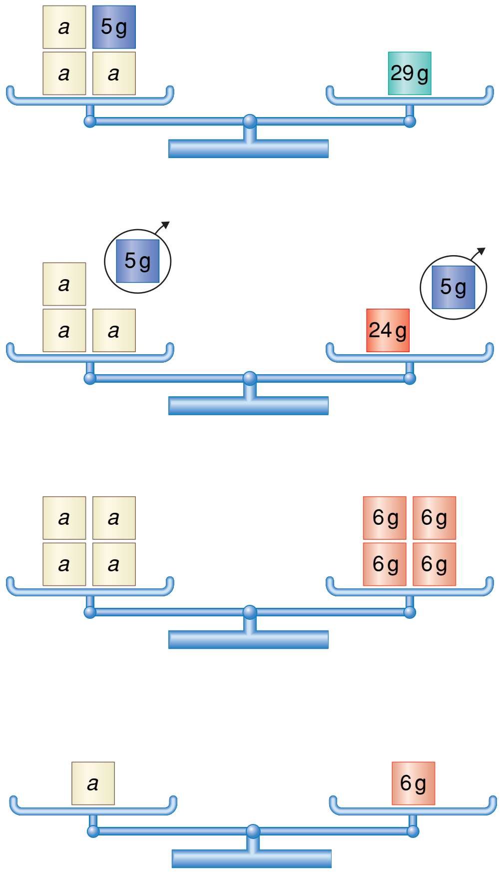
a) $-2x = -6$

b) $-15 = 3x$

c) $-24 = -4x$

d) $9x = -27$

12. **Assessment Focus** Breanna and 3 friends need \$29 to buy a game. Breanna has \$5. Each friend will contribute an equal amount of money. Breanna wants to know how much money each friend should contribute. She uses a to represent this amount in dollars.
Breanna is trying to model and solve the equation: $3a + 5 = 29$



Breanna's solution is $a = 6$.
But, when she checks it, she notices that 3 times \$6 plus her \$5 is \$18 + \$5, which is \$23.
The money is not enough to buy the game.

a) Find Breanna's mistake.

b) Sketch the balance scales to model the correct solution.

c) Verify the solution.

13. Use a model to solve each equation. Verify the solution.

a) $-2x + 3 = 13$

b) $-2x - 3 = -13$

c) $2x - 3 = -13$

d) $2x + 3 = -13$

14. Roger brings 4 cakes for dessert to the community potluck feast and powwow. His son brings 2 individual servings of dessert. Altogether, there will be 34 people at the feast. Each person has 1 serving of dessert.

a) Choose a variable to represent the number of pieces into which each cake must be cut.
Write an equation to describe this situation.

b) Use a model to solve the equation.

c) Verify the solution.
Show your work.

15. **Take It Further** A pattern rule for a number pattern is represented by $5 - 8n$, where n is the term number. What is the term number for each term value?

a) -3

b) -35

c) -155

16. **Take It Further**

a) Write an equation you could solve with balance scales and with algebra tiles.

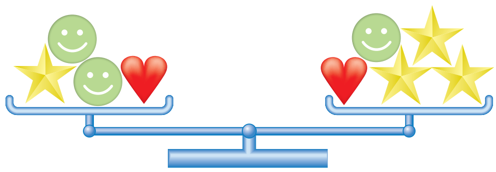
b) Write an equation you could solve with algebra tiles but not with balance scales.

c) Is there an equation that could be solved with balance scales but not with algebra tiles?

Justify your answer each time.

17. **Take It Further**

a) Write an equation in words to describe these balance scales.



b) i) The mass of a star is 11 g. What is the mass of a smiley face?
What is the mass of a heart?
Justify your answers.

ii) Compare the strategy you used in part a with that of another classmate. If the strategies are different, is one more efficient than the other?

c) Verify your answers. Write to explain your thinking.

Reflect

You have learned two models to solve an equation.
Are there situations where you prefer one model over the other?
Give an example.

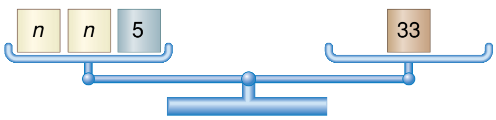
6.2 Solving Equations Using Algebra

Focus Solve a linear equation symbolically.

Investigate

Work with a partner to solve this problem.

Asuka invites some friends over to celebrate her spelling bee award.
Her mom has some veggie burgers, but she doesn't have enough.
Asuka's mom tells her that she needs twice as many veggie burgers, plus 5 more.
Her mom leaves to run errands, and Asuka has forgotten to ask how many veggie burgers her mom has.
There will be 33 people at the party—Asuka and 32 friends.
Asuka uses this diagram to model the situation:



What does n represent?
Write an equation for the balance-scales model.
How many different ways can you solve the equation?
Show each way.

Reflect & Share

Compare the equation you wrote with that of another pair of classmates.
If the equations are different, is each equation correct?
How can you find out?
Discuss the strategies you used to solve the equation.
If you did not solve the equation using algebra, work together to do that now.

Connect

We can solve an equation using algebra.
To help us visualize the equation, we think about a model, such as balance scales or algebra tiles.

To solve an equation, we need to isolate the variable on one side of the equation.
To do this, we get rid of the numbers on that side of the equation.

When we solve an equation using algebra, we must also preserve the equality.
Whatever we do to one side of the equation, we must do to the other side, too.

Example 1

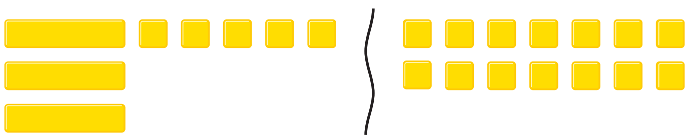
Fabian charges $3 for each bag of leaves he rakes, and $5 for mowing the lawn.
On Sunday, Fabian mowed 1 lawn and raked leaves. He earned $14.
How many bags of leaves did Fabian rake?

a) Write an equation to represent this problem.
b) Use algebra tiles to solve the equation.
   Use symbols to record each step.
c) Verify the solution using algebra.

A Solution

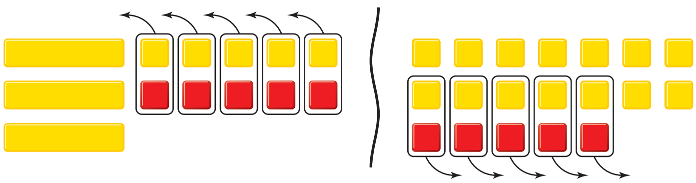
a) Let b represent the number of bags of leaves Fabian raked.
   An equation is: $3b + 5 = 14$

b) Model the equation with tiles.



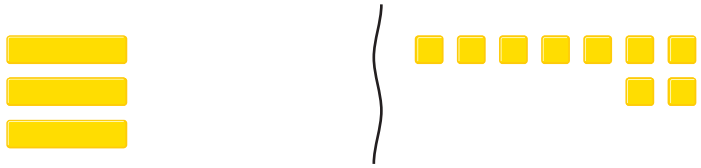
$3b + 5 = 14$

Add 5 negative unit tiles to each side to isolate the variable.
Remove zero pairs.



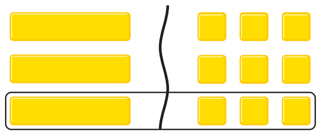
$3b + 5 - 5 = 14 - 5$

These tiles remain.



$3b = 9$

Divide each side into 3 equal groups.



$$\frac{3b}{3} = \frac{9}{3}$$
$$b = 3$$

Fabian raked 3 bags of leaves.

c) To verify the solution, substitute $b = 3$ into $3b + 5 = 14$.

Left side $= 3b + 5$ Right side $= 14$

$= 3(3) + 5$

$= 9 + 5$

$= 14$

Since the left side equals the right side, $b = 3$ is correct.

Fabian raked 3 bags of leaves.

It can be inconvenient to model an equation using algebra tiles when large numbers are involved, or when the solution is a fraction or a decimal.

Example 2

a) Use algebra to solve the equation:
$16t - 69 = -13$

b) Verify the solution.

A Solution

a) $16t - 69 = -13$

To isolate the variable term, add 69 to each side.

$16t - 69 + 69 = -13 + 69$

$16t = 56$

To isolate the variable, divide each side by 16.

$\frac{16t}{16} = \frac{56}{16}$

At this stage in the solution, we can continue in 2 ways.

As a fraction:

$t = \frac{56}{16}$

Divide by the common factor 8.

$t = \frac{56 \div 8}{16 \div 8}$

$t = \frac{7}{2}$

As a decimal:

$t = \frac{56}{16}$

Use a calculator.

$t = 3.5$

b) To verify the solution, substitute $t = \frac{7}{2}$ into:

$$\begin{aligned} 16t - 69 &= -13 \\ \text{Left side} &= 16t - 69 \\ &= 16\left(\frac{7}{2}\right) - 69 \\ &= {}^{8}\cancel{16}\left(\frac{7}{\cancel{2}_{1}}\right) - 69 \\ &= 56 - 69 \\ &= -13 \\ \text{Right side} &= -13 \end{aligned}$$

Since the left side equals the right side, $t = \frac{7}{2}$ is correct.

To verify the solution, substitute $t = 3.5$ into:

$$\begin{aligned} 16t - 69 &= -13 \\ \text{Left side} &= 16t - 69 \\ &= 16(3.5) - 69 \\ &= 56 - 69 \\ &= -13 \\ \text{Right side} &= -13 \end{aligned}$$

Since the left side equals the right side, $t = 3.5$ is correct.

Discuss the ideas

1. When is it easier to solve an equation using algebra instead of using algebra tiles or a balance-scales model?
2. When is it easier to verify a solution using substitution instead of algebra tiles or a balance-scales model?
3. When you solve an equation using algebra, why do you add or subtract the same number on each side? Why do you divide each side by the coefficient of the variable term?
4. Suppose the solution to an equation is a fraction or a decimal. Which would you prefer to use to verify the solution? Give reasons for your answer.

Practice

Check

5. Model each equation. Then solve it using concrete materials.
Use algebra to record each step you take. Verify each solution.

a) $2x - 1 = 7$
b) $11 = 4a - 1$
c) $5 + 2m = 9$
d) $1 = 10 - 3x$
e) $13 - 2x = 5$
f) $3x - 6 = 12$

6. Use algebra to solve each equation. Verify the solution.

a) $4x = -16$
b) $12 = -3x$
c) $-21 = 7x$
d) $6x = -30$

7. Check each student's work.
Rewrite a correct and complete algebraic solution where necessary.

a) Student A:

$$\begin{aligned} -3x + 15 &= 30 \\ -3x + 15 - 15 &= 30 + 15 - 15 \\ -3x &= 30 \\ \frac{-3x}{-3} &= \frac{30}{-3} \\ x &= -10 \end{aligned}$$

b) Student B:

$$\begin{aligned} 7 &= 1 + 2n \\ 7 - 1 &= 1 - 1 + 2n \\ 8 &= 2n \\ \frac{8}{2} &= \frac{2n}{2} \\ 4 &= n \\ n &= 4 \end{aligned}$$

c) Student C:

$$\begin{aligned} 3 + 2t &= 4 \\ 3 + 2t - 3 &= 4 - 3 \\ 2t &= 1 \\ t &= 2 \end{aligned}$$

d) Student D:

$$\begin{aligned} -5 &= -8 + 5f \\ -5 + 8 &= -8 + 8 + 5f \\ 3 &= 5f \\ \frac{3}{5} &= \frac{5f}{5} \\ \frac{3}{5} &= f \\ f &= \frac{3}{5} \end{aligned}$$

Apply

8. Solve each equation.
Verify the solution.

a) $2x + 5 = -7$ **b)** $-3x + 11 = 2$
c) $-9 = 5 + 7x$ **d)** $18 = -4x + 2$

9. Navid now has \$72 in her savings account. Each week she will save \$24. After how many weeks will Navid have a total savings of \$288?

a) Write an equation you can use to solve the problem.
b) Solve the equation.
When will Navid have \$288 in her savings account?
c) Verify the solution.

