Unit One: Linear Inequalities

Unit One Linear Inequalities

(a) $4x-7y=6$, $2x+y=12$

① $4x-7y=6$

②×7 $14x+7y=84$

①+② $18x=90$, $x=5$

sub $4(5)-7y=6$, $20-7y=6$, $-7y=74$, $y=2$

$(5, 2)$

(b) $6x-5y=-3$, $5x+3y=19$

①×3 $18x-15y=-9$

②×5 $25x+15y=95$

①+② $43x=86$, $x=2$

sub $x=2$: $5(2)+3y=19$, $10+3y=19$, $3y=9$, $y=3$

$(2,3)$

(c) $3(x+4)-4(y+1)=7$, $x-9=y$

① $3x+12-4y-4=7$

① $3x-4y=-1$

② $x-y=9$

① $3x-4y=-1$

②×3 $3x-3y=27$

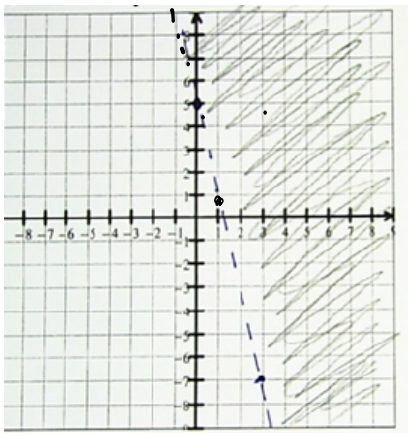
①−② $-y=-28$, $y=28$

sub ② $x-28=9$, $x=37$

$(37,28)$

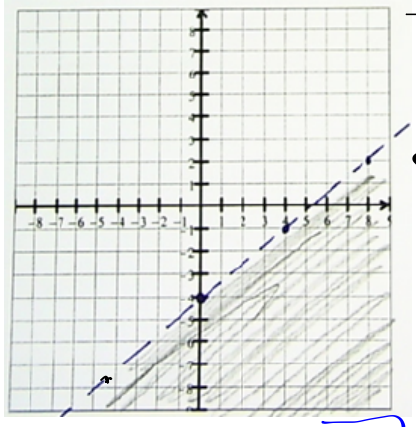
2. Graph the inequality:

a. $y>-4x+5$



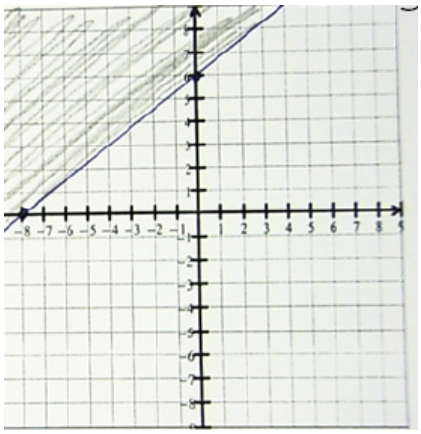
x	y
1	1
0	5
3	−7
−1	9

c.. $y<\frac{3}{4}x-4$



x	y
−4	−7
0	−4
4	−1
8	2

b. $3x-4y\leq -24$

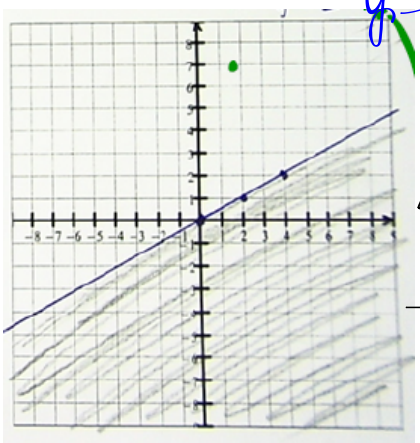


(0, 6)

(−8, 0)

