

4.1

Estimating Roots

MATH LAB



LESSON FOCUS

Explore decimal representations of different roots of numbers.

Make Connections

Since $3^2 = 9$, 3 is a square root of 9.

We write: $3 = \sqrt{9}$

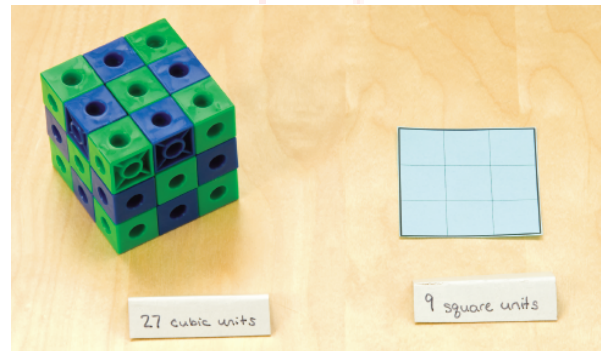
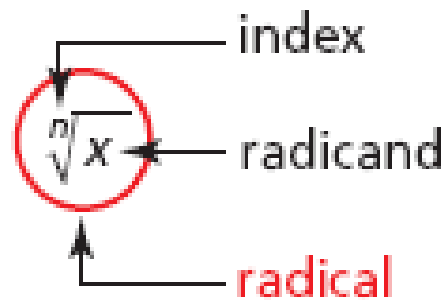
Since $3^3 = 27$, 3 is the cube root of 27.

We write: $3 = \sqrt[3]{27}$

Since $3^4 = 81$, 3 is a fourth root of 81.

We write: $3 = \sqrt[4]{81}$

How would you write 5 as a square root?
A cube root? A fourth root?



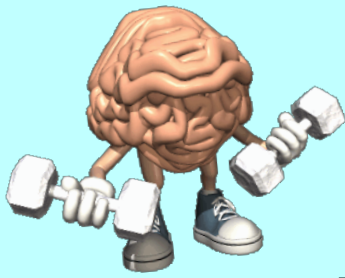


Warm Up

1) $12 - 6 \times 7 - (5-3)^2 + 24 \div 3$

2) To attend the local fair the cost for admission is \$5.25.

If you plan to go on rides it is an additional \$2.00 per ticket. How many rides could you go on if you have \$47.00?



Warm Up

Review From Grade 9

1) $(2 + (5+1)^3 + (-2)^7) \div [2(-1 + 4^2)]$

2) $\frac{(6 \times 14)}{7} + 100 \times 4 \div 5^2$

3) A taxi driver charges a flat fee of \$9.00 and \$3.00 for every kilometre travelled.

a) Write an equation that represents the scenario.

b) If you travel 18 km how much would you have to pay the taxi driver? (use your equation from part a)

b) If you have \$66.00 how far can you travel in the taxi? (use your equation from part a)

What do you know???



$$x^2 =$$

$$x^3 =$$

$$x^4 =$$

Let's try some examples:

a) 3^2

b) 4^3

c) $\sqrt{81}$

d) $\sqrt{49}$

Exponent button

$$\boxed{x^y} \text{ or } \boxed{y^x} \text{ or } \boxed{\wedge}$$

$$x^2 \quad x^3$$

$$\sqrt{\quad}$$



Exponents

★ Exponents are shorthand for multiplication:
 $(5)(5) = 5^2$, $(5)(5)(5) = 5^3$.

★ The "exponent" stands for however many times the term is being multiplied.

Exponent

→
 5^3

(3 times) $5 \times 5 \times 5 = 125$

★ The term that's being multiplied is called the "base".

Base → 5^3

Write each power as a product, then evaluate.

#1 a) 3^4 b) 5^3 c) $\left(\frac{2}{3}\right)^3$ d) $\left(\frac{4}{5}\right)^2$

$$\begin{aligned} \text{a) } 3^4 \\ &= 3 \cdot 3 \cdot 3 \cdot 3 \\ &= 81 \end{aligned}$$

$$\begin{aligned} \text{b) } 5^3 \\ &= 5 \cdot 5 \cdot 5 \\ &= 125 \end{aligned}$$

$$\begin{aligned} \text{c) } \left(\frac{2}{3}\right)^3 & \quad \frac{2^3}{3^3} \\ &= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \\ &= \frac{8}{27} \end{aligned}$$

$$\begin{aligned} \text{d) } \left(\frac{4}{5}\right)^2 \\ &= \frac{4}{5} \times \frac{4}{5} \\ &= \frac{16}{25} \end{aligned}$$

Write each product as a power, then evaluate.