10. **Assessment Focus**

The Grade 8 students had an end-of-the-year dance.
The disc jockey charged \$85 for setting up the equipment, plus \$2 for each student who attended the dance.
The disc jockey was paid \$197.
How many students attended the dance?

a) Write an equation you can use to solve the problem.

b) Solve the equation.

c) Check your answer and explain how you know it is correct.

11. Solve each equation.
Verify your solution.

a) $-8x + 11 = 59$ b) $11c + 21 = -34$

c) $23 = -5b + 3$ d) $-45 = 6a - 15$

e) $52 = 25 - 9f$ f) $-13 + 4d = 31$

12. Solve each equation.
Verify your solution.

a) $3n + 7 = 8$ b) $6x + 6 = 15$

c) $-23 = 5p - 27$ d) $5p + 6 = 7$

e) $8e - 9 = -3$ f) $-17 + 10g = -9$

13. The high temperature today is 7°C higher than twice the high temperature yesterday.
The high temperature today is –3°C. What was the high temperature yesterday?

a) Write an equation you can use to solve the problem.

b) Solve the equation.
Verify the solution.

14. **Take It Further**

Use this information.
Boat rental: \$300
Fishing rod rental: \$20

a) Write a problem that can be solved using an equation.

b) Write the equation, then solve the problem.

c) How could you have solved the problem without writing an equation? Explain.

15. **Take It Further**

Use this information:
Water is pumped out of a flooded basement at a rate of 15 L/min.

a) Write a problem that can be solved using an equation.

b) Write, then solve, the equation.

Reflect

Which types of equations do you prefer to solve using algebra?
Explain why you may not want to use algebra tiles or a balance-scales model.

6.3 Solving Equations Involving Fractions

Focus Solve an equation that involves a fraction.

Which number could you multiply $\frac{3}{7}$ by to get the product 3?

Which number could you multiply $\frac{5}{6}$ by to get the product 5?
How did you find out?

Investigate

Work with a partner.

A fair comes to Behchoko, NWT.
Nicole has some tickets for the midway.
She shares the tickets with 2 friends so that they have 9 tickets each.
How many tickets did Nicole begin with?

Let t represent the number of tickets Nicole began with.
Write an equation you can use to solve for t.
Use any strategy to solve the problem.

Reflect & Share

Compare the equation you wrote with that of another pair of classmates.

If the equations are different, are both equations correct?
How do you know?

If you did not write an equation that involved division, work together to do so now.
What strategy would you use to solve this equation?
How could you check your answer?

Equations that involve fractions cannot be easily modelled with algebra tiles or balance scales.
We can write and solve these equations using algebra.

Example 1

Grandpa has enough gift certificates to give the same number to each of his 4 grandchildren.
After Grandpa gives them the gift certificates, each grandchild has 5 gift certificates.
How many gift certificates does Grandpa have?

a) Write an equation to represent this problem.
b) Solve the equation.
c) Verify the solution.

A Solution

a) Let n represent the number of gift certificates Grandpa has.
He shares them equally among 4 grandchildren.
Each grandchild will receive $\frac{n}{4}$ gift certificates.
Each grandchild will then have 5 gift certificates.
One possible equation is: $\frac{n}{4} = 5$

b) Solve the equation using algebra.

$\frac{n}{4} = 5$ To isolate the variable, multiply each side by 4.

$$\frac{n}{4} \times 4 = 5 \times 4$$

$$n = 20$$

Grandpa has 20 gift certificates.

c) To verify the solution,
substitute $n = 20$ into $\frac{n}{4} = 5$.

Left side $= \frac{n}{4}$ Right side $= 5$

$= \frac{20}{4}$

$= 5$

Since the left side equals the right side, $n = 20$ is correct.
Grandpa has 20 gift certificates.

Example 2

The school's student council sold T-shirts for charity.
The council bought the T-shirts in boxes of 40.
The student council added \$6 to the cost of each T-shirt.
Each T-shirt sold for \$26.
What did the student council pay for 1 box of T-shirts?

a) Write an equation to represent this problem.
Solve the equation.

b) Verify the solution.

A Solution

a) Let c dollars represent what the student council paid for 1 box of T-shirts.
The cost of each T-shirt the student council bought was: $\frac{c}{40}$
\$6 was added to the cost of each T-shirt: $\frac{c}{40} + 6$
Each T-shirt sold for \$26.
So, an equation is: $\frac{c}{40} + 6 = 26$

Solve the equation using algebra.

$\frac{c}{40} + 6 = 26$ To isolate the variable term, subtract 6 from each side.

$\frac{c}{40} + 6 - 6 = 26 - 6$

$\frac{c}{40} = 20$ To isolate the variable, multiply each side by 40.

$\frac{c}{40} \times 40 = 20 \times 40$

$c = 800$

The student council paid \$800 for 1 box of T-shirts.

b) To verify the solution,
substitute $c = 800$ into $\frac{c}{40} + 6 = 26$.

Left side $= \frac{c}{40} + 6$ Right side = 26

$= \frac{800}{40} + 6$

$= 20 + 6$

$= 26$

Since the left side equals the right side, $c = 800$ is correct.

Discuss the ideas

1. Why would it be difficult to model and solve equations like those in *Examples 1* and *2* with algebra tiles or balance scales?
2. In *Example 2*, why did we subtract 6 from each side before we multiplied by 40?

Practice

Check

3. Solve each equation.
   Verify the solution.
   a) $\frac{t}{5} = 6$ b) $\frac{a}{7} = 8$
   c) $\frac{b}{6} = 3$ d) $\frac{c}{3} = 9$

4. Solve each equation.
   Verify the solution.
   a) $\frac{d}{-4} = 5$ b) $\frac{f}{8} = -5$
   c) $\frac{k}{9} = -4$ d) $\frac{m}{-5} = -7$

5. One-quarter of the golf balls in the bag are yellow.
   There are 8 yellow golf balls.
   How many golf balls are in the bag?
   a) Write an equation you can use to solve the problem.
   b) Solve the equation.
   c) Verify the solution.

6. For each sentence, write an equation.
   Solve the equation to find the number.
   a) A number divided by 6 is 9.
   b) A number divided by -4 is -3.
   c) A number divided by -5 is 7.

7. Solve each equation.
   Verify the solution.
   a) $\frac{n}{4} + 3 = 10$ b) $\frac{m}{3} - 2 = 9$
   c) $13 + \frac{x}{2} = 25$ d) $-9 + \frac{s}{2} = 2$

Apply

8. Solve each equation.
   Verify the solution.
   a) $\frac{p}{-3} + 9 = 3$ b) $\frac{t}{-6} + 12 = 18$
   c) $-24 + \frac{w}{5} = -29$ d) $-17 + \frac{e}{-7} = -8$

9. For each sentence, write an equation.
   Solve the equation to find the number.
   a) Add 1 to a number divided by -3 and the sum is 6.
   b) Subtract a number divided by 9 from 3 and the difference is 0.
   c) Add 4 to a number divided by -2 and the sum is -3.

10. One-half of the team's supply of baseballs was taken from the dressing room to the dugout. During the game, 11 baseballs were caught by fans. At the end of the game, there were 12 baseballs left in the dugout. What was the team's original supply of baseballs?
    a) Write an equation you can use to solve the problem.
    b) Solve the equation.
    c) Verify the solution.

11. **Assessment Focus** Five students in Mrs. Lamert's tutorial class after school are solving equations. She brought a bag of treats. Mrs. Lamert explained that if the 5 students shared the bag of treats equally, then gave one treat each to the teacher, each student would still have 9 treats. How many treats were in the bag? Here is the equation Jerry suggested: $\frac{n}{5} - 1 = 9$

a) Is Jerry's equation correct? Explain why or why not.

b) If your answer to part a is yes, solve the equation using algebra. If your answer to part a is no, correct the equation, then solve the equation using algebra.

c) Verify the solution.

12. One-third of the Grade 8 students went to the track-and-field meet. Five track coaches went too. There were 41 people on the bus, not including the driver. How many students are in Grade 8?

a) Write an equation you can use to solve the problem.

b) Solve the equation.

c) Verify the solution.

13. Check each student's work below. Rewrite a correct and complete algebraic solution where necessary.

a) Student A:

$$\frac{h}{-9} = 4$$
$$\frac{h}{-9} \times (-9) = 4 \times (-9)$$
$$\frac{-9h}{-9} = -36$$
$$h = -36$$

b) Student B:

$$\frac{t}{8} - 2 = 4$$
$$8 \times \frac{t}{8} - 2 = 4 \times 8$$
$$\frac{8t}{8} - 2 = 32$$
$$t - 2 = 32$$
$$t - 2 + 2 = 32 + 2$$
$$t = 34$$

c) Student C:

$$\frac{r}{-4} + 3 = 13$$
$$\frac{r}{-4} + 3 - 3 = 13 - 3$$
$$\frac{r}{-4} = 10$$
$$4 \times \frac{r}{-4} = 10 \times 4$$
$$r = 40$$

14. **Take It Further** Jonah used the equation $3 + \frac{n}{7} = 18$ to solve a word problem.

a) What might the word problem be?

b) Solve the problem.

c) Verify the solution.

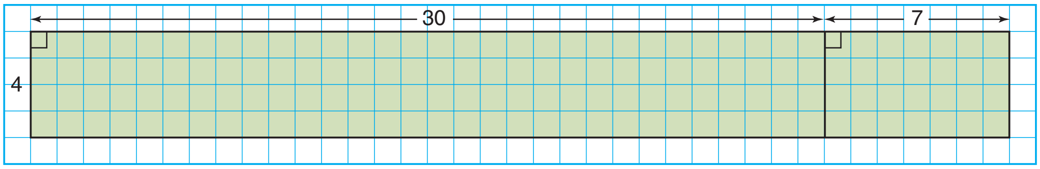
Reflect

How did your knowledge of multiplying fractions help you in this lesson? Include examples in your explanation.

6.4 The Distributive Property

Focus Relate the distributive property to algebra.

How does this diagram model the product of 4×37?
What is the product?



Investigate

Work with a partner.

Gina's Trattoria Ristorante has only round tables.
Each table seats 5 people.
Some tables are in the lounge and some are on the patio.

➤ There are 20 tables in the lounge and 8 tables on the patio.
Draw a diagram to show how to calculate the total number of people who can be seated.

➤ Gina removes some tables from the lounge.
Use a variable to represent the number of tables that remain.
Write an algebraic expression for the number of people the restaurant can now seat.