d.. $x\geq 2y$

Rearrange $y\leq \frac{x}{2}$



x	y
6	3
4	2
0	0
−2	−1

3. Given the following constraints, graph each:

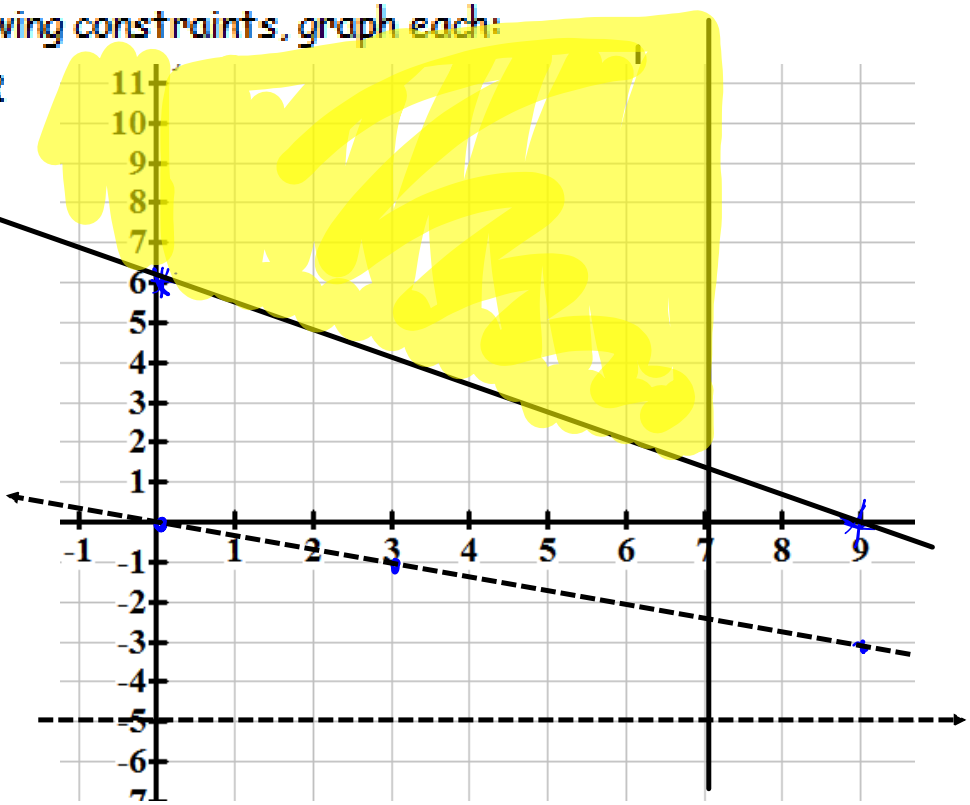
a. $x \in R$, $y \in R$

$x \leq 7$

$y > -5$

$y > -\frac{1}{3}x$

$2x + 3y \geq 18$



3b. $x \geq 0$

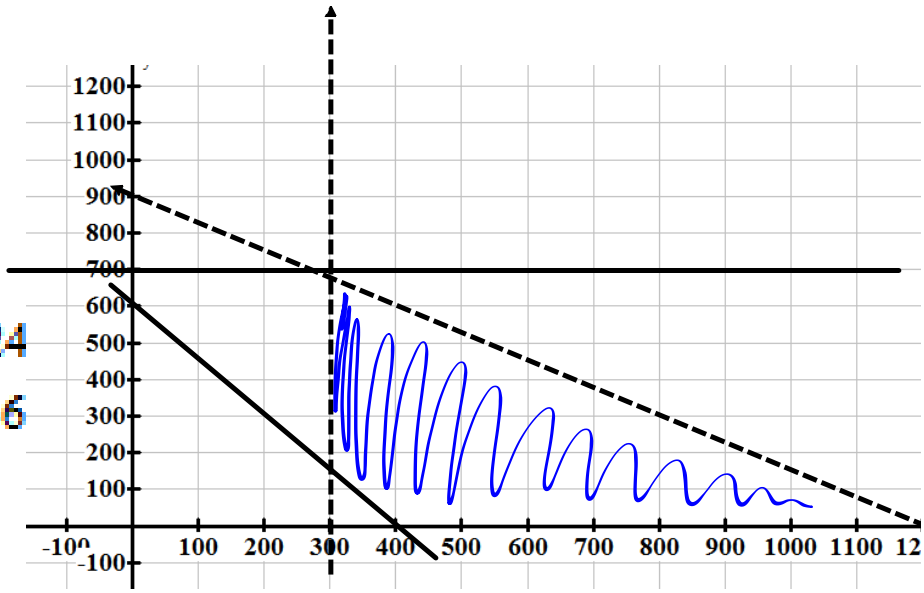
$y \geq 0$

$x > 300$

$y \leq 700$

$6x + 4y \geq 24$

$3x + 4y < 36$



4. In order to ensure optimal health for your puppy a lab technician recommends to feed the pup a daily diet containing a minimum of 24 grams (g) of fat, 36 g of carbohydrates, and 4 g of protien.
The pup should be fed no more than five ounces of food a day.
Rather than order food that is custom-blended, it is cheaper to order Food A and Food B, and blend them for an optimal mix.
Food X contains 6 g of fat, 12 g of carbohydrates, and 2 g of protein per ounce, and costs $0.20 per ounce.
Food Y contains 12 g of fat, 12 g of carbohydrates, and 1 g of protein per ounce, at a cost of $0.30 per ounce.
What is the optimal blend?

Constraints

fat: $6x + 12y \geq 24$

carbs: $12x + 12y \geq 36$

protein: $2x + 1y \geq 4$

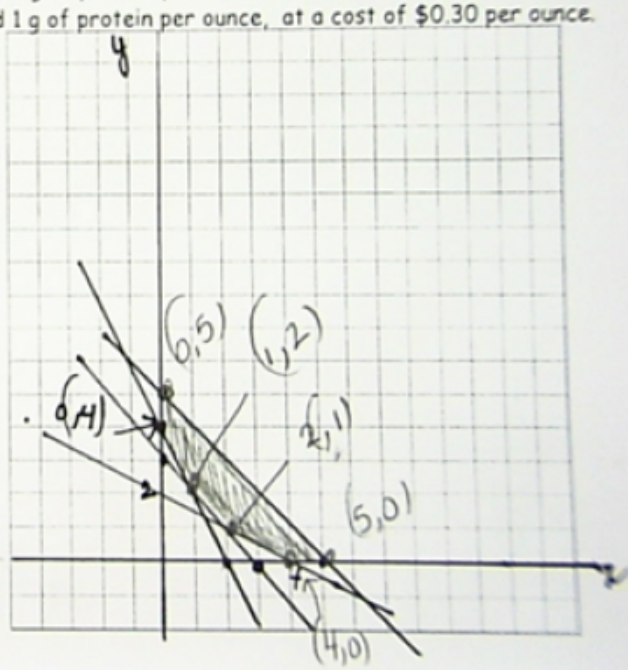
the maximum weight of the food is five ounces, so: $x + y \leq 5$

Optimization

optimization equation will be the cost relation $C = 0.2x + 0.3y$,
and we need the minimum value

(0,5) = 1.5
(0,4) = 1.2
(1,2) = 0.8
(2,1) = 0.7
(5,0) = 1.0
(4,0) = 0.8

Minimum



2 of Food X

1 of Food Y

4.

Constraints

fat: $6x + 12y \geq 24$

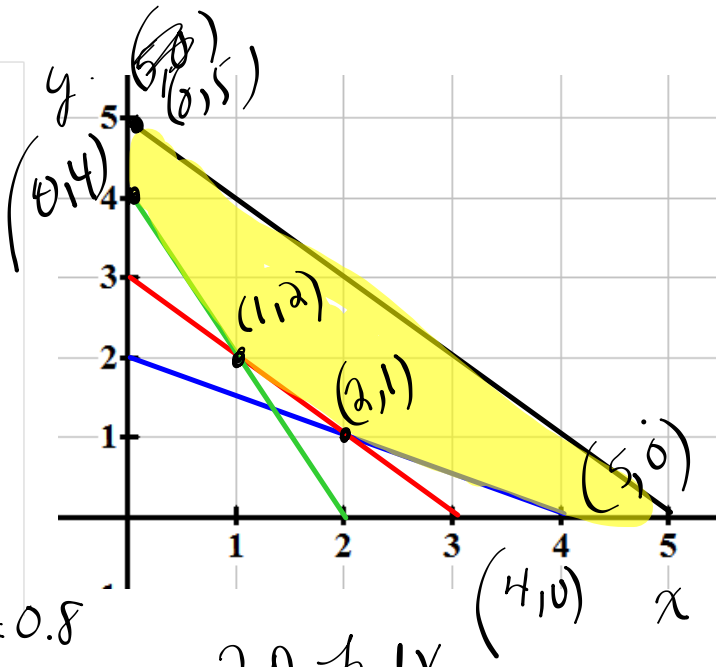
carbs: $12x + 12y \geq 36$

protein: $2x + 1y \geq 4$

the maximum weight of the food is five ounces, so: $x + y \leq 5$

Optimization

optimization equation will be the cost relation $C = 0.2x + 0.3y$, and we need the minimum value



(0,4) = 1.2

(0,5) = 1.5

(1,2) = 1.0

(2,1) = 0.7

(4,0) = 0.8

(5,0) = 1.6

2 of foodX

1 of foodY

5. For every bouquet that is sold at a fundraising banquet, $5 goes to charity. For every ticket that is sold, $18 goes to charity. The organizers' goal is to raise at least $8000. The organizers need to know how many bouquets and tickets must be sold to meet their goal.

a) Define the variables and write a linear inequality to represent the situation.