#2

a) $(4)(4)(4)$

$$= 4^3$$

$$= 64$$

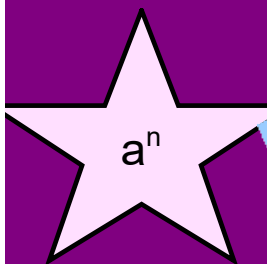
b) $(-6)(-6)(-6)(-6)(-6)$

$$= (-6)^5 \leftarrow \text{odd exponent}$$

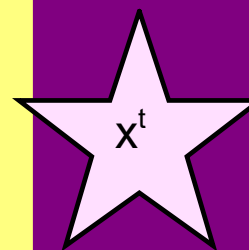
$$= -7776$$

Watch brackets

$$\begin{array}{ccc} -3^2 & \text{vs} & (-3)^2 \\ -3 \cdot 3 & & (-3)(-3) \\ -9 & & +9 \end{array}$$



POWERS



$$\begin{aligned} 1^0 &= 1 \\ 1^1 &= 1 \\ 1^2 &= 1 \\ 1^3 &= 1 \\ 1^4 &= 1 \\ 1^5 &= 1 \end{aligned}$$

$$\begin{aligned} 2^0 &= 1 \\ 2^1 &= 2 \\ 2^2 &= 4 \\ 2^3 &= 8 \\ 2^4 &= 16 \\ 2^5 &= 32 \end{aligned}$$

$$\begin{aligned} 3^0 &= 1 \\ 3^1 &= 3 \\ 3^2 &= 9 \\ 3^3 &= 27 \\ 3^4 &= 81 \\ 3^5 &= 243 \end{aligned}$$

$$\begin{aligned} 4^0 &= 1 \\ 4^1 &= 4 \\ 4^2 &= 16 \\ 4^3 &= 64 \\ 4^4 &= 256 \\ 4^5 &= 1024 \end{aligned}$$

$$\begin{aligned} 5^0 &= 1 \\ 5^1 &= 5 \\ 5^2 &= 25 \\ 5^3 &= 125 \\ 5^4 &= 625 \\ 5^5 &= 3125 \end{aligned}$$

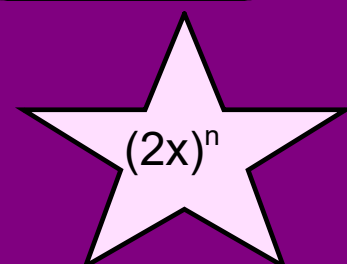
$$\begin{aligned} 6^0 &= 1 \\ 6^1 &= 6 \\ 6^2 &= 36 \\ 6^3 &= 216 \\ 6^4 &= 1296 \\ 6^5 &= 7776 \end{aligned}$$

$$\begin{aligned} 7^0 &= 1 \\ 7^1 &= 7 \\ 7^2 &= 49 \\ 7^3 &= 343 \\ 7^4 &= 2401 \\ 7^5 &= 16807 \end{aligned}$$

$$\begin{aligned} 8^0 &= 1 \\ 8^1 &= 8 \\ 8^2 &= 64 \\ 8^3 &= 512 \\ 8^4 &= 4096 \\ 8^5 &= 32768 \end{aligned}$$

$$\begin{aligned} 9^0 &= 1 \\ 9^1 &= 9 \\ 9^2 &= 81 \\ 9^3 &= 729 \\ 9^4 &= 6561 \\ 9^5 &= 59049 \end{aligned}$$

$$\begin{aligned} 10^0 &= 1 \\ 10^1 &= 10 \\ 10^2 &= 100 \\ 10^3 &= 1000 \\ 10^4 &= 10000 \\ 10^5 &= 100000 \end{aligned}$$







Can you see the difference?

$$-4^2$$

Negative is not
being repeated
Only the #4

$$- 4 \times 4$$

$$- 16$$

$$(-4)^2$$

Here both (-) and (4)
are repeated

$$(-4)(-4)$$

$$+ 16$$

THINK

$$(-1)^2 = (-1) \cdot (-1) = +1$$

$$(-1)^3 = (-1) \cdot (-1) \cdot (-1) = -1$$

$$(-1)^4 = +1$$

$$(-1)^5 = -1$$

odd exponent on (-) makes answer (-)

Did you see a pattern??

$$(-1)^{10247} = -1$$

$$(-1)^{29584} = +1$$

$$(-243)^{301} \rightarrow \text{odd} \rightarrow \text{neg(ans)}$$

$$(-1)^{10247} = -1$$

$$(-1)^{29584} = 1$$

THINK

😊 Evaluating powers when the base is negative...

