Multiple Choice

1. C
2. $t_n=6n+9$
3. D
4. B
5. B
6. D
7. B
8. C
9. A
10. A
11. C
12. D
13. D
14. A

____9. On the first day of the month, Michael places 5¢ in a jar. The next day, he places 7¢ in the jar. The third day, he places 9¢ in the jar, and so on for 24 days. What amount will be in the jar at the end of this period of time?

A. $6.72 B. $6.96 C. $6.36 D. $6.12

5¢, 7¢, 9¢, ... for 24 days

$t_{24} = 5+(24-1)2$
$= 5+23(2)$
$= 5+46$
$= 51¢$

$S_{24} = \frac{24}{2}[2(5)+(24-1)2]$
$= 12[10+23(2)]$
$12[10+46]$
$12[56]$
$672¢$

______10. The population of a community was 82 000 at the beginning of 2000. Assuming a rate of growth of 1.6% per year since 2000, what will the population be at the beginning of 2025?

A. 123894 B. 2082800 C. 121943 D. 120023

2000 2001
82000 ×1.016
r=

$t_n = ar^{n-1}$
$= 82000(1.016)^{25-1}$
$82000(1.016)^{26-1}$

____ 14. Which of the following best describes the series -50 + (-45) + (-81/2) + (-729/20) + ⋯?

A. The series is convergent and has a sum of -500
B. The series is divergent and hasa sum of -500
C. The series is divergent and has no sum
D. The series is convergent and hasa no sum

diverges to ∞

geometric appr

convergent

$S_n = \frac{a[r^n-1]}{[r-1]}$

$\frac{-50\left[\left(\frac{9}{10}\right)^{\infty}-1\right]}{\left[\frac{9}{10}-1\right]}$

$\frac{-50[0-1]}{[-1/10]}$

$= -500$

FORMULAS:

$$t_n = a + (n-1)d$$

$$s_n = \frac{n}{2}[2a + (n-1)d]$$

$$t_n = ar^{n-1}$$

$$S_n = \frac{a[r^n - 1]}{r-1}$$

$$S_\infty = \frac{a}{1-r}$$

$$\sum_{i=1}^{n} c = cn$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

1. $t_8 = \frac{13}{3}$ $t_2 = \frac{1}{3}$

(a)

① $\frac{13}{3} = a + 7d$

② $\frac{1}{3} = a + 1d$

$\frac{12}{3} = 6d$

$4 = 6d$

$\frac{4}{6} = d$

$d = \frac{2}{3}$

$\frac{1}{3} = a + \frac{4}{6}$

$\frac{1}{3} = a + \frac{2}{3}$

$\frac{1}{3} - \frac{2}{3} = a$

$-\frac{1}{3} = a$

(b) $t_n = a + (n-1)d$

$t_n = -\frac{1}{3} + (n-1)\frac{2}{3}$

$t_n = -\frac{1}{3} + \frac{2}{3}n - \frac{2}{3}$

$t_n = \frac{2}{3}n - \frac{3}{3}$

OR

$t_n = \frac{2}{3}n - 1$

(c) (i) $t_{12} = a + (n-1)d$

$= -\frac{1}{3} + (12-1)\frac{2}{3}$

$= -\frac{1}{3} + 11\left(\frac{2}{3}\right)$

$= -\frac{1}{3} + \frac{22}{3}$

$= \frac{21}{3} = 7$

(ii) $t_{26} = -\frac{1}{3} + (26-1)\frac{2}{3}$

$= -\frac{1}{3} + 25\left(\frac{2}{3}\right)$

$= -\frac{1}{3} + \frac{50}{3}$

$= \frac{49}{3}$

(d) (i) $t_n = a + (n-1)d$

$9 = -\frac{1}{3} + (n-1)\frac{2}{3}$

$9 = -\frac{1}{3} + \frac{2}{3}n - \frac{2}{3}$

$9 = -\frac{3}{3} + \frac{2}{3}n$

$9 = -1 + \frac{2}{3}n$

$10 = \frac{2}{3}n$

$30 = 2n$

$15 = n$

(ii) $t_n = a + (n-1)d$

$\frac{97}{3} = -\frac{1}{3} + (n-1)\frac{2}{3}$

$\frac{97}{3} = -\frac{1}{3} + \frac{2}{3}n - \frac{2}{3}$

$\frac{97}{3} = -\frac{3}{3} + \frac{2}{3}n$

$\frac{97}{3} + \frac{3}{3} = \frac{2}{3}n$

$\frac{100}{3} = \frac{2}{3}n$

$\frac{300}{3} = 2n$

$100 = 2n$

$50 = n$

(e) $S_{200} = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{200}{2}\left[2\left(\frac{-1}{3}\right) + (200-1)\frac{2}{3}\right]$