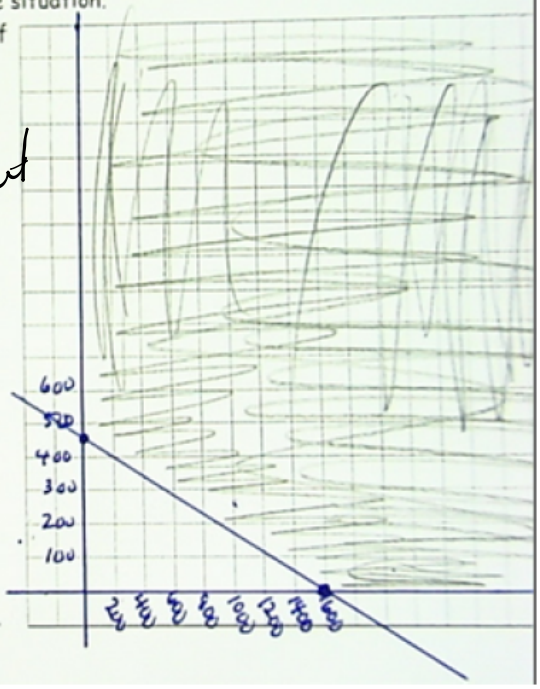
b) Graph the linear inequality , The first coordinate is the number of bouquets and the second is the number of tickets.

x = # bouquet
y = # tickets

← $5 for every
← $18 for ticket

$5x + 18y \geq 8000$

x	y
0	444.4
1600	0



6. x = # high school friends
y = # university friends

$$x + y \leq 375$$
$$x \geq 2y$$

7. x = # hot dogs
y = # hamburgers

$$x + y \leq 300$$
$$x \leq 250$$
$$y \leq 125$$

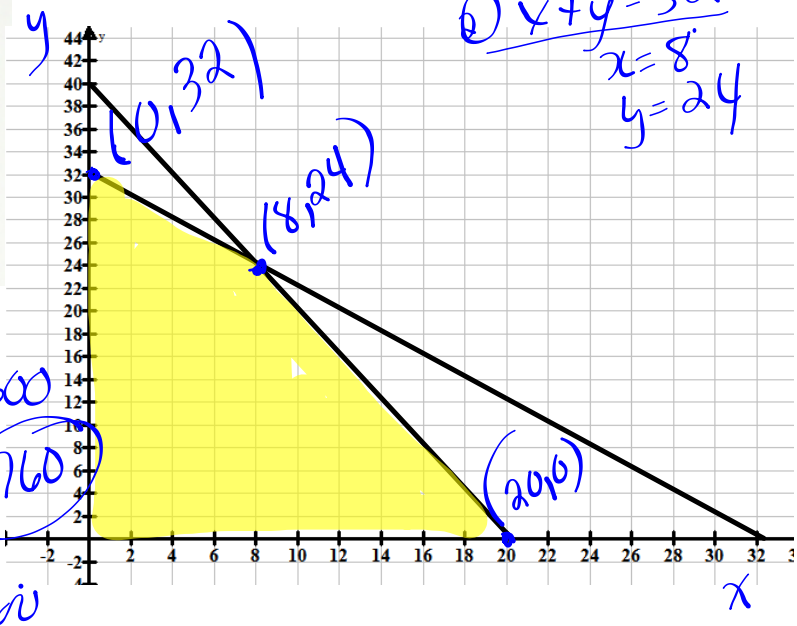
Optimization $3x+2y$

8.

	Downhill x	Cross Country y	
Assembly	$2x$	$1y$	≤ 40
Finish	$1x$	$1y$	≤ 32

$2x + y \leq 40$
$x + y \leq 32$
Optimization $70x + 50y$

$x \geq 0$
$y \geq 0$



① $2x + y = 40$
② $x + y = 32$
$x = 8$
$y = 24$

$70x + 50y$

$(0,32) = 70(0) + 50(32) = 1600$

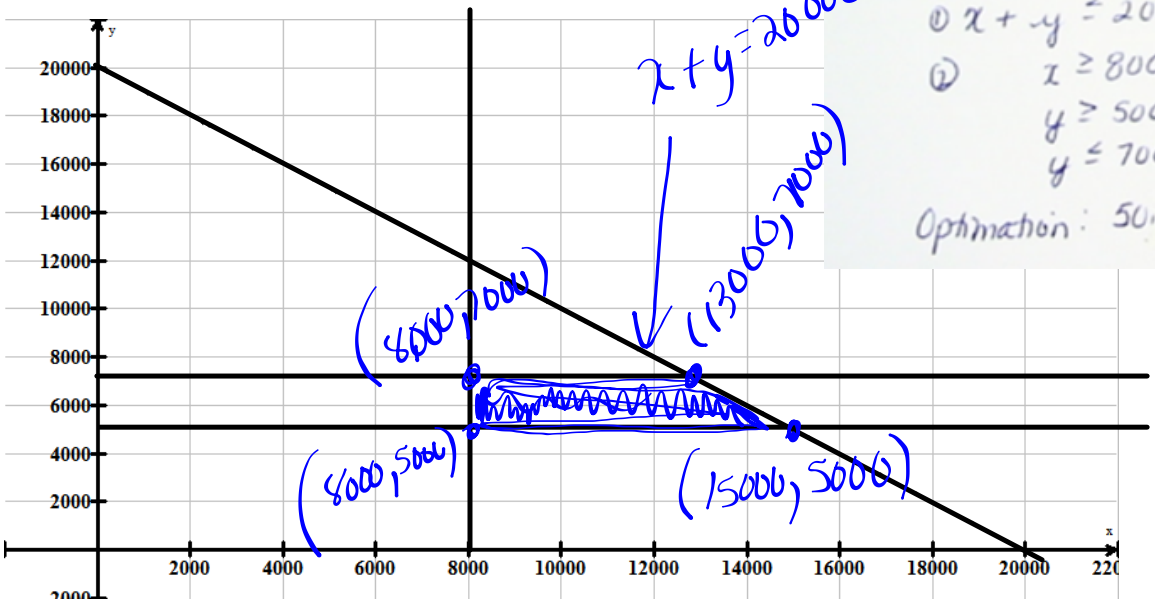
$(8,24) = 70(8) + 50(24) = 1760$

$(20,0) = 70(20) + 50(0) = 1400$

9.

	DVD (x)	Book (y)
①	$x + y \leq 20000$	
②	$x \geq 8000$	
	$y \geq 5000$	
	$y \leq 7000$	