Study
X

If the exponent is even the answer will be positive.

If the exponent is odd the answer will be negative.



Perfect Squares



$$(1)^2 = 1 \times 1 = 1$$

$$(2)^2 = 2 \times 2 = 4$$

$$(3)^2 = 3 \times 3 = 9$$

$$\text{Ex) } \sqrt{9} = 3$$

$$(4)^2 = 4 \times 4 = 16$$

$$(5)^2 = 5 \times 5 = 25$$

$$(6)^2 = 6 \times 6 = 36$$

$$(7)^2 = 7 \times 7 = 49$$

$$(8)^2 = 8 \times 8 = 64$$

$$(9)^2 = 9 \times 9 = 81$$

$$(10)^2 = 10 \times 10 = 100$$

$$(11)^2 = 11 \times 11 = 121$$

$$(12)^2 = 12 \times 12 = 144$$

$$(13)^2 = 13 \times 13 = 169$$

$$(14)^2 = 14 \times 14 = 196$$

$$(15)^2 = 15 \times 15 = 225$$

$$(16)^2 = 16 \times 16 = 256$$

$$(17)^2 = 17 \times 17 = 289$$

$$(18)^2 = 18 \times 18 = 324$$

$$(19)^2 = 19 \times 19 = 361$$

$$(20)^2 = 20 \times 20 = 400$$

$$(21)^2 = 21 \times 21 = 441$$

$$(22)^2 = 22 \times 22 = 484$$

$$(23)^2 = 23 \times 23 = 529$$





Perfect Cubes



$$(1)^3 = 1 \times 1 \times 1 = 1$$

$$(2)^3 = 2 \times 2 \times 2 = 8$$

$$(3)^3 = 3 \times 3 \times 3 = 27 \rightarrow \text{Ex) } \sqrt[3]{27} = 3$$

$$(4)^3 = 4 \times 4 \times 4 = 64$$

$$(5)^3 = 5 \times 5 \times 5 = 125$$

$$(6)^3 = 6 \times 6 \times 6 = 216$$

$$(7)^3 = 7 \times 7 \times 7 = 343$$

$$(8)^3 = 8 \times 8 \times 8 = 512$$

$$(9)^3 = 9 \times 9 \times 9 = 729$$

$$(10)^3 = 10 \times 10 \times 10 = 1000$$

$$(11)^3 = 11 \times 11 \times 11 = 1331$$

$$(12)^3 = 12 \times 12 \times 12 = 1728$$

$$(13)^3 = 13 \times 13 \times 13 = 2197$$

$$(14)^3 = 14 \times 14 \times 14 = 2744$$

$$(15)^3 = 15 \times 15 \times 15 = 3375$$

$$(16)^3 = 16 \times 16 \times 16 = 4096$$

$$(17)^3 = 17 \times 17 \times 17 = 4913$$

$$(18)^3 = 18 \times 18 \times 18 = 5832$$

$$(19)^3 = 19 \times 19 \times 19 = 6859$$

$$(20)^3 = 20 \times 20 \times 20 = 8000$$

$$(21)^3 = 21 \times 21 \times 21 = 9261$$

$$(22)^3 = 22 \times 22 \times 22 = 10648$$

$$(23)^3 = 23 \times 23 \times 23 = 12167$$

$$(24)^3 = 24 \times 24 \times 24 = 13824$$

$$(25)^3 = 25 \times 25 \times 25 = 15625$$





What do you know???