Reflect & Share

Compare your diagram and expression with those of another pair of classmates.
If your diagrams and expressions are different, are both of them correct? How can you check?
If your expressions are the same, work together to write a different expression for the number of people the restaurant can now seat.

Connect

➤ A charity sells pots of flowers for \$10 each to raise money.
8 people pay with a \$10 bill.
5 people pay with a \$10 cheque.
The total amount of money collected, in dollars, is:

- Add the total number of bills and cheques, then multiply by 10:
  $10 \times (8 + 5) = 130$

Or

- Multiply the number of bills by 10 and the number of cheques by 10, then add:
  $10 \times 8 + 10 \times 5 = 130$

We write: $10 \times (8 + 5) = 10 \times 8 + 10 \times 5$

We can use the same strategy to write two equivalent expressions for any numbers of bills and cheques.

➤ Suppose you are selling event tickets for \$20 each.
Some people pay with a \$20 bill and some pay with a \$20 cheque.
Suppose b is the number of \$20 bills you receive,
and c is the number of \$20 cheques you receive.
The total amount of money you collect, in dollars, is:

- Add the total number of bills and cheques, then multiply by 20:
  $20 \times (b + c)$

Or

- Multiply the number of bills by 20 and the number of cheques by 20, then add:
  $20b + 20c$

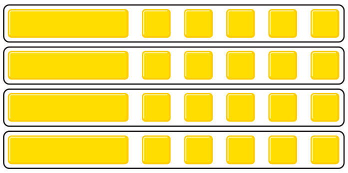
We write: $20(b + c) = 20b + 20c$

Multiplication has been *distributed* over addition.

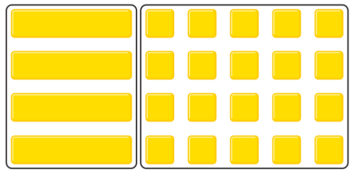
➤ We can also model the distributive property with algebra tiles.

For example:

To model $4(x + 5)$, you need 4 groups of 1 positive variable tile and 5 positive unit tiles.

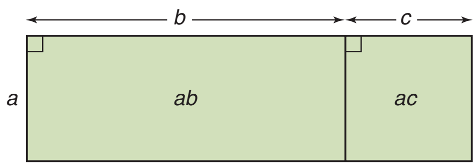


To model $4x + 20$, you need the same tiles, but their arrangement is grouped differently.



We can see that $4(x + 5) = 4x + 20$ because the two diagrams show the same numbers of tiles.

When an expression for the distributive property uses only variables, we can illustrate the property with this diagram.



$a(b + c) = ab + ac$
That is, the product $a(b + c)$ is equal to the sum $ab + ac$.

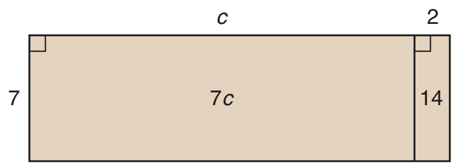
Example 1

Use the distributive property to write each expression as a sum of terms.
Sketch a diagram in each case.

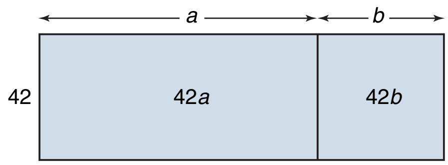
a) $7(c + 2)$ **b)** $42(a + b)$

A Solution

a) $7(c + 2) = 7(c) + 7(2)$
$= 7c + 14$



b) $42(a + b) = 42(a) + 42(b)$
$= 42a + 42b$



In *Example 1*, when we use the distributive property, we **expand**.

We can also use the distributive property with any integers.

Example 2

Expand.

a) $-3(x + 5)$

b) $-4(-5 + a)$

A Solution

a) $-3(x + 5) = -3(x) + (-3)(5)$
$= -3x - 15$

b) $-4(-5 + a) = -4(-5) + (-4)(a)$
$= 20 - 4a$

Subtraction can be thought of as "adding the opposite."
For example, $3 - 4 = 3 + (-4)$

Example 3

Expand.

a) $6(x - 3)$

b) $5(8 - c)$

A Solution

Rewrite each expression using addition.

a) $6(x - 3) = 6[x + (-3)]$
$= 6(x) + 6(-3)$
$= 6x - 18$

b) $5(8 - c) = 5[8 + (-c)]$
$= 5(8) + 5(-c)$
$= 40 - 5c$

Discuss the ideas

1. Look at the algebra tile diagrams on page 340.
Why are the tiles grouped in different ways?

2. Can you draw a diagram to model each product in *Example 2*?
Justify your answer.

3. Do you think the distributive property can be applied when there is a sum of 3 terms, such as $2(a + b + c)$?
Draw a diagram to illustrate your answer.

Practice

Check

4. Evaluate each pair of expressions.
What do you notice?

a) **i)** $7(3 + 8)$
ii) $7 \times 3 + 7 \times 8$

b) **i)** $5(7 - 2)$
ii) $5 \times 7 + 5 \times (-2)$

c) **i)** $-2(9 - 4)$
ii) $(-2) \times 9 + (-2) \times (-4)$

5. Use algebra tiles to show that $5(x + 2)$ and $5x + 10$ are equivalent.
Draw a diagram to record your work.
Explain your diagram in words.

6. Draw a rectangle to show that $7(4 + s)$ and $28 + 7s$ are equivalent.
Explain your diagram in words.

7. Expand.

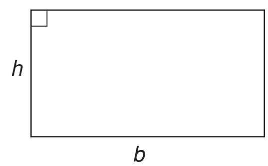
a) $2(x + 10)$ **b)** $5(a + 1)$
c) $10(f + 2)$ **d)** $6(12 + g)$
e) $8(8 + y)$ **f)** $5(s + 6)$
g) $3(9 + p)$ **h)** $4(11 + r)$
i) $7(g + 15)$ **j)** $9(7 + h)$

Apply

8. Expand.

a) $3(x - 7)$ **b)** $4(a - 3)$
c) $9(h - 5)$ **d)** $7(8 - f)$
e) $5(1 - s)$ **f)** $6(p - 2)$
g) $8(11 - t)$ **h)** $2(15 - v)$
i) $10(b - 8)$ **j)** $11(c - 4)$

9. Write two formulas for the perimeter, P, of a rectangle.
Explain how the formulas illustrate the distributive property.



10. Explain how you know $hb = bh$.
Use an example to justify your answer.

11. Which expression is equal to $9(6 - t)$?
How do you know?

a) $54 - 9t$
b) $96 - 9t$
c) $54 - t$

12. Expand.

a) $-6(c+4)$ b) $-8(a-5)$
c) $10(f-7)$ d) $3(-8-g)$
e) $-8(8-y)$ f) $-2(-s+5)$
g) $-5(-t-8)$ h) $-9(9-w)$

13. **Assessment Focus** Which pairs of expressions are equivalent?
Explain your reasoning.

a) $2x+20$ and $2(x+20)$
b) $3x+7$ and $10x$
c) $6+2t$ and $2(t+3)$
d) $9+x$ and $x+9$

14. There are 15 players on the Grade 8 baseball team. Each player needs a baseball cap and a team jersey.
A team jersey costs \$25. A baseball cap costs \$14.

a) Write 2 different expressions to find the cost of supplying the team with caps and jerseys.
b) Evaluate each expression. Which expression did you find easier to evaluate? Explain.

15. Five friends go to the movies.
They each pay \$9 to get in, and \$8 for a popcorn and drink combo.

a) Write 2 different expressions to find the total cost of the outing.
b) Evaluate each expression. Which expression was easier to evaluate? Justify your choice.

16. Match each expression in Column 1 with an equivalent expression in Column 2.

Column 1	Column 2
a) $6(t-6)$	i) $6t+36$
b) $-6(t-6)$	ii) $-6t+36$
c) $-6(t+6)$	iii) $-6t-36$
d) $6(6+t)$	iv) $6t-36$

17. **Take It Further**
Harvey won some money on a scratch-and-win ticket. Then, he won a \$2 bonus. When he arrived at the counter, he noticed that he had also won a "triple your winnings" ticket. As Harvey was cashing in his prize, the cashier told him he was the 100th customer, so his total winnings were automatically doubled.
Write two algebraic expressions to describe Harvey's winnings.

18. **Take It Further**

a) Expand.
   i) $7(5+y-2)$
   ii) $-3(-t+8-3)$
   iii) $-8(-9+s+5)$
   iv) $12(-10-p+7)$
b) Choose an expression in part a. How many different ways can you expand the expression? Show your work.

19. **Take It Further** Expand.

a) $2(7+b+c)$ b) $11(-6+e-f)$
c) $-(-r+s-8)$ d) $-10(-6-v-w)$
e) $5(j-15-k)$ f) $-4(-g+12-h)$

Reflect

How did your knowledge of operations with integers help you in this lesson?

6.5 Solving Equations Involving the Distributive Property

Focus Apply the distributive property to solve an equation.

The distributive property is needed to solve some algebraic equations.

Investigate

Work with a partner.

➤ Alison thought of her favourite number.
She subtracted 2.
Then Alison multiplied the difference by 5.
The product was 60.
What is Alison's favourite number?

Use any strategy to solve the problem.

➤ Write a similar number problem.
Trade problems with another pair of classmates.
Write an algebraic equation, then solve it.

Reflect & Share

Compare your strategy and answer with the same pair of classmates.
Are the equations the same?
How can you check the solution is correct?
How did you solve your classmates' problem?

Connect

These examples show how we can use the distributive property to solve some algebraic equations.

Example 1

John and Lorraine are landscaping their yard. They are buying pyramidal cedars that cost \$12 each. John and Lorraine need 11 cedars to shade their patio on two adjacent sides. They would like to purchase as many more cedars as they can for the far end of their lot. John and Lorraine have \$336 to buy cedars. How many more cedars can they buy?

a) Write an equation that models this problem.
b) Solve the equation.
c) Verify the solution.

A Solution

a) Let e represent how many more cedars John and Lorraine can buy.
Then they will buy a total of $(e + 11)$ cedars. Since the cedars are \$12 each, an equation is: $12(e + 11) = 336$

b)

$$\begin{aligned} 12(e + 11) &= 336 \\ 12(e) + 12(11) &= 336 \\ 12e + 132 &= 336 \\ 12e + 132 - 132 &= 336 - 132 \\ 12e &= 204 \\ \frac{12e}{12} &= \frac{204}{12} \\ e &= 17 \end{aligned}$$

Use the distributive property to remove the brackets.

Use a calculator.

John and Lorraine can buy 17 more cedars.

c) To verify the solution, substitute $e = 17$ into $12(e + 11) = 336$.

$$\begin{aligned} \text{Left side} &= 12(e + 11) \\ &= 12(17 + 11) \\ &= 12(28) \\ &= 336 \end{aligned}$$

Right side $= 336$

Since the left side equals the right side, $e = 17$ is correct.

Example 2

Solve: $14 = 3(x + 4)$
Verify the solution.

A Solution

$14 = 3(x + 4)$
Expand.

$$\begin{aligned}
14 &= 3(x + 4)\\
14 &= 3(x) + (3)(4)\\
14 &= 3x + 12\\
14 - 12 &= 3x + 12 - 12\\
2 &= 3x\\
\frac{2}{3} &= \frac{3x}{3}\\
\frac{2}{3} &= x\\
x &= \frac{2}{3}
\end{aligned}$$

To verify the solution, substitute $x = \frac{2}{3}$ into $14 = 3(x + 4)$.