$100\left[-\frac{2}{3} + 199\left(\frac{2}{3}\right)\right]$

$100\left[-\frac{2}{3} + \frac{398}{3}\right]$

$100\left[\frac{396}{3}\right]$

$100\left[132\right]$

13200

2.(a) $t_n = ar^{n-1}$

$162 = ar^{5-1}$ $\quad$ $13122 = ar^{9-1}$

$162 = ar^4$ $\quad$ $13122 = ar^8$

① $13122 = ar^8$

② $162 = ar^4$

① ÷ ② $\quad 81 = r^4$

$\sqrt[4]{81} = r$

$3 = r$

② $162 = a(3)^4$

$162 = a(81)$

$2 = a$

(b) $t_n = ar^{n-1}$

$t_n = 2(3)^{n-1}$

(c) ⓘ $t_3 = ar^{n-1}$

$= 2(3)^{3-1}$

$= 2(3)^2$

$= 18$

(ii) $t_{10} = 2(3)^{10-1}$

$= 2(3)^9$

$= 39366$

(d) ⓘ $t_n = ar^{n-1}$

$9565938 = 2(3)^{n-1}$

$\frac{9565938}{2} = 3^{n-1}$

$4782969 = 3^{n-1}$

$3^{14} = 3^{n-1}$

$14 = n-1$

$15 = n$

(ii) $1458 = 2(3)^{n-1}$

$729 = 3^{n-1}$

$3^6 = 3^{n-1}$

$6 = n-1$

$7 = n$

(e) $S_{10} = \frac{a[r^n - 1]}{[r-1]}$

$= \frac{2[3^{10} - 1]}{[3-1]}$

$= 59048$

$n^{10} = 1024^{(1/10)}$

$n = (1024)$

3. $t_n = 15 - 4n + 2n^2$

$t_1 = 15 - 4(1) + 2(1)^2 = 13$

$t_2 = 15 - 14(2) + 2(2)^2 = 15$

$t_3 = 21$

$t_4 = 31$

$t_5 = 45$

4. Need to find n first:

(a) $12582912 + 6291456 + 3145728 + 1572864 + \ldots + 3$

geometric

$a = 12582912$

$r = \frac{1}{2}$

$t_n = ar^{n-1}$

$3 = 12582912\left(\frac{1}{2}\right)^{n-1}$

$\frac{3}{12582912} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{4194304} = \left(\frac{1}{2}\right)^{n-1}$

$22 = n - 1$

$23 = n$

now find

$S_{23} = \frac{a[r^n - 1]}{[r-1]}$

$= \frac{12582912\left[\left(\frac{1}{2}\right)^{23} - 1\right]}{\left[\frac{1}{2} - 1\right]} = 25165821$

4. (b) $24+30+36+42+\ldots+12030$ arithmetic
$a=24$
$d=6$

find n first

$t_n = a+(n-1)d$

$12030 = 24+(n-1)6$

$12030 = 24+6n-6$

$12030 = 18+6n$

$12012 = 6n$

$2002 = n$

Now find

$S_{2002} = \frac{n}{2}[2a+(n-1)d]$

$= \frac{2002}{2}[2(24)+(2002-1)6]$

$= 12066054$

5. (a) $t_{35}=-411$ $t_{20}=-231$

$-411 = a+(35-1)d$ $-231 = a+(20-1)d$

① $-411 = a+34d$ ② $-231 = a+19d$

① $-411 = a+34d$
② $-231 = a+19d$
①-② $-180 = 15d$
$-12 = d$

Sub $d=-12$

$-231 = a+19(-12)$

$-231 = a-228$

$-3 = a$

$S_{300} = \frac{300}{2}[2(-3)+(300-1)(-12)]$

$= 150[-6+299(-12)]$

$= -539100$

5(b) $t_n = ar^{n-1}$:

$1835008 = ar^{10-1}$ $112 = ar^{3-1}$

$1835008 = ar^9$ $112 = ar^2$

$1835008 = ar^9$
$\div$ $112 = ar^2$
$16384 = r^7$

$\sqrt[7]{16384} = r$

$4 = r$

$112 = a(4)^2$

$112 = 16a$

$\frac{112}{16} = a$

$7 = a$

$S_{14} = \frac{7[4^{14}-1]}{[4-1]} = \boxed{626349395}$

6.(a) $5+15+45+135+$

$$S_\infty = \frac{a[r^n-1]}{[r-1]}$$

$$= \frac{5[3^\infty-1]}{[3-1]}$$

$$S_n = \frac{5[3^\infty-1]}{2}$$

$r>1$ gets bigger

$\therefore$ diverges

$3^\infty \Rightarrow \infty$

(b) $144, 96, 64, \ldots$

$$r = \frac{t_2}{t_1} = \frac{96}{144} = \frac{2}{3}$$

$$t_n = ar^{n-1}$$

$$t_\infty = 144\left(\frac{2}{3}\right)^{\infty-1}$$

$$= 144\left(\frac{2}{3}\right)^\infty$$

$$= 144(0)$$

$$= 144$$

$\left(\frac{2}{3}\right)^\infty \Rightarrow 0$

$$S_n = \frac{a[r^n-1]}{[r-1]}$$

$$S_\infty = \frac{144\left(\left(\frac{2}{3}\right)^\infty - 1\right)}{\left(\frac{2}{3}-1\right)}$$

$$= \frac{144(0-1)}{-\frac{1}{3}}$$

$$= \frac{-144}{-\frac{1}{3}}$$

$$= 432$$

term 1 $\quad t_2 \quad t_3$

7. 5000, 5125, 5253.125, . . .

geometric $\quad r = 1.025$

$t_n = ar^{n-1}$

$t_n = 5000(1.025)^{n-1}$

(b) $t_1 = $ yr 2000 $= 5000$

$t_2 = $ yr 2001 $= 5125$

$t_3 = $ yr 2002 $= 5253.125$

$t_4 = $ yr 2003 $= 5384.45\ldots$

(c) 2008 is term 9

$t_9 = 5000(1.025)^{8-1}$

$= \$6092.01$

(d) $t_n = ar^{n-1}$

$11866 = 5000(1.025)^{n-1}$

$2.3732 = (1.025)^{n-1}$

$\log_{1.025} 2.3732 = n-1$

$35 = n-1$

$36 = n$

In the yr 2036

8. $S_8 = -3280 \quad r = -3$

$$-3280 = \frac{a\left[(-3)^8 - 1\right]}{[-3-1]}$$

$$-3280 = \frac{a[6560]}{[-4]}$$

$$2 = a$$

$$\therefore t_1 = 2$$

9. (a) $\sum_{K=1}^{122} 4k^3$

$$4\left[\frac{n(n+1)}{2}\right]^2$$

$$4\left[\frac{122(123)}{2}\right]^2$$

$$= 225180036$$

(b) $\sum_{K=1}^{1500} 12$

$$= 12n$$

$$= 12(1500)$$

$$= 18000$$

(c) $\sum_{K=1}^{75}(5K^2-10K+2)$

$$5\left[\frac{n(n+1)(2n+1)}{6}\right]-10\,\frac{n(n+1)}{2}+2n$$

$$5\left[\frac{75(76)(151)}{6}\right]-10\left[\frac{75(76)}{2}\right]+2(75)$$

$$= 688900$$

(d) $\sum_{K=50}^{150}(K^3-3)$

$$\sum_{K=1}^{150}(K^3-3) - \sum_{K=1}^{49}(K^3-3)$$

$$\left[\left[\frac{n(n+1)}{2}\right]^2-3n\right]_{K=150} - \left[\left[\frac{n(n+1)}{2}\right]^2-3n\right]_{K=49}$$

