

Laws of Exponents

Review From Gr.9

Product of powers law: $a^m \cdot a^n = a^{m+n}$

Quotient of powers law: $\frac{a^m}{a^n} = a^{m-n}$

Power of a power law: $(a^m)^n = a^{mn}$

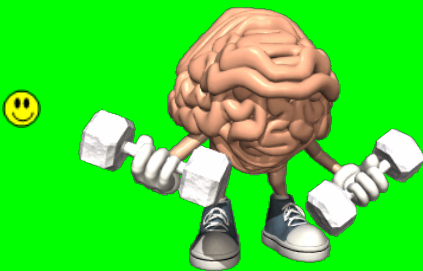
Write as a single power.

a) $3^2 \cdot 3^5 = 3^7$

b) $(4^2)^5 = 4^{10}$

c) $(-5)^{10} \div (-5)^8 = (-5)^2$





Name: _____

Period: _____

Simplify then evaluate

$$1) (2^4)^3$$

power of power

$$2^{12}$$

$$= 4096$$

$$2) [(-5)^2 \times 2]^3$$

$$(-5)^6 \times 2^3$$

$$15625 \times 8$$

$$125000$$

$$3) [(-1)^{11}]^3 = (-1)^{33}$$

$$= -1$$

Write each expression as a product or quotient of powers. Then evaluate.

$$1) [(-3) \times (5)]^2$$

$$(-3)^2 \times (5)^2$$

$$9 \times 25$$

$$225$$

$$2) \left(\frac{6}{5}\right)^4 = \frac{6^4}{5^4} = \frac{1296}{625}$$

4.4 Fractional Exponents and Radicals



LESSON FOCUS

Relate rational exponents and radicals.

Make Connections

Coffee, tea, and hot chocolate contain caffeine. The expression $100(0.87)^{\frac{1}{2}}$ represents the percent of caffeine left in your body $\frac{1}{2}$ h after you drink a caffeine beverage.

Given that $0.87^1 = 0.87$ and $0.87^0 = 1$, how can you estimate a value for $0.87^{\frac{1}{2}}$?



★ Use a calculator to complete the table.

Column 1	Column 2
x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} = 1^{0.5} = 1$
4	$4^{\frac{1}{2}} = 4^{0.5} = 2$
9	$9^{0.5} = 3$
16	$16^{0.5} = 4$
25	

a) What do you notice about the numbers in the first column?

b) Compare the numbers in the first and second columns. What conclusions can you make?



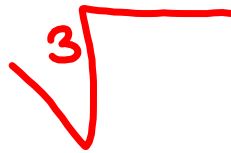
c) What do you think the exponent $\frac{1}{2}$ means?

★ Use a calculator to complete the table.

Column 1 Column 2 Column 3

x	$x^{\frac{1}{3}}$
1	
8	
27	
64	
125	

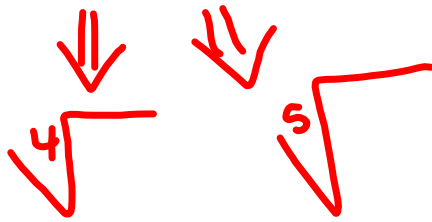
a) What do you notice about the numbers in the first column?



b) Compare the numbers in the first and second columns. What conclusions can you make?

c) What do you think the exponent $\frac{1}{3}$ means?

★ What do you think $a^{\frac{1}{4}}$ and $a^{\frac{1}{5}}$ mean?



What does $a^{\frac{1}{n}}$ mean? Explain your reasoning.

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Rational Exponents and Radicals ☆

Let's examine radicals...

$$\sqrt{5} \times \sqrt{5} = 5$$

How would this play out with exponent laws?

$$5^{0.5} \times 5^{0.5} = 5^1$$

Study

$$\text{RULE: } \sqrt{x} = x^{\frac{1}{2}}$$

Study

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

What about other rational exponents and radicals?

$$8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^1$$

$\downarrow \quad \downarrow \quad \downarrow$
 $2 \times 2 \times 2$
 $= 8$

Rule: $\sqrt[3]{x} = x^{\frac{1}{3}}$

In general... $(\sqrt[n]{x})^m$ OR $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Rational Exponents

- To evaluate exponents that are rational (fractions), the denominator of the fraction indicates which root to take and the numerator indicates which power the entire base is to be raised.