Left side $= 14$ Right side $= 3(x + 4)$

$$\begin{aligned}
&= 3(\tfrac{2}{3} + 4)\\
&= 3(\tfrac{2}{3} + \tfrac{12}{3})\\
&= 3(\tfrac{14}{3})\\
&= 14
\end{aligned}$$

Since the left side equals the right side, $x = \frac{2}{3}$ is correct.

Discuss the ideas

1. How could you solve the equation in *Example 1* without using the distributive property?
   What is the first step in the solution?
2. Why would it be more difficult to use this method to solve the equation in *Example 2*?
3. For the solution in *Example 2*, could you use the decimal form of the fraction to verify the solution?
   Justify your answer.

Practice

Check

4. Solve each equation using the distributive property.
Verify the solution.

a) $3(x + 5) = 36$
b) $4(p - 6) = 36$
c) $5(y + 2) = 25$
d) $10(a + 8) = 30$

5. Solve each equation.
Verify the solution.

a) $-2(a + 4) = 18$
b) $-3(r - 5) = -27$
c) $7(-y + 2) = 28$
d) $-6(c - 9) = -42$

6. Marc has some hockey cards.
His friend gives him 3 more cards.
Marc says that if he now doubles the number of cards he has, he will have 20 cards. How many cards did Marc start with?

a) Choose a variable to represent the number of cards Marc started with. Write an equation to model this problem.
b) Solve the equation using the distributive property.
c) Verify the solution. Explain your thinking in words.

7. A student wrote this equation to solve the problem in question 6:
$2n + 3 = 20$
How would you explain to the student why this is incorrect?

Apply

8. The perimeter of a rectangle is 26 cm.
The rectangle has length 8 cm.
What is the width of the rectangle?

a) Write an equation that can be solved using the distributive property.
b) Solve the equation.
c) Verify the solution.

9. **Assessment Focus** The price of a souvenir T-shirt was reduced by \$5. Jason bought 6 T-shirts for his friends. The total cost of the T-shirts, before taxes, was \$90. What was the price of a T-shirt before it was reduced?

a) Write an equation to model this problem.
b) Solve the equation.
c) Verify the solution. Explain how you know it is correct.

10. Chuck and 7 friends went to Red Deer's Westerner Days fair. The cost of admission was \$6 per person. They each bought an unlimited midway ride ticket. The total cost of admission and rides for Chuck and his friends was \$264. What was the price of an unlimited midway ride ticket?

a) Write an equation to model this problem.
b) Solve the equation. Verify the solution.

11. Inge chose an integer. She added 9, then multiplied the sum by -5. The product was 15. Which integer did Inge choose?
   a) Write an equation you can use to solve the problem.
   b) Solve the equation.
   c) Verify the solution.

12. Mario chose an integer. He subtracted 7, then multiplied the difference by -4. The product was 36. Which integer did Mario choose?
   a) Write an equation you can use to solve the problem.
   b) Solve the equation.
   c) Verify the solution.

13. Kirsten used the distributive property to solve this equation: $8(-x + 3) = 8$
   a) Check Kirsten's work. Is her solution correct?

$$\begin{aligned} 8(-x + 3) &= 8 \\ 8(-x) + 8(3) &= 8 \\ -8x + 24 &= 8 \\ -8x + 24 - 24 &= 8 - 24 \\ -8x &= -16 \\ \frac{-8x}{-8} &= \frac{-16}{8} \\ x &= -2 \end{aligned}$$

   b) If your answer is yes, verify the solution. If your answer is no, describe the error, then correct it.

14. Solve each equation using the distributive property. Verify the solution.
   a) $-10 = 5(t - 2)$ b) $7 = 2(p - 3)$
   c) $4(r + 5) = 23$ d) $-3(s + 6) = 18$

15. **Take It Further**
   Amanda's office has 40 employees. The employees want to have a catered dinner. They have found a company that will provide what they need for \$25 per person. Amanda knows that some people will bring a guest. The company has budgeted \$1500 for this event. How many guests can they invite? Assume the price of \$25 includes all taxes.
   a) Write an equation for this problem.
   b) Solve the equation.
   c) Verify the solution.

16. **Take It Further**
   Glenn used the equation $7(n - 2) = 42$ to solve a word problem.
   a) Create a word problem that can be solved using this equation.
   b) Solve the problem. Verify the solution.

17. **Take It Further**
   Solve each equation using the distributive property. Verify the solution.
   a) $7(2 + p - 5) = 14$
   b) $8(x - 9 + 7) = -13$
   c) $-2(10 - s + 1) = -21$

Reflect

How do you know when to use the distributive property to help you solve an equation? Include examples in your explanation.

Make the Number

In this game, an ace is 1, a jack is 11, a queen is 12, and a king is 13.

HOW TO PLAY

1. Shuffle the cards.
   Place 4 cards face up. These are the "*seed*" *cards*.
   Place the remaining cards face down in a pile.
   Turn over the top card. This is the "*target*" *card*.

2. Each player chooses a "*secret integer*" between –9 and 9.
   Give the integer a name, such as *x*.

3. Use your secret integer, the "seed" cards, and any operations or brackets to write an expression equal to the "target" number.
   You may use up to 5 integers in your expression.

   You score 5 points for each "seed" card number used, and 2 additional points if you use a "seed" card number more than once.

4. When all players have written their equations, share the equations.
   Each player solves everyone else's equations.
   If a player writes an incorrect equation so the solution is not the secret integer, that player loses all the points scored for writing the equation.

5. Play continues. The player with the most points after 3 rounds wins.

YOU WILL NEED

A standard deck of cards with the jokers removed; paper and pencil

NUMBER OF PLAYERS

2 to 4

GOAL OF THE GAME

To get the most points

Example
"Seed" cards: 4♥, Q♠, 7♥, 7♣
"Target" card: 2♦
"Secret Integer": 5
Expression: $12(5 - 4) - 7 - 4 + 1$
Equation: $12(x - 4) - 7 - 4 + 1 = 2$
Score: 3×5 points (3 "seed" cards used) +
2 points (one "seed" card used twice)
= 17 points

Mid-Unit Review

LESSON

6.1

1. Use a model to solve each equation. Verify the solution.

a) $4x = -36$ **b)** $-7x = 63$

c) $4x + 7 = 19$ **d)** $-3x + 5 = 17$

2. Alice has some granola bars in her backpack. If she triples the number of granola bars then adds 4, she will get 13. How many granola bars does Alice have?

a) Choose a variable. Write an equation for this situation.

b) Use a model to solve the equation.

c) Verify the solution. Show how you did this.

6.2

3. Solve each equation. Verify the solution.

a) $4x + 9 = -27$ **b)** $-5x + 8 = 23$

c) $3x - 4 = -3$ **d)** $10 = 6x + 5$

4. The school's sports teams held a banquet. The teams were charged $125 for the rental of the hall, plus $12 for each meal served. The total bill was $545. How many people attended the banquet?

a) Write an equation you could use to solve the problem.

b) Solve the equation. Verify the solution.

6.3

5. Solve each equation. Verify the solution.

a) $\frac{n}{4} = -8$ **b)** $\frac{m}{3} - 2 = 3$

c) $\frac{b}{-3} = 6$ **d)** $\frac{f}{-8} + 8 = 12$

6. For each sentence, write an equation. Solve the equation to find the number.

a) A number divided by –7 is 4.

b) A number divided by –9 is –3.

c) Add 5 to a number divided by –2 and the sum is 0.

6.4

7. Draw a rectangle to show that: $6(3 + a) = 18 + 6a$

8. Expand.

a) $3(x + 11)$ **b)** $5(12 + y)$

c) $-7(a - 4)$ **d)** $-12(-t + 6)$

6.5

9. Use the distributive property to solve each equation. Verify the solution.

a) $3(x + 2) = 21$ **b)** $4(p - 3) = 16$

c) $-5(r + 4) = -15$ **d)** $6(-s - 3) = 24$

10. Jon is playing a game. He starts with some points. On his first turn, Jon wins 6 points. On his second turn, Jon's points are doubled. He then has 26 points. How many points did Jon start with?

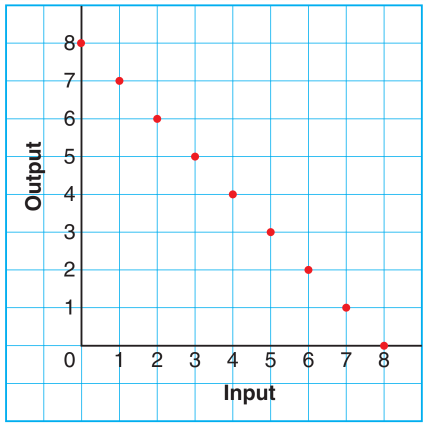
a) Write an equation to model this problem.

b) Solve the equation. Verify the solution.

6.6 Creating a Table of Values

Focus Create a table of values from the equation of a linear relation.

How many different ways can you describe the relation shown in this graph?



Investigate

Work with a partner.

At the country fair, Mischa sells hot dogs for \$3 each, and drinks for \$2 each. A meal consists of hot dogs and one drink.

The number of hot dogs in a meal, h, is related to the total cost of the meal in dollars. The relation is: h is related to $3h + 2$.

➤ Copy and complete the a table of values for the relation.

Input h	Output $3h + 2$
1	
2	

➤ How can you use the table of values to find:
- the cost of a meal when a person orders 9 hot dogs?
- the number of hot dogs ordered when a meal costs \$35?

Reflect & Share

Compare your answers with those of another pair of classmates.
When you know the total cost of a meal, how can you determine the number of hot dogs ordered?
What helped you solve this problem?
What else can you find out using the table or the relation?
Work together to write, then answer, 3 questions about this relation.

Connect

We can represent a relation in different ways.
For example, consider this relation: x is related to $20 - 3x$

➤ We can create a table of values.

When $x = 1$,

$$\begin{aligned} 20 - 3x &= 20 - 3(1) \\ &= 20 - 3 \\ &= 17 \end{aligned}$$

When $x = 2$,

$$\begin{aligned} 20 - 3x &= 20 - 3(2) \\ &= 20 - 6 \\ &= 14 \end{aligned}$$

A table of values is:

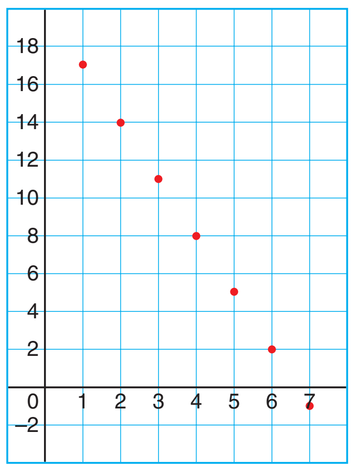
	Input x	Output $20 - 3x$	
	1	17	
+1	2	14	−3
+1	3	11	−3
+1	4	8	−3
+1	5	5	−3
+1	6	2	−3
+1	7	−1	−3

When the input increases by 1, the output decreases by 3.

When the change in the input is constant and the change in the output is constant, the relation is a **linear relation**.

➤ We can draw a graph.
Use the data in the table.
The input is plotted on the horizontal axis.
The output is plotted on the vertical axis.
On the horizontal axis, the scale is
1 square represents 1 unit.
On the vertical axis, the scale is
1 square represents 2 units.