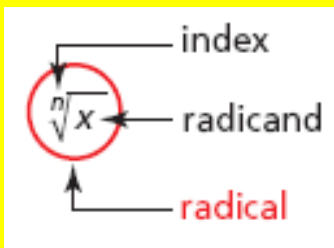
$$\sqrt{8 \times 8}$$

$$\sqrt{8^2} \text{ cancel out } \sqrt{}$$

$$\sqrt{64} = 8$$

$$\sqrt[3]{27}$$

$$\Rightarrow \sqrt[3]{3 \times 3 \times 3} = \sqrt[3]{3^3} = 3$$



$$\sqrt[4]{64}$$

$$\begin{array}{c} \sqrt{98} \\ \swarrow \quad \searrow \\ \sqrt{81} \quad \sqrt{100} \\ \Downarrow \quad \Downarrow \\ 9 \quad 10 \\ \text{closer} \rightarrow \\ \approx 9.9 \end{array}$$

$$\begin{array}{c} \sqrt[4]{64} \\ \swarrow \quad \searrow \\ \sqrt[4]{16} \quad \sqrt[4]{81} \\ \Downarrow \quad \Downarrow \\ 2 \quad 3 \\ \approx 2.6 \end{array}$$

Buttons
→ \sqrt{x}

$$\sqrt[3]{27} = 3$$

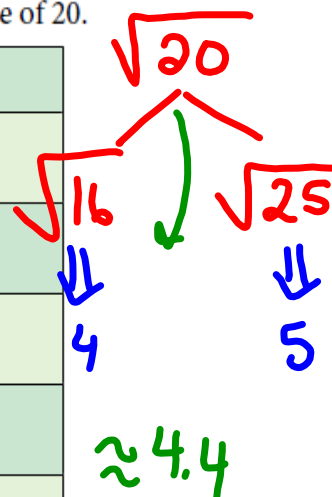
→ $x^{\frac{1}{2}}$ above his x^y

TRY THIS

Write the two consecutive perfect squares closest to 20.

Fill in the table until the square of the estimate is within 1 decimal place of 20.

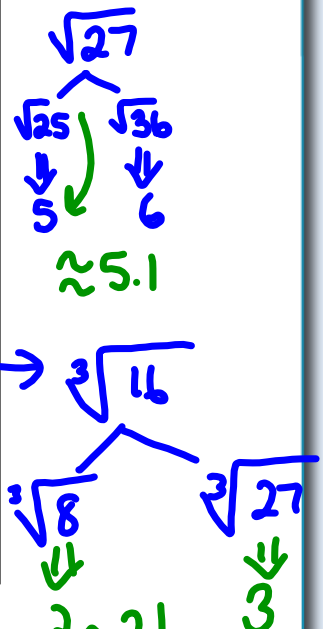
Estimated value of $\sqrt{20}$	Square of estimate



TRY THIS

Determine the value of each radical.

Radical	Value	Is the Value Exact or Approximate?
$\sqrt{16}$	4	Exact
$\sqrt{27}$	≈ 5.1	Approximate
$\sqrt{\frac{16}{81}}$	$\frac{\sqrt{16}}{\sqrt{81}} = \frac{4}{9} = 0.\bar{4}$	Exact
$\sqrt{0.64}$	$\frac{\sqrt{64}}{\sqrt{100}} = \frac{8}{10} = 0.8$	Exact
$\sqrt[3]{16}$		Approx
$\sqrt[3]{27}$	= 3	Exact
$\sqrt[3]{\frac{16}{18}}$		Approx



$\sqrt[3]{\frac{16}{18}} = \frac{\sqrt[3]{16}}{\sqrt[3]{18}}$ ← est
 $\frac{2.1}{2.6} < 1$ ← estimate

0.789 \leftarrow stops in thousandths \rightarrow use place value for denominator

$$\frac{789}{1000}$$

$$\sqrt{\frac{16}{81}} = \frac{\sqrt{16}}{\sqrt{81}} = \frac{4}{9} \text{ Exact}$$

List of Perfect 4^{ths}

USE
Calculators

$$1^4 = 1 \times 1 \times 1 \times 1 = 1$$

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$3^4$$

$$4^4$$

$$5^4$$

.

.

.

.

$$25^4$$

then do
perfect
5^{ths}

$$\begin{aligned}\sqrt{5184} &= \sqrt{144 \times 36} \\ &= \sqrt{144} \times \sqrt{36} \\ &= 12 \times 6 \\ &= 72\end{aligned}$$

1
4
9
16
25
36
49
64
81
100
121
144
169
196
225