Optimation: $50x + 30y$



Optimation: $50x + 30y$

$(8000, 7000) = 610000$

$(8000, 5000) = 550000$

$(15000, 5000) = 900000$ ✓

$(13000, 7000) = 860000$

Unit 2: Quadratics

1. $y = -7x^2 - 126x - 700$

$y = -7(x^2 + 18x) - 700$

$y = -7(x^2 + 18x + 81 - 81) - 700$

$y = -7(x^2 + 18x + 81) + 567 - 700$

$y = -7(x + 9)^2 - 133$

opens down

$(-9, -133)$

range $y \leq -133$

max of -133

y-int -700

b. $y = 9.5x^2 - 76x + 141$

$y = 9.5(x^2 - 8x) + 141$

$y = 9.5(x^2 - 8x + 16 - 16) + 141$

$y = 9.5(x^2 - 8x + 16) - 152 + 141$

$y = 9.5(x - 4)^2 - 11$

opens up

$(4, -11)$

range $y \geq -11$

min of -11

y-int 141

c. $y = \frac{1}{5}x^2 + 4x + 24$

$y = \frac{1}{5}(x^2 + 20x) + 24$

$y = \frac{1}{5}(x^2 + 20x + 100 - 100) + 24$

$y = \frac{1}{5}(x^2 + 20x + 100) - 20 + 24$

$y = \frac{1}{5}(x + 10)^2 + 4$

opens up

$(-10, 4)$

range $y \geq 4$

min of 4

y-int 24

d. $y = 11x^2 - 22x - 4$

$y = 11(x^2 - 2x) - 4$

$y = 11(x^2 - 2x + 1 - 1) - 4$

$y = 11(x^2 - 2x + 1) - 11 - 4$

$y = 11(x - 1)^2 - 15$

opens up

$(1, -15)$

range $y \geq -15$

min of -15

y-int -4

2. Fill in the following: Show your work on a separate sheet when you change to standard form

Function remember $y=a(x-h)2+k$	a	Opens Up or down	Vertex (h,k)	Axis of symmetry	Range	Standard form	Max/ min	y-intercept
$y=-\frac{1}{3}(x-4)^2-11$	1/3	Down	(4,-11)	$x=4$	$y\leq -11$	$y=-\frac{1}{3}x^2+\frac{8}{3}x-\frac{49}{3}$	max -11	$-\frac{49}{3}$
$y=1.9(x+1)^2+18$	1.9	up	(-1,18)	$x=-1$	$y\geq 18$	$y=1.9x^2+3.8x+19.9$	min 18	19.9
$y=-x^2+6$	1	Down	(0,6)	$x=0$	$y\leq 6$	$y=-x^2+6$	max 6	6
$y=2(x-13)^2$	2	up	(13,0)	$x=13$	$y\geq 0$	$y=2x^2-52x+338$	min 0	0
$y=14(x-10)^2-3.2$	14	up	(10,-3.2)	$x=10$	$y\geq -3.2$	$y=14x^2-280x+1432$	min -3.2	1432

3. $y=-3(x+4)^2+32$

vertex $(-4,32)$

opens

$y=-3(x+4)^2+32$

$y=-3(x^2+8x+16)+32$

$y=-3x^2-24x-48+32$

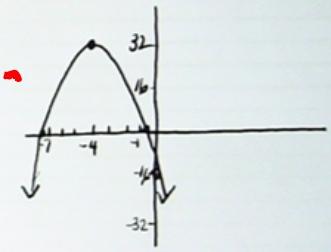
$y=-3x^2-24x-16$

y-int -16

x-int $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\frac{24\pm\sqrt{384}}{-6}$

-7.3 $\quad -0.73$



b. $y=\frac{1}{4}(x-12)^2-60$

Vertex $(12,-60)$

opens up

$y=\frac{1}{4}(x-12)(x-12)-60$

$y=\frac{1}{4}(x^2-24x+144)-60$

$y=\frac{1}{4}x^2-6x+36-60$

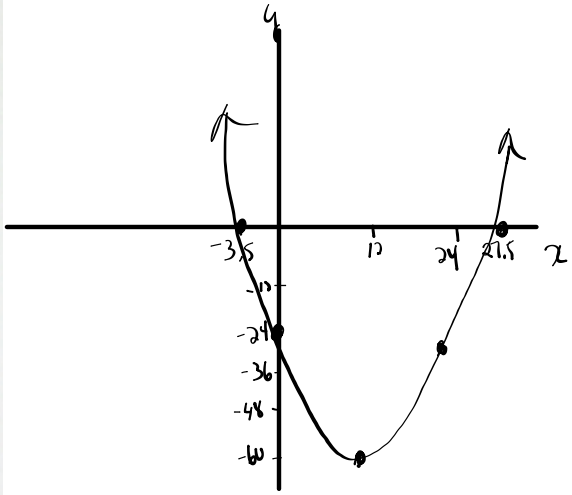
$y=\frac{1}{4}x^2-6x-24$

y-int -24

x-int $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\frac{6\pm\sqrt{60}}{0.5}$

27.5 $\quad -3.5$



4. vertex (9,3)

(a) pt (7,-13)

$y = a(x-9)^2 + 3$

$-13 = a(7-9)^2 + 3$

$-13 = a(-2)^2 + 3$

$-16 = 4a$

$a = -4$

$y = -4(x-9)^2 + 3$

vertex (14,12)

(b) pt (8,660)

$y = a(x-14)^2 + 12$

$660 = a(8-14)^2 + 12$

$660 - 12 = a(-6)^2$

$648 = 36a$

$18 = a$

$y = 18(x-14)^2 + 12$

$y = a(x-h)^2 + k$

5. (a) $x^2 - 14x + 45$ ST

$(x-9)(x-5)$

(b) $3x^2 + 16x - 12$ HT

$3x^2 + 18x - 2x - 12$

$3x(x+6) - 2(x+6)$

$(x+6)(3x+2)$

(c) $x^2 - 144$ DS

$(x-12)(x+12)$

(d) $9x^2 - 100$ DS

$(3x-10)(3x+10)$

(e) $x^2 + x - 56$ ST

$(x+8)(x-7)$

(f) $8x^2 - 2x - 3$ HT

$8x^2 - 6x + 4x - 3$

$2x(4x-3) + 1(4x-3)$

$(4x-3)(2x+1)$

(g) $9x^2 - 6x - 8$ HT

$9x^2 - 12x + 6x - 8$

$3x(3x-4) + 2(3x-4)$

$(3x-4)(3x+2)$

(h) $12x^2 + 16x + 5$ HT

$12x^2 + 10x + 6x + 5$

$2x(6x+5) + 1(6x+5)$

$(6x+5)(2x+1)$

Simple trinomial

hard trinomials (decomposition)