Graph of $20 - 3x$ against x



➤ We can write an equation for the relation.
We introduce a second variable, y.
Then, an efficient way to write the relation x is related to $20 - 3x$ is:
$y = 20 - 3x$
We say the equation of the linear relation is $y = 20 - 3x$.

We write the table of values as:

x	1	2	3	4	5	6	7
y	17	14	11	8	5	2	–1

A related pair of x and y values is called an **ordered pair**.
Some ordered pairs for this relation are:
(1, 17), (2, 14), (3, 11), (4, 8), (5, 5), (6, 2), (7, –1), (x, y)

Example 1

Saskatoon Pizza charges $11 for a medium cheese pizza, plus $2 for each topping.
An equation for this relation is $c = 11 + 2n$, where n represents the number of toppings and c represents the cost of the pizza in dollars.

a) Use the equation to create a table of values.
b) Use the equation to find the cost of a pizza with 5 toppings. Check the answer.
c) Use the equation to find how many toppings are on a pizza that costs $27.

A Solution

a) Since it is possible to order a pizza with no toppings, start the table of values with $n = 0$.
When $n = 0$, the cost is:

$$\begin{aligned} c &= 11 + 2n \\ &= 11 + 2(0) \\ &= 11 + 0 \\ &= 11 \end{aligned}$$

A pizza with no toppings costs $11.

When $n = 1$,
$$\begin{aligned} c &= 11 + 2n \\ &= 11 + 2(1) \\ &= 11 + 2 \\ &= 13 \end{aligned}$$

When $n = 2$,
$$\begin{aligned} c &= 11 + 2n \\ &= 11 + 2(2) \\ &= 11 + 4 \\ &= 15 \end{aligned}$$

When $n = 3$,
$$\begin{aligned} c &= 11 + 2n \\ &= 11 + 2(3) \\ &= 11 + 6 \\ &= 17 \end{aligned}$$

A table of values is:

n	*c*
0	11
1	13
2	15
3	17

b) To find the cost of a pizza with 5 toppings, substitute $n = 5$.

$$\begin{aligned} c &= 11 + 2n \\ &= 11 + 2(5) \\ &= 11 + 10 \\ &= 21 \end{aligned}$$

A five-topping pizza will cost $21.
To check the answer, extend the table.
In the second column, the value of c increases by 2 each time.
So, when $n = 4, c = 19$; and when $n = 5, c = 21$.

c) To find out how many toppings are on a pizza that costs $27, substitute $c = 27$.

$$27 = 11 + 2n$$

Solve for n.

$$\begin{aligned} 27 - 11 &= 11 + 2n - 11 \\ 16 &= 2n \\ \frac{16}{2} &= \frac{2n}{2} \\ 8 &= n \end{aligned}$$

There are 8 toppings on a pizza that costs $27.

To check the answer, calculate the cost of a pizza with 8 toppings.
For 8 toppings, the cost is $11 plus $8 \times \$2$, which is $\$11 + \$16 = \$27$.
Since this matches the given cost, the number of toppings is correct.

Example 2

The equation of a linear relation is: $y = -5x - 3$
Some ordered pairs in the relation are:
$(0, -3), (1, -8), (2, -13), (3, \quad), (4, -23), (\quad, -28)$
Find the missing numbers in the ordered pairs.

A Solution

The first missing number is in the ordered pair $(3, \quad)$.
The missing number is the value of y when $x = 3$.
Substitute $x = 3$ in the equation $y = -5x - 3$.

$$\begin{aligned} y &= -5(3) - 3 \\ &= -15 - 3 \\ &= -18 \end{aligned}$$

The ordered pair is $(3, -18)$.
The second missing number is in the ordered pair $(\quad, -28)$.
The missing number is the value of x when $y = -28$.
Substitute $y = -28$ in the equation $y = -5x - 3$.

$$-28 = -5x - 3$$

Solve for x.

$$\begin{aligned} -28 + 3 &= -5x - 3 + 3 \\ -25 &= -5x \\ \frac{-25}{-5} &= \frac{-5x}{-5} \\ 5 &= x \end{aligned}$$

The ordered pair is $(5, -28)$.

Example 2 Another Solution

To find the missing number in $(3, \quad)$:
There is a pattern in the x-values.
So, list the y-values in order, and
look for a pattern in the y-values.
$-3, -8, -13, ?, -23, -28$
The numbers decrease by 5 each time.
So, the first missing number is: $-13 - 5 = -18$

To find the missing number in $(\quad, -28)$:
The first ordered pair is $(0, -3)$.
-28 is in the 6th ordered pair.
So the missing number is 5.

Discuss the ideas

1. Look at the tables of values in *Connect*.
   Give some examples of pairs of numbers that will never appear in these tables.
2. Why do you think the numbers (4, 8) are called an ordered pair, and not simply a pair?
3. To check that a solution to an equation is correct, you can either extend the table of values or substitute in the equation.
   a) When would it be easier to extend the table of values?
   b) When would it be easier to substitute?

Practice

Check

4. Copy and complete each table of values.

a) $y = x + 1$

x	y
1	
2	
3	
4	
5	

b) $y = x + 3$

x	y
1	
2	
3	
4	
5	

c) $y = 2x$

x	y
1	
2	
3	
4	
5	

5. Make a table of values for each relation.
   a) $y = 2x + 1$
   b) $y = 2x - 1$
   c) $y = -2x + 1$

6. The equation of a linear relation is:
   $y = 9x - 7$
   Some ordered pairs in the relation are:
   (0, –7), (1, 2), (2,), (3, 20),
   (4,), (, 38)
   Find the missing numbers in the ordered pairs.

7. Melanie earns $7 per hour when she baby-sits. An equation for this relation is $w = 7h$, where h represents the number of hours and w represents Melanie's wage in dollars.
   a) Use the equation to create a table of values.
   b) In one week, Melanie earned $105. How many hours did she baby-sit?
   c) In one month, Melanie baby-sat for 24 h. How much did she earn from baby-sitting in that month?

Apply

8. Copy and complete each table of values.

a) $y = x + 2$

x	y
–3	
–2	
–1	
0	
1	
2	
3	

b) $y = x - 3$

x	y
–3	
–2	
–1	
0	
1	
2	
3	

c) $y = x + 4$

x	y
–3	
–2	
–1	
0	
1	
2	
3	

9. Make a table of values for each relation.

a) $y = -2x + 3$

b) $y = -5x - 4$

c) $y = 8x - 3$

10. The equation of a linear relation is:
$y = -3x + 5$
Some ordered pairs in the relation are:
(–3, 14), (–1, 8), (1,), (3, –4),
(5,), (, –16)
Find the missing numbers in the ordered pairs.

11. The equation of a linear relation is:
$y = -2x + 7$
Find the missing number in each ordered pair.

a) (–8,) **b)** (12,)

c) (, 31) **d)** (, –23)

12. **Assessment Focus** Herbie has a mass of 100 kg.
His personal trainer sets a goal for him to lose 2 kg per month until he reaches his goal mass. An equation for this relation is $m = 100 - 2n$, where n represents the number of months and m represents Herbie's mass in kilograms.

a) Use the equation to create a table of values.

b) At some time, Herbie should have a mass of 60 kg. How many months will he have trained?

c) By his birthday, Herbie had trained for 7 months.
What was his mass then?

13. Candice is experimenting with divisibility rules. She can't remember the rule for "divisibility by 9", so she makes a table of values to study the multiples of 9.
She uses the equation $m = 9t$ to find multiples of 9.

a) In the equation $m = 9t$, what does m represent?
What does t represent?

b) Make a table of values for this equation.

c) What patterns do you see in the table?
Use these patterns to write a rule for divisibility by 9.

d) Is 126 divisible by 9?
How do you know?

e) What is the 17th multiple of 9?
How do you know?

14. **Take It Further**
These ordered pairs are in the same linear relation:
$(-2, -6)$, $(0, 2)$, $(2, 10)$, $(4, 18)$
The ordered pairs below are also in this relation. Find the missing number in each ordered pair.
Describe the strategy you used.

a) $(-4, \quad)$ **b)** $(\quad, -26)$
c) $(3, \quad)$ **d)** $(\quad, -2)$

15. **Take It Further**
These ordered pairs are in the same linear relation: $(0, -8)$, $(-4, -28)$, $(4, 12)$
The ordered pairs below are also in this relation. Find the missing number in each ordered pair.
Describe the strategy you used.

a) $(-2, \quad)$ **b)** $(\quad, -48)$
c) $(6, \quad)$ **d)** $(\quad, -3)$

Math Link

Science

Pressure is force per unit area.
Pressure is measured in pascals (Pa).
A formula for pressure is:

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

When we know the pressure in pascals and the area in square metres, we can use this formula to find the force newtons (N).

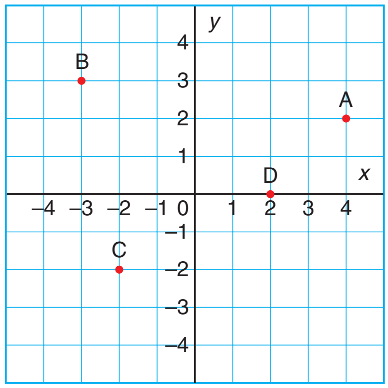
Reflect

You have learned 2 ways to find the missing number in an ordered pair.
Which strategy do you prefer?
When might you use one strategy rather than the other?

6.7 Graphing Linear Relations

Focus Construct a graph from the equation of a linear relation and describe the graph.

Look at the coordinate grid.
Point A has coordinates (4, 2).
What are the coordinates of Point B? Point C? Point D?



Investigate

Work with a partner.

Benny is designing banners for a school event. On a banner, the school motto is 50 cm long and the school logo is 20 cm long.

In each design, the banner has the school logo at each end, and the school motto in the middle. The motto can be repeated any number of times.

An equation that relates the length of the banner to the number of mottos is $\ell = 40 + 50n$, where ℓ is the length of the banner in centimetres with n mottos.

➤ Explain each term in the equation of the relation.
What does each term represent?
➤ Use the equation to make a table of values for the relation.
➤ Graph the relation.
➤ What can you find out from the table of values? From the graph?

Reflect & Share

Compare your table of values and graph with those of another pair of classmates.
Should you draw a line through the points? Justify your decision.
Is the relation linear? How can you tell?

Connect

Sylvia works at a garden nursery. She is paid \$6 for every tray of tomatoes she plants. Let n represent the number of trays Sylvia plants. Let p represent her pay in dollars. An equation that relates Sylvia's pay to the number of trays she plants is: $p = 6n$

Substitute values for n to find corresponding values of p.

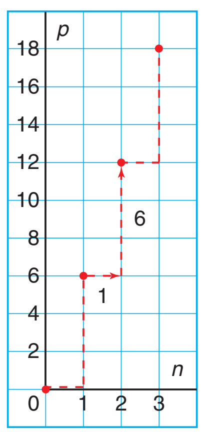
When $n = 0$, $p = 6(0)$
$= 0$

When $n = 1$, $p = 6(1)$
$= 6$

Here is a table of values.

n	p
0	0
1	6
2	12
3	18

+1 +1 +1 / +6 +6 +6



To graph the relation, plot n along the horizontal axis and p along the vertical axis.
Label the axes and write the equation of the relation on the graph.
The points lie on a straight line, so the relation is linear.
Since Sylvia only gets paid for whole numbers of trays planted, we do not join the points. For example, Sylvia is *not* paid for 1.5 trays planted.
These data are **discrete**. When data are discrete, there are numbers between those given that are not meaningful in the context of the problem.

