Estimating Radicals

Use list of perfect squares, cubes, 4^{ths}, and 5^{th} and Find out which 2 perfects does the radicand fall between.

look in perfect square list

$\sqrt{74}$

$\sqrt{64}$ $\sqrt{81}$

closer

evaluate evaluate

8 9

≈ 8.7

You try $\sqrt{226}$

look perfect cubes

$\sqrt[3]{66}$

$\sqrt[3]{64}$ $\sqrt[3]{125}$

closer

4 5

≈ 4.1

You try $\sqrt[3]{6575}$

You try $\sqrt[4]{240}$

$\sqrt[4]{4070}$

$\sqrt[4]{2401}$ $\sqrt[4]{4096}$

closer

7 8

≈ 7.7

Entire to Mixed (Simplifying)

→ You must use perfect squares, cubes, 4ths and 5th list

→ check to see if radicand appears in the "right" list

$\sqrt{}$ look in perfect squares

$\sqrt[3]{}$ look in cube list

$\sqrt[4]{}$ look in perfect 4ths list

$\sqrt[5]{}$ look in perfect 5th list

→ if it is not in the list look for largest n^{th} root that factors (divides) into it. Locate where the radicand would fit in your list and work backwards until you divide out and find a whole number.

Ex) $\sqrt[3]{750}$ — 750 is not in perfect square list but falls between 1000, 729 so now check to see if divisible by any perfect cubes below 729

Break up

$\underbrace{\sqrt[3]{125}}_{\text{perfect cube that worked}} \times \underbrace{\sqrt[3]{6}}_{\text{number un-cubed not perfect}}$

evaluate ⇓ leave ⇓

$5\sqrt[3]{6}$

750 ÷ 729 ⇒ Decimal ⇒ No

750 ÷ 512 ⇒ Decimal ⇒ No

750 ÷ 343 ⇒ Decimal ⇒ NO

750 ÷ 216 ⇒ Decimal ⇒ NO

750 ÷ 125 = 6 Yes

↑ perfect cube

Ex) ① $\sqrt[4]{8192}$ ② $\sqrt[3]{1536}$ ③ $\sqrt{405}$

Modified

Mixed to Entire

$5\sqrt{3}$ ← this is a square root (understood index of 2)

Move coefficient under radical sign you must raise it to the index and multiply by the radicand already there

$\sqrt{\underbrace{5^2}_{\text{calculate}}\times 3}$

$\sqrt{25 \times 3}$

$= \sqrt{75}$

Ex2) $4\sqrt[3]{5}$

$\sqrt[3]{\underbrace{4^3}_{\text{calculate}}\times 5}$

$= \sqrt[3]{\underbrace{64 \times 5}_{\text{mult}}}$

$= \sqrt[3]{320}$

Practice

a) $3\sqrt{7}$ b) $5\sqrt[3]{7}$ c) $2\sqrt[4]{3}$

d) $6\sqrt{3}$ e) $2\sqrt[3]{9}$ f) $3\sqrt[4]{8}$

Exponent Laws (Grade 9)

1) • Product law → when multiplying like bases you add exponents

$2^4 \cdot 2^7$

like base

2^{4+7}

2^{11}

Ex2) $3x^2y^2 \cdot 6x^3y^1$

$\underbrace{3 \cdot 6}\ x^2 \cdot x^3\ y^2 \cdot y$

$18\ x^{2+3}\ y^{2+1}$

$18\ x^5\ y^3$

2) Quotient Law → when dividing like bases you subtract exponents

Ex) $\dfrac{x^7}{x^4}$

$= x^{7-4}$

$= x^3$

Ex2) $\dfrac{14x^3\ y^{10}}{7x^1\ y^2}$

divide numbers if there

$2\ x^{3-1}\ y^{10-2}$

$\boxed{2\ x^2\ y^8}$

3) power of a power → the power applies to all numbers and letters inside. You multiply each exponent by the exponent outside

Ex) $(3x^2y^5)^2$

remember exponent of 1 on 3 is understood

$\underbrace{3^2}_{\text{evaluate}}\ x^{2\times2}\ y^{5\times2}$

$9\ x^4\ y^{10}$

5

4) Power of Quotient → Power gets multiplied by each exponent inside bracket both on top and bottom

Ex) $\left(\frac{3^{1}x^{7}}{y^{3}}\right)^2$ understood 1

$$\frac{3^{1\times2}x^{7\times2}}{y^{3\times2}}$$

$$\frac{3^{2}x^{14}}{y^{6}}$$

evaluate any number

$$9\frac{x^{14}}{y^{6}}$$

5) Power of Product → the power outside bracket applies to every exponent inside includ. ones on numbers. (Multiply exponents

Ex) $(2^{1}x^{4}y^{2})^{3}$

$$2^{1\times3}x^{4\times3}y^{2\times3}$$

$$\underbrace{2^{3}}_{\text{evaluate on calc}}x^{12}y^{6}$$

$$8x^{12}y^{6}$$

6) Zero law → anything raised to a exponent of zero is 1

Ex) $x^{0}=1$ or Ex2) $(x^{3}y^{7})^{0}=1$

• Every Radical can be expressed as a power

$$(\sqrt[n]{x})^{m} = \underset{\text{base}}{x}^{\frac{m}{n}\,\leftarrow\,\text{power},\ \text{index}}$$

Ex1) $(\sqrt[3]{5})^2 = 5^{\frac{2}{3}}$

Ex2) $(\sqrt{5})^7 = 5^{7/2}$

understood "2" there

• Every Power can be expressed as a Radical

$$8^{\frac{2}{3}} = (\sqrt[3]{8})^2$$

can evaluate

$$(2)^2$$

$$= 4$$

New

$\boxed{\text{Law 7}}$

• Negative exponents

$(x)^{-m} = \left(\frac{1}{x}\right)^m$ then apply power law $\frac{1^m}{x^m} = \frac{1}{x^m}$

write the base as a reciprocal (flip base) then exponent is positive

$\left(\frac{1}{x}\right)^{-m}$ Flip base $\left(\frac{x}{1}\right)^{m}$ ← now pos

don't need

Ex) $\frac{m^2}{m^5} \Rightarrow m^{-3} \Rightarrow \frac{1}{m^3}$ ← now positive exponent

$x^{-2}y^3 \Rightarrow$ to make positive $\frac{y^3}{x^2}$