diff. of squares

6. (a) $8x(x-5)-7(2-3x)=3x+7$

$8x^2-40x-14+21x=3x+7$

$8x^2-22x-21=0$

→ Quad Form

$$\frac{22\pm\sqrt{1156}}{16}$$

$$\frac{22\pm34}{16}$$

3.5

-0.75

6. (b) $-x^2+3x+2=-3x^2-2x+4$

$2x^2+5x-2=0$

OR Quad Form

$$\frac{-5\pm\sqrt{41}}{4}$$

0.35 -2.85

7. (a) $h=5t^2-40t+83.4$

$h=5(t^2-8t)+83.4$

$h=5(t^2-8t+16-16)+83.4$

$h=5(t^2-8t+16)-80+83.4$

$h=5(t-4)^2+3.4$

min height $=3.4$ m

(b) When $t=0$

$h=5(0)^2-40(0)+83.4$

$h=83.4$ m

(c) $5=5t^2-40t+83.4$

$0=5t^2-40t+78.4$

$$\frac{40\pm\sqrt{32}}{10}$$

3.4 sec

4.6 sec

8. (a) $h = -9.8t^2 + 58.8t + 67.2$

$h = -9.8(t^2 - 6t) + 67.2$

$h = -9.8(t^2 - 6t + 9 - 9) + 67.2$

$h = -9.8(t^2 - 6t + 9) + 88.2 + 67.2$

$h = -9.8(t-3)^2 + 155.4$

max $h = 155.4$ m

(b) at $t = 3$ sec

(c) at $t = 5$

$h = -9.8(5)^2 + 58.8(5) + 67.2$

$h = 116.2$ m

9. (a) $h = -7t^2 + 7t + 2.25$

$h = -7(t^2 - t) + 2.25$

$h = -7(t^2 - t + 0.25 - 0.25) + 2.25$

$h = -7(t^2 - t + 0.25) + 1.75 + 2.25$

$h = -7(t - 0.5)^2 + 4$

max $h = 4$ m

(b) 0.5 sec

(c) 2.25 m

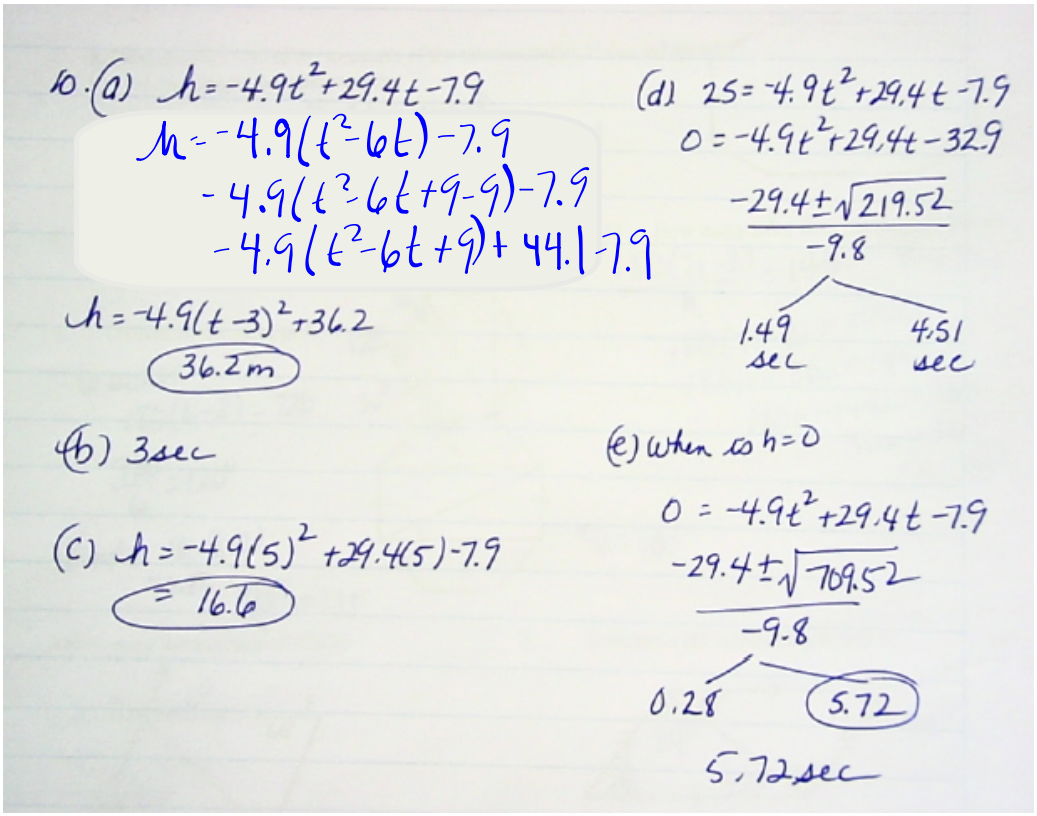
(d) $1 = -7t^2 + 7t + 2.25$

$0 = -7t^2 + 7t + 1.25$

$\frac{-7 \pm \sqrt{84}}{-14}$

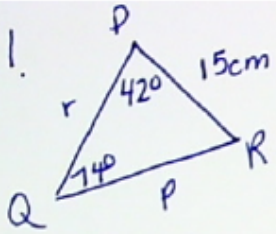
-0.15 1.15

1.15 sec


10. (a) $h = -4.9t^2 + 29.4t - 7.9$
$h = -4.9(t^2 - 6t) - 7.9$
$-4.9(t^2 - 6t + 9 - 9) - 7.9$
$-4.9(t^2 - 6t + 9) + 44.1 - 7.9$
$h = -4.9(t-3)^2 + 36.2$
36.2 m
(b) 3 sec
(c) $h = -4.9(5)^2 + 29.4(5) - 7.9$
$= 16.6$
(d) $25 = -4.9t^2 + 29.4t - 7.9$
$0 = -4.9t^2 + 29.4t - 32.9$
$\frac{-29.4 \pm \sqrt{219.52}}{-9.8}$
1.49 sec
4.51 sec
(e) When is $h = 0$
$0 = -4.9t^2 + 29.4t - 7.9$
$\frac{-29.4 \pm \sqrt{709.52}}{-9.8}$
0.28
5.72
5.72 sec

Trigonometry

1.



$\frac{p}{\sin 42^\circ} = \frac{15}{\sin 74^\circ}$

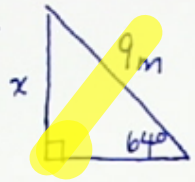
$p = 10.4$

$\angle R = 64^\circ$

$\frac{r}{\sin 64^\circ} = \frac{15}{\sin 74^\circ}$

$r = 14.03$

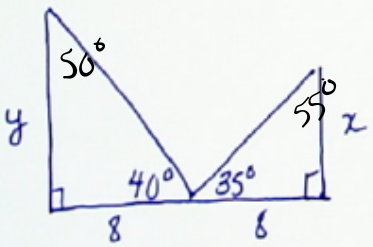
2.



$\sin 64^\circ = \frac{x}{9}$ OR $\frac{x}{\sin 64^\circ} = \frac{9}{\sin 90^\circ}$

$x = 8.1\text{m}$

3.



$y = 6.7$ $x = 5.6$

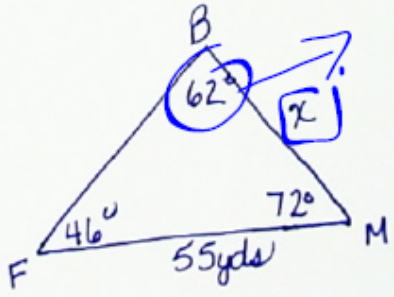
1.1

$6.7 - 5.6 = 1.1$

$\frac{y}{\sin 46^\circ} = \frac{8}{\sin 50}$

$y = 6.7$

4.

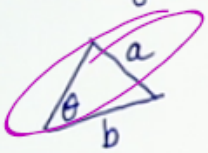


$\frac{x}{\sin 46} = \frac{55}{\sin 62^\circ}$

$x = 44.8\text{ yds}$

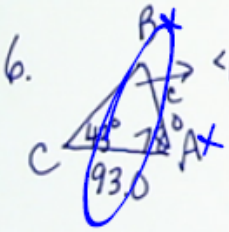
5. (a) a side and angle opposite.
b) SAS OR SSS

(c) Law of Sines: Given 2 sides and an angle



when $a < b$

6.