The graph shows that for every tray Sylvia plants, her pay increases by \$6. As the number of trays increases, so does her pay.

Example 1

A Grade 8 class is going on a field trip. The bus seats 24 students. An equation that relates the number of boys on the bus to the number of girls is $b = 24 - g$, where g represents the number of girls and b represents the number of boys.

a) Create a table of values for the relation.

b) Graph the relation.

c) Describe the relationship between the variables in the graph.

A Solution

a) Substitute values for g to find corresponding values of b.

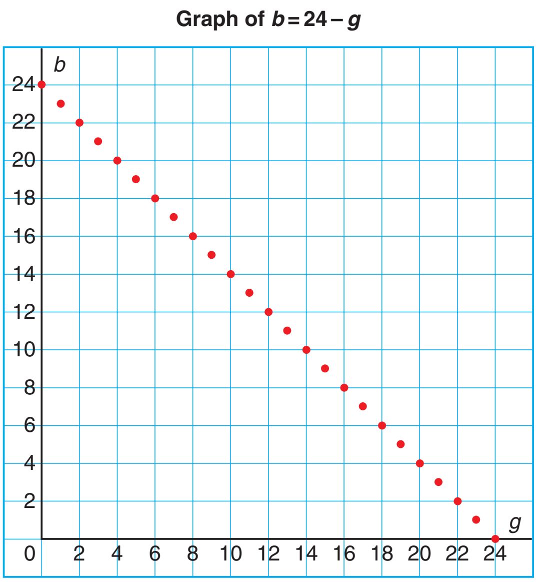
When $g = 0$, $b = 24 - 0$
$= 24$

When $g = 1$, $b = 24 - 1$
$= 23$

A table of values is:

g	b
0	24
1	23
2	22
3	21
4	20
...	...
24	0

b)



c) The variables represent the number of boys and the number of girls.
As the number of girls increases by 1, the number of boys decreases by 1.
The graph begins and ends at 24 on each axis. It is not possible to have more than either 24 boys or 24 girls on the bus.

Example 2

The equation of a linear relation is: $y = -4x + 1$

a) Create a table of values for the relation for integer values of x from -4 to 4.

b) Graph the relation.

c) Describe the relationship between the variables in the graph.

A Solution

a) When $x = -4$,
$$\begin{aligned} y &= -4x + 1 \\ &= -4(-4) + 1 \\ &= 16 + 1 \\ &= 17 \end{aligned}$$

When $x = -3$,
$$\begin{aligned} y &= -4x + 1 \\ &= -4(-3) + 1 \\ &= 12 + 1 \\ &= 13 \end{aligned}$$

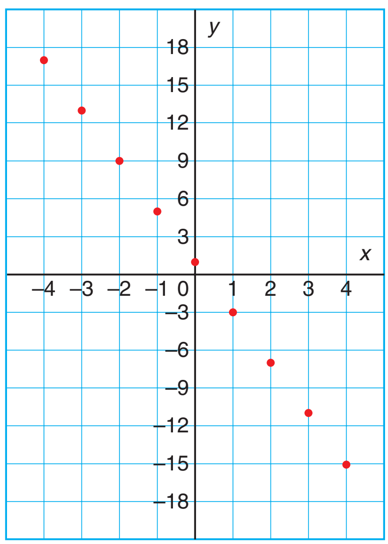
When $x = -2$,
$$\begin{aligned} y &= -4x + 1 \\ &= -4(-2) + 1 \\ &= 8 + 1 \\ &= 9 \end{aligned}$$

A table of values is:

x	y
–4	17
–3	13
–2	9
–1	5
0	1
1	–3
2	–7
3	–11
4	–15

b)

Graph of $y = -4x + 1$



c) The variables are x and y.
When x increases by 1, y decreases by 4.
The points lie on a line that goes down to the right.

Discuss the ideas

1. In *Example 1*, is it possible to have negative values for b and g? Justify your answer.
2. In *Example 1*, the relation is linear and the points lie on a line. Why did we not draw a line through the points?
3. In *Example 1*, the equation for the relation could have been written as $g = 24 - b$.
How would the table of values change?
How would the graph change?

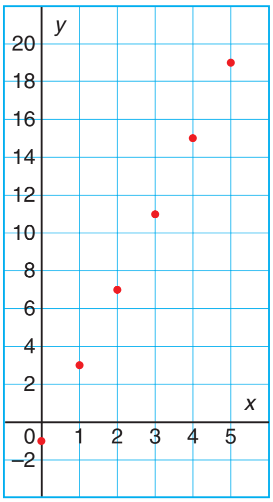
Practice

Check

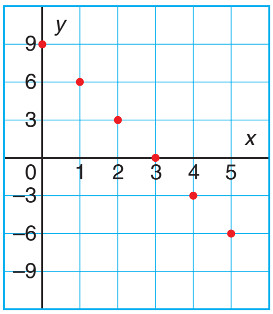
You will need grid paper.

4. Each graph below is a graph of a linear relation. Describe the relationship between the variables in each graph.
   a) $y = 4x - 1$ b) $y = -3x + 9$

Graph of $y = 4x - 1$



Graph of $y = -3x + 9$



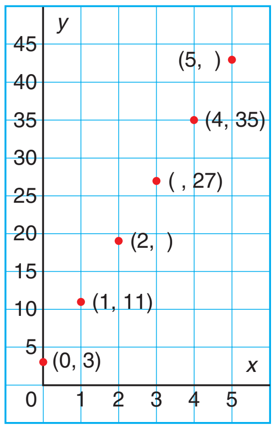
5. Graph each relation for integer values of x from 0 to 5.
   a) $y = 2x$ b) $y = 3x$
   c) $y = 4x$ d) $y = 5x$
   e) $y = -2x$ f) $y = -3x$
   g) $y = -4x$ h) $y = -5x$

6. Graph each relation for integer values of x from 0 to 5.
   a) $y = 2x + 1$ b) $y = 2x - 1$
   c) $y = -2x + 1$ d) $y = -2x - 1$
   e) $y = 3x + 1$ f) $y = 3x - 1$
   g) $y = -3x + 1$ h) $y = -3x - 1$

Apply

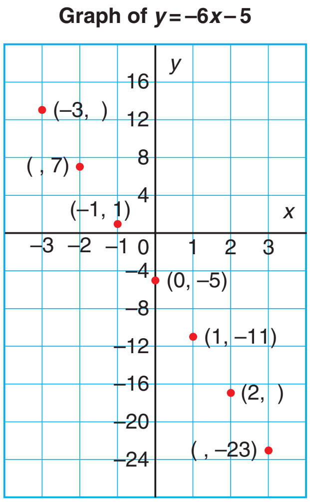
7. Here is a graph of the linear relation $y = 8x + 3$.

Graph of $y = 8x + 3$



Each point on the graph is labelled with an ordered pair.
Some numbers in the ordered pairs are missing. Find the missing numbers.
Explain how you did this.

8. Here is a graph of the linear relation $y = -6x - 5$.



Each point on the graph is labelled with an ordered pair.
Some numbers in the ordered pairs are missing. Find the missing numbers.
Explain how you did this.

9. Use the data from *Example 1*, page 361.
An equation for the linear relation is:
$c = 11 + 2n$,
where n is the number of toppings on the pizza, and c is the total cost of the pizza in dollars. Here is a table of values.

n	0	1	2	3	4	5	6	7	8
c	11	13	15	17	19	21	23	25	27

a) Construct a graph for the data.
b) Describe the relationship between the variables in the graph.
c) Find the ordered pair on the graph that shows the cost of a pizza with 6 toppings.

10. Use the data from Lesson 6.6 *Practice* question 12, page 357.
An equation for the linear relation is:
$m = 100 - 2n$,
where n is the number of months that Herbie trains and m is his mass at any time in kilograms.
Here is a table of values.

n	0	2	4	6	8	10
m	100	96	92	88	84	80

a) Construct a graph for the data.
b) Describe the relationship between the variables in the graph.
c) Find the ordered pair on the graph that indicates Herbie's mass after 7 months. Explain how you did this.

11. **Assessment Focus** Regina plans a marshmallow roast. She will buy 8 marshmallows for each person who attends, and 12 extra marshmallows in case someone shows up unexpectedly.
Let n represent the number of people who attend. Let m represent the number of marshmallows Regina must buy. An equation that relates the number of marshmallows to the number of people is: $m = 8n + 12$

a) Create a table of values for the relation.
b) Graph the relation.
c) Describe the relationship between the variables in the graph.
d) Is the relation linear? How do you know?

12. Graph each relation for integer values of x from -4 to 4.

a) $y = 8x + 2$ b) $y = -8x - 2$
c) $y = -7x + 4$ d) $y = 5x - 4$

13. Peter's Promoting is organizing a concert. The cost of the venue and the rock band is \$15 000. Each concert ticket sells for \$300. Peter's profit is the money he makes from selling tickets minus the cost. Let n represent the number of tickets sold. Let p represent Peter's profit. An equation that relates the profit to the number of tickets sold is:
$p = 300n - 15\ 000$

a) Create a table of values for the relation. Use these values of n: 10, 20, 30, 40, 50, 60, 70, 80
b) Graph the relation. What do negative values of p represent?
c) Describe the relationship between the variables in the graph.
d) How can you use the graph to find the profit when 75 tickets are sold?

14. Take It Further A computer repair company charges \$60 to make a house call, plus an additional \$40 for each hour spent repairing the computer. An equation that relates the total cost to the time in hours for a house call is $C = 60 + 40n$, where n represents the time in hours, and C represents the total cost of the house call in dollars.

a) Graph the relation.
b) Describe the relationship between the variables in the graph.
c) Does the point $(-1, 20)$ lie on the graph? What does this point represent? Does this point make sense in the context of the problem? Explain.

15. Take It Further

a) Graph each relation.
Describe the relationship between the variables in the graph.
i) $y = -9x + 4$ ii) $y = 6x - 3$
iii) $y = -7x - 2$ iv) $y = 4x + 11$
v) $y = 7x + 5$ vi) $y = 3x - 8$
vii) $y = -9x - 6$ viii) $y = -8x + 7$
b) Which graphs go up to the right? Which graphs go down to the right?
c) How can you use the equation of a linear relation to tell if its graph goes up to the right or down to the right?

Reflect

You now know these ways to represent a relation:
- table of values
- equation
- graph

Which way do you think tells you the most about the relation?

Using Spreadsheets to Graph Linear Relations

Focus Use technology to construct a graph from the equation of a linear relation.

Spreadsheet software can be used to create a table of values from the equation of a linear relation.
The software can then be used to graph the data in the table of values.

Chris and her family belong to an outdoor club.
They are able to rent ATVs at reasonable rates.
It costs \$55 to rent a "Rambler,"
plus \$3 for each hour rented.
It costs \$60 to rent a "Northern,"
plus \$2 for each hour rented.
Chris wants to rent an ATV for a few hours over the weekend.
She writes these equations to help her decide which ATV to rent.