128255175-1500478
= -126754697

Assignment solutions

1. (a) not geometric. $\to \infty$

(b) $S_\infty = \dfrac{4.5\left(\left(\frac{2}{3}\right)^{\infty} - 1\right)}{\left(\frac{2}{3} - 1\right)}$

$= \dfrac{4.5(0-1)}{-\frac{1}{3}}$

$= \dfrac{-4.5}{-\frac{1}{3}} = 13.5$ conv to

(c) $t_\infty = 1(5)^{\infty}$ $r > 1$

$= \infty$ $\therefore$ diverge

diverge

(d) $t_\infty = 6\left(\frac{1}{2}\right)^{\infty - 1}$

$= 6(0)$

$= 0$

conv to 0

$S_\infty = \dfrac{6\left[\left(\frac{1}{2}\right)^{\infty} - 1\right]}{\left[\frac{1}{2} - 1\right]}$

$\dfrac{6[0-1]}{-\frac{1}{2}}$

$= 12$ conv

(e) $t_\infty = 200\left(\frac{1}{100}\right)^{\infty - 1}$

$= 200(0)$

conv to 0

$S_\infty = \dfrac{200\left[\left(\frac{1}{100}\right)^{\infty} - 1\right]}{\left[\frac{1}{100} - 1\right]}$

$= \dfrac{200(0-1)}{-\frac{99}{100}} = 202.02\ldots$

conv to:

2.(a) Write the Series and find the sum

$$\sum_{K=1}^{8} \frac{k}{K+1} \Rightarrow \frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}+\frac{7}{8}+\frac{8}{9}$$

$$\sim 6.17$$

(b) $$\sum_{k=3}^{13}(k+2)^2 = \underset{k=3}{25}+\underset{k=4}{36}+49+64+81+100+121+144+169+\underset{k=12}{196}$$

$$= 985$$

3.(a) $\sum_{k=1}^{15} k^2$ quad

$\frac{n(n+1)(2n+1)}{6}$

$\frac{15(16)(31)}{6}$

$= 1240$

(b) $\sum_{k=1}^{60} 9$ constant

$= 9n$

$9(60$

$= 540$

(c) $\sum_{k=1}^{12}(3k^2-5k+4)$

Quad Linear constant

$$3\frac{n(n+1)(2n+1)}{6}-5\frac{n(n+1)}{2}+4n$$

$$3\frac{(12)(13)(25)}{6}-5\frac{(12)(13)}{2}+4(12)$$

1608

(d) $\sum_{k=5}^{15}(5k^3-3)$ b/c it does not start at 1

$$\left[\sum_{k=1}^{15}(5k^3-3)\right]-\left[\sum_{k=1}^{4}(5k^3-3)\right]$$

cubic Linear cubic Linear

$$\left(5\left[\frac{n(n+1)}{2}\right]^2-3n\right)-\left(5\left[\frac{n(n+1)}{2}\right]^2-3n\right)$$

$n=15$ $n=4$

$$\underbrace{\left[5\left[\frac{15(16)}{2}\right]^2-3(15)\right]}_{71955}-\underbrace{\left[5\left[\frac{4(5)}{2}\right]^2-3(4)\right]}_{488}$$

$=71467$

(e) $\sum_{k=70}^{200}(7k-2k^2)$

Linear Quadratic

$$\sum_{k=1}^{200}(7k-2k^2)-\sum_{k=1}^{69}(7k-2k^2)$$

(e) $\sum_{K=70}^{200} (7K - 2K^2)$

Linear Quadratic

5

$$\sum_{K=1}^{200} (7K - 2K^2) - \sum_{K=1}^{69} (7K - 2K^2)$$

$n = 200$ $n = 69$

$$\left[7\,\frac{n(n+1)}{2} - 2\,\frac{n(n+1)(2n+1)}{6}\right] - \left[7\,\frac{n(n+1)}{2} - 2\,\frac{n(n+1)(2n+1)}{6}\right]$$

$$\left[7\,\frac{(200)(201)}{2} - 2\,\frac{(200)(201)(401)}{6}\right] - \left[7\,\frac{(69)(70)}{2} - 2\,\frac{(69)(70)(139)}{6}\right]$$

$$(-5232700) - (-206885)$$

$$= -5025815$$

Attachments

Practice test 2013.doc

Practice test 2017.doc