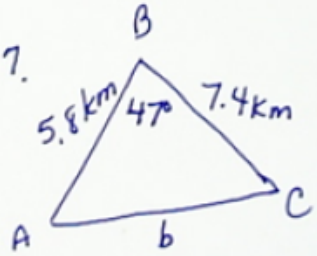


$\angle B = 59°$

$$\frac{c}{\sin 43°} = \frac{93}{\sin 59°}$$

$c = 73.99$

7.

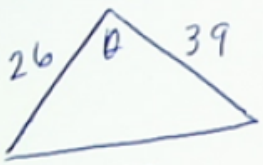


$b^2 = 5.8^2 + 7.4^2 - 2(5.8)(7.4)\cos 47°$

$b^2 = 29.857$

$b = 5.46\text{km}$

8.

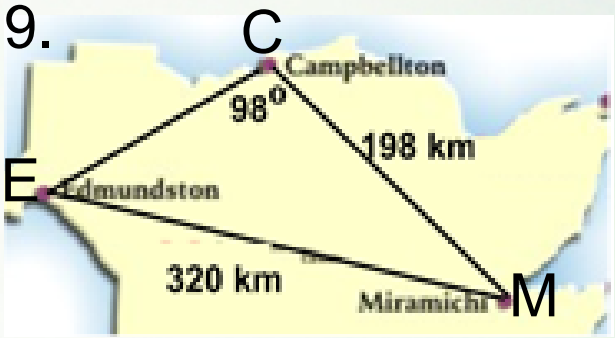


$\cos\theta = \frac{26^2 + 39^2 - 52^2}{2(26)(39)}$

$\cos\theta = -0.25$

$\theta = 104.5°$

9.



~~find~~ E

$\frac{198}{\sin E} = \frac{320}{\sin 98°}$

$\sin E = \frac{198 \sin 98}{320}$

$\sin E = 0.6127$

$E = 37.8°$

$\angle E = 37.8°$

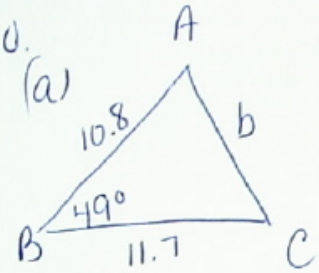
$\angle M = 44.2°$

$\frac{x}{\sin 44.2°} = \frac{320}{\sin 98}$

$x = 225.3\text{km}$

10.

(a)



$b^2 = 10.8^2 + 11.7^2 - 2(10.8)(11.7)\cos 49^\circ$

$b^2 = 87.7308$

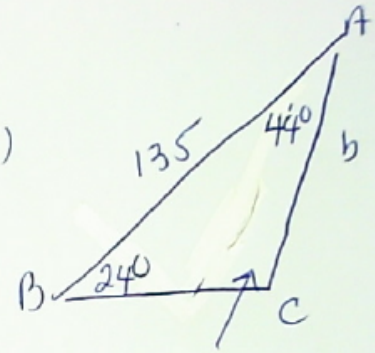
$b = 9.4$

$\frac{\sin C}{10.8} = \frac{\sin 49}{9.4}$

$\sin C = 0.8671$

$C = 60^\circ \quad A = 71^\circ$

(b)



$C = 112^\circ$

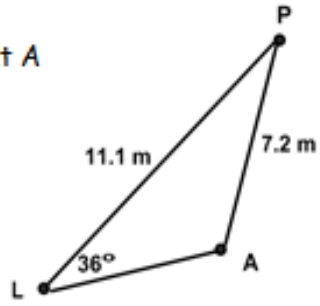
$\frac{b}{\sin 24^\circ} = \frac{135}{\sin 112^\circ}$

$b = 59.2$

$\frac{a}{\sin 44^\circ} = \frac{135}{\sin 112^\circ}$

$a = 101.1$

11. Determine the measure of the obtuse angle at A in triangle PAL



$\frac{\sin A}{11.1} = \frac{\sin 36^\circ}{7.2}$

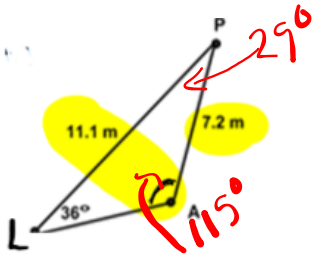
$\sin A = \frac{11.1 \sin 36^\circ}{7.2}$

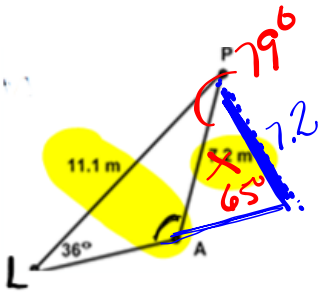
$\sin A = 0.9062$

$A = \sin^{-1} 0.9062$

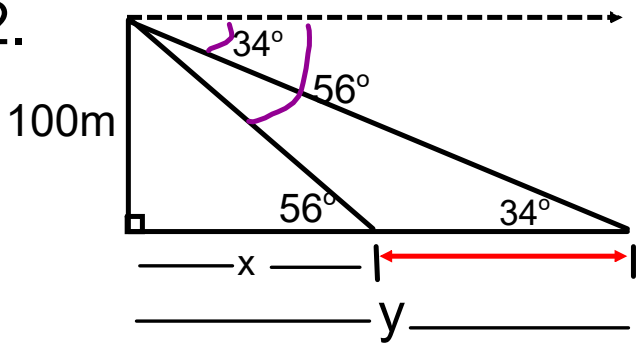
$A = 65^\circ$

other $\angle$: $180 - 65 = 115^\circ$ obtuse





12.



$\tan 56^\circ = \frac{100}{x}$

$x = \frac{100}{\tan 56^\circ}$

$x = 67.5$ m

$\tan 34^\circ = \frac{100}{y}$

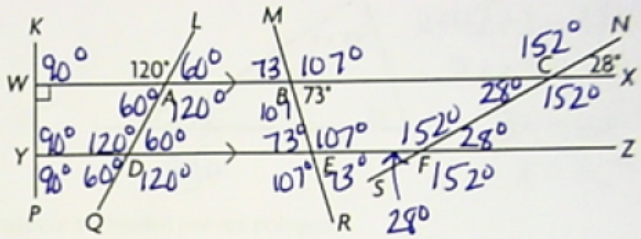
$y = \frac{100}{\tan 34^\circ}$

$y = 148.3$ m

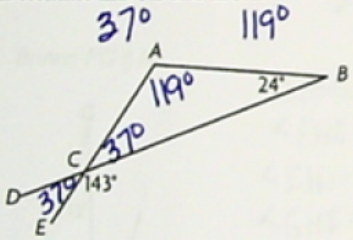
148.3m - 67.5m = 80.8m

Unit Four: Geometry

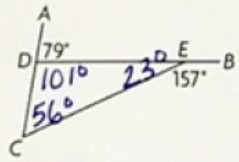
1. Determine the measure of all unknown angles