Let h represent the number of hours.
Let c represent the total cost of the rental in dollars.
An equation for the total cost to rent a "Rambler" is:
$c = 55 + 3h$
An equation for the total cost to rent a "Northern" is:
$c = 60 + 2h$

To create a table of values

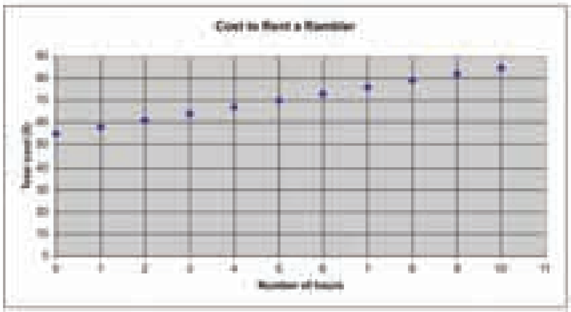
➤ Open a spreadsheet program.
Make a table of values for the "Rambler" relation using spreadsheet operations.

This is what you might see:

A	B	C
h	c	
0	55	
1	58	
2	61	
3	64	
4	67	
5	70	
6	73	
7	76	
8	79	
9	82	
10	85	

To graph the relation

➤ Highlight the data.
Click the graph/chart icon.
Select *XY* (*Scatter*).
Label the graph and the axes.
Your graph may look similar to this:



The points lie on a line so the graph represents a linear relation.
When the input increases by 1, the output increases by 3.
The graph goes up to the right.
This is because the total cost increases for every hour the "Rambler" is rented.

Check

1. Create a table of values for the "Northern" ATV.
Use the table of values to graph the relation.
Describe the relationship between the two variables in the graph.

2. Suppose Chris likes both ATVs equally.
What conclusions can you make from the tables or the graphs to help Chris save money?

Choosing a Strategy

Have you ever solved a problem, looked back, and realized you could have solved the problem a different way?
There are many different strategies for solving problems.

Try to solve this problem in at least two different ways.

The intramural dodgeball league at your school has 10 teams.
Each team must play every other team exactly once.
How many games need to be scheduled?

Try these Steps

What information are you given in the problem?
Is there any information that is not needed?
What are you asked to find?
Is an estimate okay or do you need an exact answer?

What strategies might work for this problem?

Try the strategy you think will work best.
If you have trouble solving the problem, try a different strategy.
You might have to try 3 or 4 strategies.

Have you answered the question?
Does your answer seem reasonable?
How do you know you have found all the answers?

Strategies

- **Make a table.**
- **Use a model.**
- **Draw a diagram.**
- **Solve a simpler problem.**
- **Work backward.**
- **Guess and test.**
- **Make an organized list.**
- **Use a pattern.**
- **Draw a graph.**
- **Use logical reasoning.**

Strategies for Success

Use at least two different strategies to solve each problem.

1. A rectangular field has length 550 m and width 210 m. Fence posts are placed 10 m apart along the perimeter of the field, with one post in each corner. How many fence posts are needed?

2. A basketball tournament starts at 10:00 a.m. The winners play every 1.5 h and the losers are eliminated. The winning team finishes their last game at 4:00 p.m. How many teams are in the tournament?

3. Marsha and Ivan have money to spend at the Raven Mad Days Celebration in Yellowknife.
If Marsha gives Ivan $5, each person will have the same amount.
If, instead, Ivan gives Marsha $5, Marsha will have twice as much as Ivan.
How much money does each person have?

4. Briony wants to print copies of her new brochure.
The local print shop charges 15¢ a copy for the first 25 copies, 12¢ a copy for the next 50 copies, and 8¢ a copy for any additional copies.
How much would Briony pay for each number of copies?

a) 60 copies **b)** 240 copies

5. How many different necklaces can you make with:

a) one red bead, one yellow bead, and one green bead?

b) two red beads, one yellow bead, and one green bead?

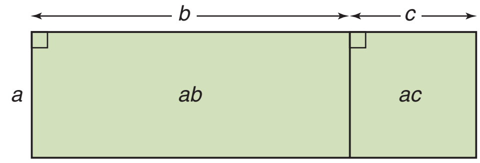
Justify your answers.

Unit Review

What Do I Need to Know?

Distributive Property

The product of a number and the sum of two numbers can be written as a sum of two products:
$a(b + c) = ab + ac$

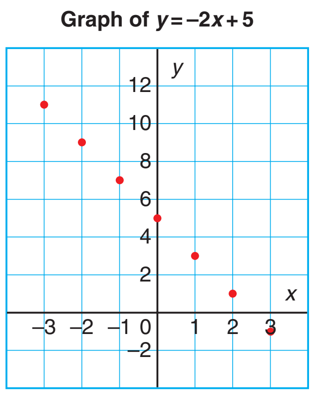


The distributive property can be used to solve some algebraic equations.

We can represent a linear relation in different ways:
- as a two-variable equation
- as a table of values
- as a graph

$y = -2x + 5$

x	y
–3	11
–2	9
–1	7
0	5
1	3
2	1
3	–1



A related pair of x and y values is an *ordered pair*.
When data are *discrete*, the numbers between those given do not make sense in the context of the problem.
When the graph of a relation is a straight line, the relation is *linear*.

What Should I Be Able to Do?

LESSON

6.1

1. Tracey put one coin in the candy machine, and 7 candies were dispensed.
When she put 2 coins in the machine, 14 candies were dispensed.
How many coins would Tracey have to put in to have 56 candies dispensed?

a) Choose a variable to represent the number of coins Tracey would have to put in. Write an equation to describe the situation.

b) Use a model to solve the equation.

c) Verify the solution.

2. Use a model to solve each equation.
Verify the solution.

a) $-6x = -30$

b) $11x = -22$

c) $4x + 1 = 17$

d) $-4x + 1 = 17$

e) $11 = 3x - 7$

f) $-11 = 3x + 7$

6.2

3. Troy is taking care of Mr. Green's property. He is paid $8 for mowing the lawn, and $3 for each garden that he weeds. On Saturday, Mr. Green paid Troy $29.
How many gardens did Troy weed on Saturday?

a) Write an equation to represent this problem.

b) Solve the equation.

c) Verify the solution.

4. Solve each equation.
Verify the solution.

a) $-6x + 8 = -16$

b) $11x - 5 = 28$

c) $3x + 5 = 7$

d) $-4x - 9 = 19$

e) $13 = 3x + 12$

f) $7x - 19 = -5$

5. The high temperature today is 6°C higher than three times the high temperature yesterday. The high temperature today is 3°C. What was the high temperature yesterday?

a) Write an equation you can use to solve the problem.

b) Solve the equation.

c) Verify the solution.

6.3

6. Solve each equation.
Verify the solution.

a) $\frac{p}{-4} + 11 = 8$

b) $\frac{t}{10} + 12 = 3$

c) $-21 + \frac{w}{6} = -30$

d) $-12 + \frac{e}{-8} = -1$

7. Check this student's work.
Rewrite a correct and complete algebraic solution if necessary.

$$\frac{h}{-2} = -7$$
$$\frac{h}{-2} \times 2 = -7 \times 2$$
$$\frac{2h}{-2} = -14$$
$$h = -14$$

LESSON

8. One-fifth of the fish in the lake are pickerel. There are 52 pickerel in the lake. How many fish are in the lake?

a) Write an equation you can use to solve the problem.

b) Solve the equation.

c) Verify the solution.

6.4

9. Draw a rectangle or algebra tiles to show that $3(x + 4)$ and $3x + 12$ are equivalent.
Explain your diagram in words.

10. Expand.

a) $6(x + 9)$ b) $3(11 - 4c)$

c) $5(-7s + 5)$ d) $-4(3a - 2)$

11. Which expression is equal to $-5(-t + 4)$? How do you know?

a) $-5t - 20$ b) $5t - 20$ c) $5t + 20$

6.5

12. Solve each equation using the distributive property.
Verify the solution.

a) $7(x + 2) = 35$

b) $5(b - 6) = 30$

c) $-3(p - 9) = -24$

d) $5(s - 3) = 9$

13. Martina chose an integer.
She subtracted 7, then multiplied the difference by –4. The product was 36.
Which integer did Martina choose?

a) Write an equation you can use to solve the problem.

b) Solve the problem.

c) Verify the solution.

14. Chas used the distributive property to solve this equation: $-2(c - 5) = 28$

a) Check Chas' work.
Is his solution correct?

$$\begin{aligned} -2(c - 5) &= 28 \\ -2(c) - 2(-5) &= 28 \\ -2c - 10 &= 28 \\ -2c - 10 + 10 &= 28 + 10 \\ -2c &= 38 \\ \frac{-2c}{-2} &= \frac{38}{-2} \\ c &= -19 \end{aligned}$$

b) If your answer to part a is yes, verify the solution.
If your answer to part a is no, describe the error, then correct it.

6.6

15. Copy and complete each table of values.

a) $y = x - 8$

b) $y = -x + 5$

x	–3	–2	–1	0	1	2	3
y							

x	–3	–2	–1	0	1	2	3
y							

16. Lauree is making friendship bracelets.
She needs 6 strands of yarn for each bracelet.
An equation for this relation is $s = 6n$, where n represents the number of bracelets, and s represents the number of strands of yarn needed.

LESSON

a) Use the equation to create a table of values.

b) Suppose Lauree makes 7 bracelets. How many strands of yarn does she need?

c) Suppose Lauree has 66 strands of yarn.
How many bracelets can she make?

d) Yarn comes in packages of 20 strands. How many packages of yarn will Lauree need to make 18 bracelets? Explain your answer.

6.7

17. Use the data from question 16.

a) Construct a graph for the data.

b) Describe the relationship between the variables in the graph.

c) Find the ordered pair on the graph that shows how many bracelets can be made with 54 strands of yarn.

6.6

18. The equation of a linear relation is:
$y = -7x + 4$
Find the missing number in each ordered pair.

a) $(-2,\ \ )$ b) $(\ \ , -17)$

c) $(8,\ \ )$ d) $(\ \ , 4)$

19. Francis sells memberships to a local health club. He is paid \$200 per week, plus \$40 for each membership he sells. An equation for this relation is $p = 200 + 40n$, where n represents the number of memberships Francis sells, and p represents his pay in dollars.

a) Use the equation to create a table of values.

b) One week, Francis sold 9 memberships. What was his pay for that week?

c) One week, Francis was paid \$480. How many memberships did he sell that week?

6.7

20. Use the data from question 19.

a) Construct a graph for the data.

b) Describe the relationship between the variables in the graph.

c) Find the ordered pair on the graph that shows Francis' pay when he sells 5 memberships.

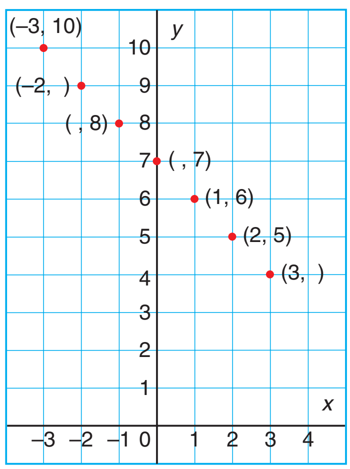
21. Graph each relation for integer values of x from -3 to 3.

a) $y = -6x$ b) $y = -4x + 3$

c) $y = 7x - 1$ d) $y = 9x + 8$

22. Here is a graph of the linear relation: $y = -x + 7$. Each point on the graph is labelled with an ordered pair. Some numbers in the ordered pairs are missing. Find the missing numbers. Explain how you did this.