Example

$$16^{\frac{1}{4}} \quad \text{Exponential form}$$

$$= \sqrt[4]{16} \quad \text{Radical form}$$

$$= 2$$

$$125^{\frac{1}{3}}$$

$$= \sqrt[3]{125}$$

$$= 5$$

\swarrow exponent
 \nwarrow index

$$125^{\frac{2}{3}}$$

$$= (\sqrt[3]{125})^2$$

$$= (5)^2$$

$$= 25$$

**Example 1**Evaluating Powers of the Form $a^{\frac{1}{n}} = \sqrt[n]{a}$

Evaluate each power without using a calculator.

a) $27^{\frac{1}{3}}$ b) $0.49^{\frac{1}{2}}$ c) $(-64)^{\frac{1}{3}}$ d) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$



CHECK YOUR UNDERSTANDING



Exponential form

$$X^{\frac{m}{n}}$$

Power (pointing to m)
Root (pointing to n)

=

Radical form

$$\left(\sqrt[n]{X}\right)^m$$

Write the following in radical form and evaluate.

Ex) $8^{\frac{2}{3}}$

$$\left(\sqrt[3]{8}\right)^2$$

$$= (2)^2$$

$$= 4$$

If Exponent is a decimal, then rewrite exponent as a fraction (Reduce Fraction)

$$32^{0.2} =$$

← write as fraction

$$32^{2/10}$$

$$32^{1/5}$$

← Reduce fraction

$$= \sqrt[5]{32}$$

$$= 2$$

Examples: Express each exponential in radical form , then evaluate.



$$1. 8^{\frac{2}{3}} =$$

$$2. 125^{-\frac{1}{3}} =$$

$$3) -32^{0.4}$$



Express as a exponent:

$$a) \sqrt[5]{32} \\ 32^{\frac{1}{5}}$$

$$b) \sqrt[3]{-64} \\ (-64)^{\frac{1}{3}}$$

under 3 and 2.

$$c) (\sqrt{144})^3 \\ 144^{\frac{3}{2}}$$

Express as a Radical:

$$a) 8^{\frac{5}{3}} \\ (\sqrt[3]{8})^5$$

$$b) 49^{\frac{3}{2}} \\ (\sqrt{49})^3$$

$$c) (-125)^{\frac{2}{3}} \\ \sqrt[3]{(-125)^2}$$

Example 2 Rewriting Powers in Radical and Exponent Form

a) Write $40^{\frac{2}{3}}$ in radical form in 2 ways.

a) Use $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ or $\sqrt[n]{a^m}$.

$$40^{\frac{2}{3}} = (\sqrt[3]{40})^2 \text{ or } \sqrt[3]{40^2}$$

b) Write $\sqrt{3^5}$ and $(\sqrt[3]{25})^2$ in exponent form.

$$3^{\frac{5}{2}} \quad 25^{\frac{2}{3}}$$

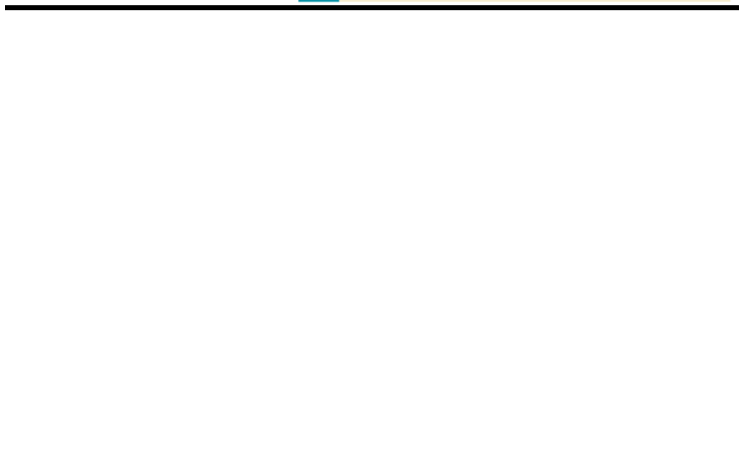
Homework

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4.4 Fractional Exponents and Radicals

Exercises

A



#3 ad
#4 ac
#5 ab
#6 ac
#8 ab
#9
#12 ab
#15

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