2. Detemine $\angle DCE$ and $\angle CAB$?



3. Determine the correct measures of the interior angles of ΔCDE



4. a. Determine the sum of the measures of the interior angles of this polygon.
b. Are each angle the same measure

(a) $180(8-2) = 1080^\circ$

(b) NO, sides are not equal

5. Each interior angle of a regular convex polygon measures 144°. How many sides does the polygon have?

$\frac{180(n-2)}{n} = 144$ 10 sides

$180(n-2) = 144n$

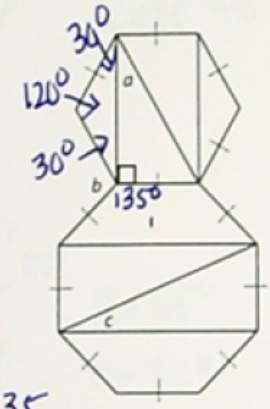
$180n - 360 = 144n$

$180n - 144n = 360$

$36n = 360$

$n = 10$

6. Determine the value of b.



6 sides $180(6-2) = 720$

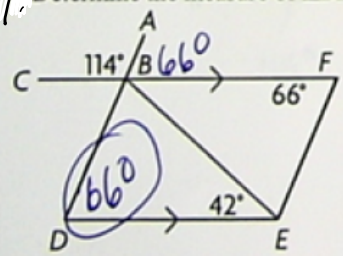
$\frac{720}{6} = 120^\circ$

8 sides $180(8-2) = 1080$

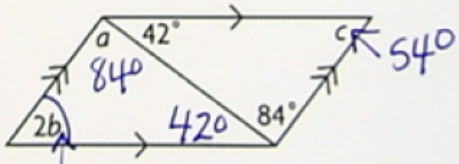
$\frac{1080}{8} = 135$

$\angle b = 105^\circ$

7. Determine the measure of $\angle BDE$.



8. Determine the values of a, b, and c.



$2b = 54$

$\therefore b = 27$

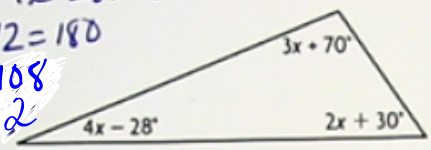
9. Determine the value of x.

$3x+70+4x-28+2x+30=180$

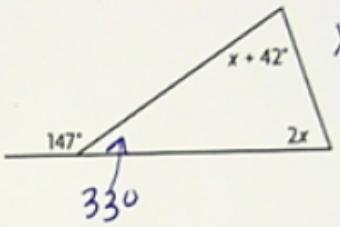
$9x+72=180$

$9x=108$

$x=12$



10. Determine the value of x.



$x+42^\circ+2x+33=180^\circ$

$3x+75=180$

$3x=105^\circ$

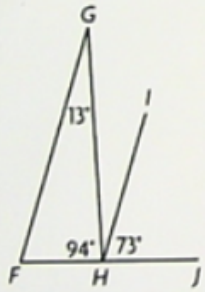
$x=35^\circ$

11. Determine the sum of the measures of the angles in a 13-sided convex polygon. Show your calculation.

$180(13-2)$

$=1980^\circ$

12. Prove: $FG \parallel HI$

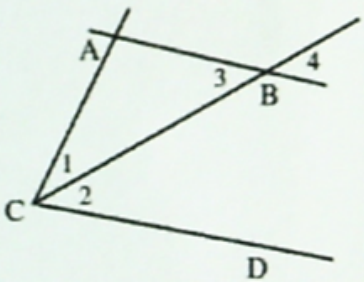


State.	Justification
$\angle FHG=94^\circ$	Given
$\angle IHJ=73^\circ$	Given
$\angle GHI=13^\circ$	Supplementary
$\angle FGH=13^\circ$	Given
$\angle FGH=\angle GHI$	proven
$FG \parallel HI$	equal alt $\angle$s

13. Given $AB \parallel CD$

$\angle 1 = \angle 4$

Prove: $\angle 1 = \angle 2$

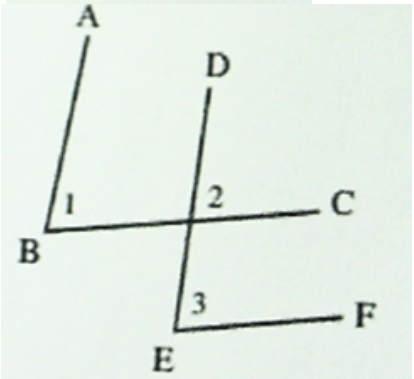


Statement	reason
$AB \parallel CD$	given
$\angle 1 = \angle 4$	given
$\angle 4 = \angle 2$	Corresp. $\angle$s.
$\angle 1 = \angle 2$	transitive

14. Given $BC \parallel EF$

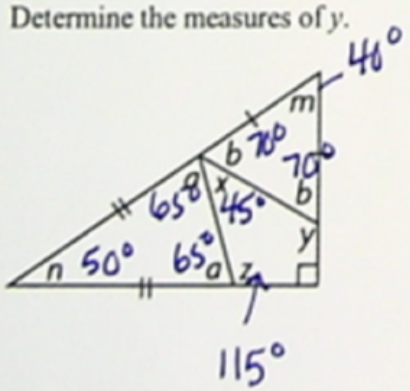
$\angle 1 = \angle 3$

Prove: $AB \parallel DE$



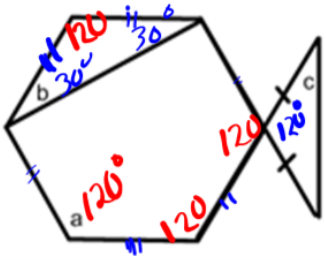
Statement	reason
$BC \parallel EF$	given
$\angle 1 = \angle 3$	given
$\angle 2 = \angle 3$	Corresp. $\angle$s
$\angle 1 = \angle 2$	transitive
$AB \parallel DE$	equal corr. $\angle$s

15. Given $\angle z = 115°$.
Determine the measures of y.



$a = 65°$
$n = 50°$
$m = 40°$
$b = 70°$
$x = 45°$
$y = 110°$

16.