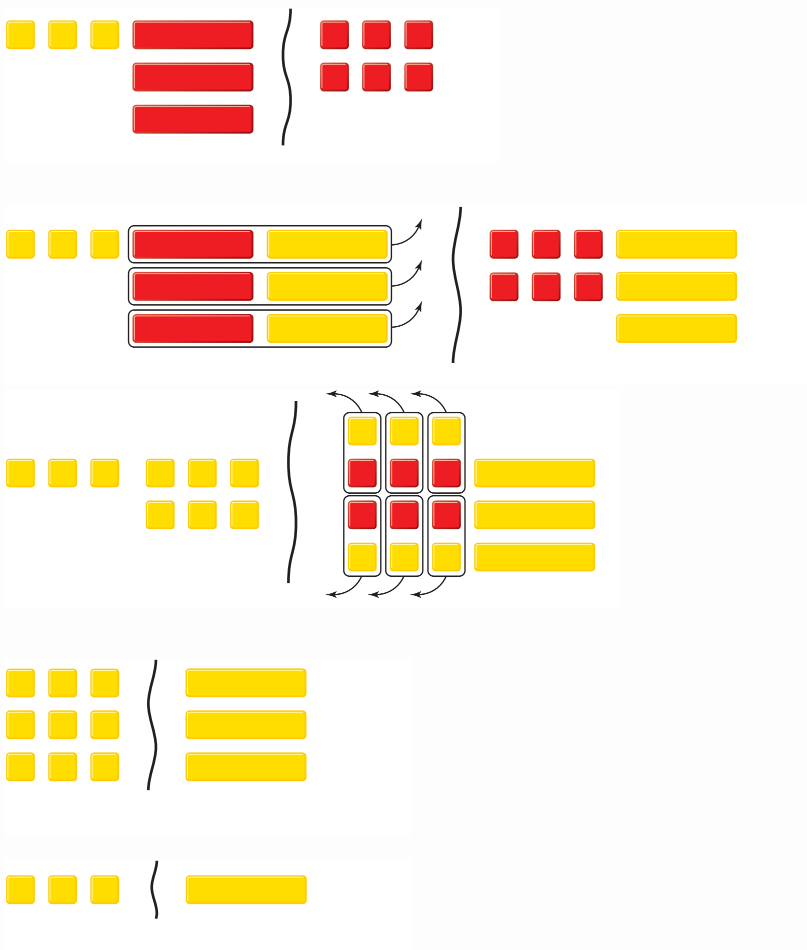
Graph of $y = -x + 7$



Practice Test

1. **a)** Use a model to solve this equation:
$-6s + 5 = -7$
Draw a picture to show the steps you took to solve the equation.
b) Verify the solution. Show how you did this.

2. Blair is using algebra tiles to model and solve equations.
Here is Blair's work for one question.



a) Use algebra to record each step Blair took.
b) Verify the solution in two different ways.

3. Use grid paper.

a) Draw a picture to show that $4(x + 3)$ and $4x + 12$ are equivalent.

b) Could you draw a picture to show that $-4(x + 3)$ and $-4x - 12$ are equivalent? Why or why not?

4. Solve each equation.
Verify the solution.

a) $5(x - 3) = 45$ **b)** $\frac{n}{7} + 7 = 4$

c) $\frac{p}{-6} = -7$ **d)** $12x - 23 = 73$

5. A school soccer team rented a bus for the day.
The bus cost \$200 for the day, plus \$14 for each person on the bus.
The total cost of the bus rental was \$424.
How many people were on the bus?

a) Write an equation you can use to solve the problem.

b) Solve the equation.

c) Check your answer. Write to explain how you know it is correct.

6. Create a table of values for each relation.

a) $y = -6x + 1$ **b)** $y = 7x - 4$

7. A chocolate bar is divided into 10 equal pieces. Both Andy and Greg want some. Let a represent the number of pieces Andy gets. Let g represent the number of pieces Greg gets. An equation that relates the number of pieces Andy gets to the number of pieces Greg gets is:
$g = 10 - a$

a) Create a table of values for the relation.

b) Graph the relation.

c) Describe the relationship between the variables in the graph.

d) Which ordered pair shows the fairest way to divide the chocolate bar? Justify your answer. Identify the point on the graph.

8. The equation of a linear relation is:
$y = -5x + 2$
Find the missing number in each ordered pair.
Explain how you did this.

a) $(\ \ , 2)$ **b)** $(-4,\ \ )$

c) $(\ \ , -13)$ **d)** $(8,\ \ )$

Unit Problem Planning a Ski Trip

In 1988, the alpine events of the Winter Olympic Games were held at Nakiska Mountain, just west of Calgary. The Grade 8 students are planning a ski trip to Nakiska. An organizing committee has been formed to research the trip.

Part 1

Two local bus companies, Company A and Company B, offer packages for school trips.
The amount each company charges is given by these equations:
Company A: $C = \$300 + \$55n$
Company B: $C = \$100 + \$75n$
where n is the number of people on the bus and C is the total cost in dollars.
Suppose 85 students and 10 adults go on the trip.
Which company should the committee choose?
What strategy did you use to find out?
Justify your answer.

Part 2

The daily cost per person for ski rental is \$23.
On one day, the total cost of ski rental was \$851.
Write an equation you can use to find out how many students rented skis that day.
Solve the equation. Verify the solution.

Part 3

When you are skiing, you must be aware of extreme temperatures.
Nakiska Mountain has an elevation of about 2250 m.
An equation for calculating the temperature at this elevation is:
$T = c - 15$, where c is the temperature in degrees Celsius at sea level, and T is the temperature at an elevation of 2250 m.

a) Suppose the temperature at sea level is 0°C.
What is the temperature at the peak of Nakiska Mountain?

b) Suppose the temperature at the peak of the mountain is −23°C.
What is the temperature at sea level?

Part 4

The hotel in Calgary is offering a group discount.
It charges a daily rate of $1500, plus $30 per person.
An equation for this relation is $h = 1500 + 30p$, where h represents the total cost of the hotel per day in dollars, and p represents the number of people.

a) Create a table of values for the relation.
Use $p = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$.

b) Graph the relation.

c) Describe the relationship between the variables in the graph.

d) Use the graph to find the total cost of the hotel when 95 people go on the trip.
What does each person have to pay for the hotel?
How did you find out?

Check List

Your work should show:

- ✓ an accurate graph and all calculations in detail
- ✓ your understanding of linear equations and linear relations
- ✓ the strategies you used to solve the problems
- ✓ a clear explanation of the thinking behind your solutions

Reflect on Your Learning

Write about equations and linear relations and how variables are used in both.
Include examples in your explanation.

Units 1–6 Cumulative Review

UNIT

1

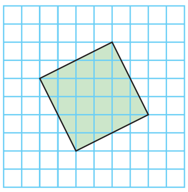
1. Find.

a) $\sqrt{1}$ b) 16^2

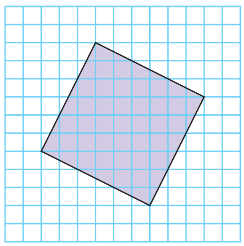
c) $\sqrt{43}$ d) the square of 11

2. Copy each square on 1-cm grid paper. Find its area, then write the side length of the square.

a)



b)



2

3. A watch loses 3 s per hour for 24 h. Use integers to find the total number of seconds the watch lost over 24 h.

4. Evaluate.

a) $(-15) \times (+4)$ b) $(-8) \times (-10)$

c) $(+57) \div (-3)$ d) $\frac{-38}{+2}$

3

5. Use an area model to find each product.

a) $\frac{5}{9} \times \frac{3}{4}$ b) $2\frac{2}{3} \times \frac{3}{5}$

c) $\frac{21}{5} \times \frac{4}{3}$ d) $1\frac{1}{4} \times 3\frac{5}{6}$

6. A dental hygienist takes $1\frac{1}{6}$ h to clean a patient's teeth.

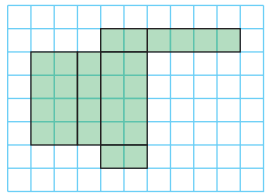
a) Estimate the number of patients the hygienist can see in $5\frac{1}{4}$ h.

b) Calculate the number of patients the hygienist can see in $5\frac{1}{4}$ h.

c) What assumptions do you make?

4

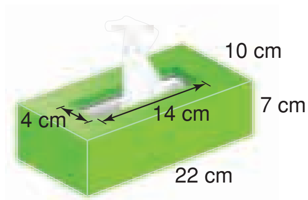
7. a) Predict the object this net will form.



b) Fold the net to verify your prediction.

c) Describe the object.

8. Find the surface area of the object.



9. A triangular prism has a base that is a right triangle. The side lengths of the triangle are 5 cm, 12 cm, and 13 cm. The prism is 20 cm high.

a) What is the volume of the prism?

b) Sketch a net for the prism. Label the net with the dimensions of the prism.

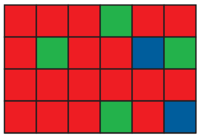
c) What is the surface area of the prism?

5

10. There were 25 people in Sebastian's Tae Kwon Doe class. Then, three people dropped out. Write this decrease as a percent. Illustrate the percent on a number line.

UNIT

11. a) Write each ratio in simplest form.



i) red squares to blue squares
ii) blue squares to green squares
iii) red squares and blue squares to total number of squares
iv) green squares to blue squares to red squares

b) Suppose the grid is increased to a rectangle measuring 9 units by 4 units. The ratios of the colours remain the same. How many red squares will there be in the new rectangle?

12. Which is the better buy?

a) 2 rolls of paper towel for \$0.99 or 12 rolls of paper towel for \$5.59
b) 500 mL of mouthwash for \$3.99 or 125 mL of mouthwash for \$1.29

13. The price of a set of headphones in Prince Albert, Saskatchewan, is \$34.99. This is a discount of 25%.

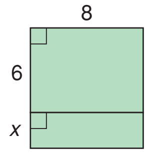
a) What is the regular price of the headphones?
b) What is the sale price of the headphones including taxes?

6

14. Solve each equation. Verify the solution.

a) $-5x - 7 = 18$ b) $4(s - 6) = -52$
c) $\frac{t}{-12} = -8$ d) $\frac{f}{8} + 9 = 4$

15. Write two expressions for the area of the shaded rectangle.



16. Rashad chose an integer. He added 11, then multiplied the sum by –2. The product was –4. Which integer did Rashad choose?

a) Write an equation you can use to solve the problem.
b) Solve the problem. Verify the solution.

17. The equation of a linear relation is $y = -9x + 5$. Find the missing number in each ordered pair.

a) (2,) b) (, 5)
c) (–3,) d) (, –31)

18. Cecilia rides an ice-cream bicycle. She charges \$50 to go to an event, plus \$3 for each ice-cream bar she sells. An equation for the relation is $p = 50 + 3n$, where n represents the number of ice-cream bars Cecilia sells, and p represents her pay in dollars.

a) Use the equation to create a table of values.
b) One day, Cecilia sold 45 ice-cream bars. What was her pay for that day?
c) One day, Cecilia was paid \$260. How many ice-cream bars did she sell that day?