Each $\angle$ $\frac{180(n-2)}{n}$

$\frac{180(6-2)}{6} = 120°$

$\angle a = 120°$
$\angle b = 30°$
$\angle c = 30°$

Unit 5: Financial

1. Patrick purchased a $15000 GIC for 12 years with a simple interest rate of 3.7%. What is his GIC worth in 12 years?

2. Wendy sold her acre of land by the river for $35 000, she plans to invest the money for 20 years. Her options are:
   - ***Option A: 20-year bond at 4..5%, compounded semi-annually.***
   - ***Option B: 10-year GIC at 3.1%, compounded semi-annually; reinvest funds in a 10-year GIC at 5.1%, compounded quarterly.***

   a. Determine the future value of each investment
   b. Determine the rate of return (round to the nearest tenth of a percent) for each investment

Option A:

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$= 35000\left(1+\frac{0.045}{2}\right)^{40}$

$= 35000\left(1+\frac{0.045}{2}\right)^{20(2)}$

$= \$85321.61$

Opt. B

first 10 yrs

$A = 35000\left(1+\frac{0.031}{2}\right)^{20}$

$= 35000\left(1+\frac{0.031}{2}\right)^{10(2)}$

$= \$47606.54$

next 10 yrs: $A = 47606.54\left(1+\frac{0.051}{4}\right)^{40}$

$= 47606.54\left(1+\frac{0.051}{4}\right)^{4(10)}$

$= \$79023.58$

b) $ROR = \frac{\text{Interest earned}}{\text{amt. invested}}$

Opt A $= \frac{50321.61}{35000} = 1.44$ Rate $= 144\%$

Opt B $= \frac{44023.58}{35000} = 1.26$ Rate $= 126\%$

3 a. How much should your parents have invested when you were born if interest rates were 7.3% compounded monthly if they wished to have 430000 for your 18th birthday for school.
b. Approximately how long would it take for a sum of money to double if it is invested at 9.5%

(a) $P = \frac{A}{\left(1+\frac{r}{n}\right)^{nt}}$

$= \frac{430000}{\left(1+\frac{0.073}{12}\right)^{12(18)}}$

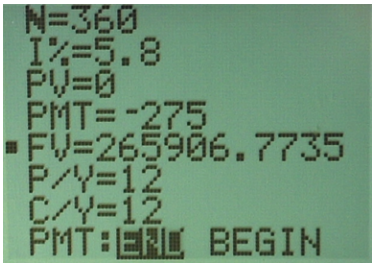
$= \$116021.38$

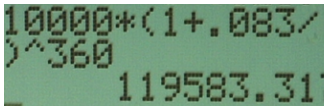
(b) Doubling Period $= \frac{72}{9.5}$ Rule of 72.

$= 7.6$ yrs

4. Sylvia opened this portfolio when she turned 25.
   - Monthly deposits of $275 into an account averaging 5.8%, compounded daily
   - A $10 000 bond earning 8.3%, compounded monthly

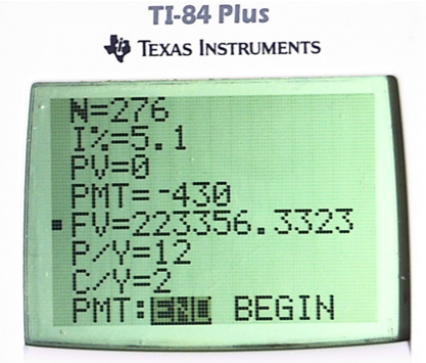
   What will be the value of the portfolio when she turns 55? Show your work.





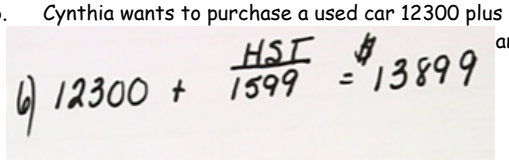
$265906.77
+ $119583.31
$385490.08

5. Barney Rubble regularly deposits $430 per month into a Registered Retirement Savings Plan (RRSP) for his retirement. How much money will he have when he retires in 23 years, knowing that the interest rate is 5.1% compounded semi-annually?

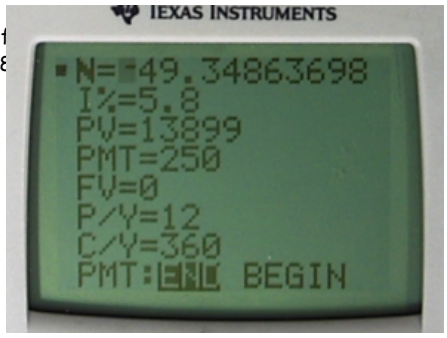


$223356.33

6. Cynthia wants to purchase a used car 12300 plus HST of an of 5.8



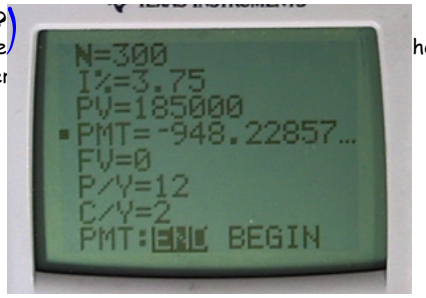
49 payments



7. The Wilsons are buying a house that costs $260 000. They will finance the purchase with a 25 year mortgage with an interest rate of 3.75%, compounded semi-annually. They must make a down payment of $75000.

a) How much will each payment be?
b) How much interest will Debbie e he loan? Show your work.
c) How much will she pay altogether

260000 (c)
- 75000
185000



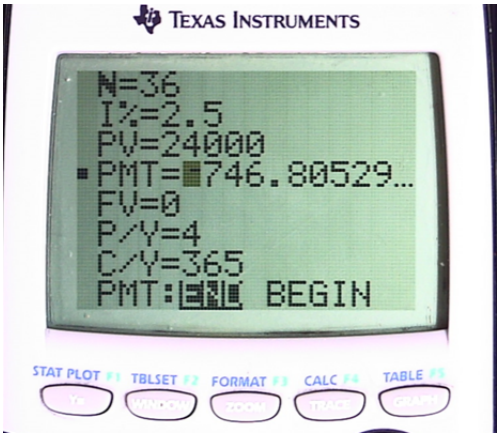
payment $948.23

(b) $948.23 x 12 x 25 = 284469 is what she paid,
She mortgaged $185000.

284469 - 185000
= $99469 interest

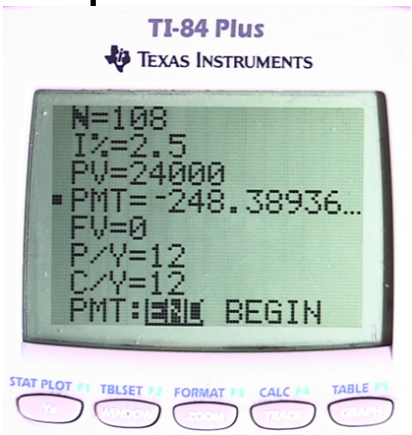
(c) 260000 OR 185000 + 75000 + 99469
99469 mortgage down payment Interest
$359469 = $359469

8. option A



payment $746.81 quarterly
for 9 years
$746.81x4x9 = $26885.16

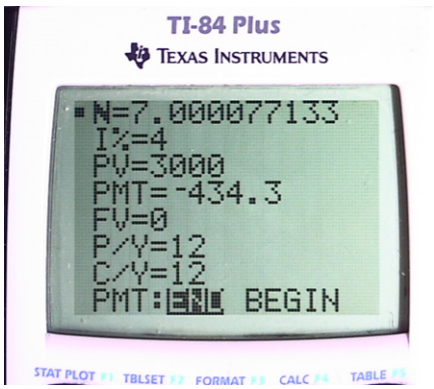
option B



payment $248.39 monthly
for 9 years
$248.39x12x9 = $26933.04

cheaper

9.



7 payments
7 months

a. 7x$434.30 = $3040.10
b. $40.10 interest