Section 6.4 Volume and Capacity of Spheres, Cones, and Pyramids, Build Your Skills, p377–378
Student Resource p264–266

Build Your Skills

1. Volume of rectangular prism $= 53 \times 25 \times 30$
$= 39\ 750\ \text{cm}^3$
Volume of rectangular pyramid $= \frac{1}{3}(53 \times 25 \times 15)$
$= 6625\ \text{cm}^3$
Total volume = 39 760 + 6625
Total volume = 46 375 cm^3
The volume of Jayne's model is 46 375 cm^3.

2. a) $V = \frac{4}{3}\pi r^3$
$V = \frac{4}{3}\pi\left(\frac{6}{2}\right)^3$
$V = 113\ \text{mm}^3$
The volume of the ball bearing is 113 mm^3.
b) Doubling the radius results in an increase in volume of 2^3 or 8.
c) The volume will increase by a factor of 4^3 or 64.

3. a) 14.70 mm
b) 7.64 mm
c) 12.48 mm

4. a) 2.325 cm
b) 4.22 cm
c) 1.14 cm

5. a) Volume of cube:
$V = \ell^3$
$V = 54^3$
$V = 157\ 464\ \text{mm}^3$
Volume of sphere $= \frac{4}{3}\pi r^3$
$V = \frac{4}{3}\pi\left(\frac{54}{2}\right)^3$
$V \approx 82\ 448\ \text{mm}^3$
Volume of marble ground away = volume of cube − volume of sphere
$V = 157\ 464 - 82\ 448$
$V = 75\ 016\ \text{mm}^3$
b) Percentage wasted (w)
$$\frac{\text{volume ground away}}{\text{volume of cube}} \times 100$$
$w = \frac{75\ 016}{157\ 464} \times 100\%$
$w \approx 47.6\%$
Almost one half of the marble is wasted.

Extend Your Thinking

6. volume of pyramid = $\frac{1}{3}\ell^2 h_p$
volume of cone = $\frac{1}{3}\pi r^2 h_c$
volume of cone = $\frac{1}{3}\pi\left(\frac{\ell}{2}\right)^2 h_c$
volume of pyramid = volume of cone
$$\frac{1}{3}\ell^2 h_p = \frac{1}{3}\pi\left(\frac{\ell}{2}\right)^2 h_c$$
Divide both sides by $\frac{1}{3}\ell^2$.
$$h_p = \left(\frac{\pi}{4}\right) h_c$$
$h_p = 0.785\ h_c$
The height of the pyramid is slightly more than $\frac{3}{4}$ of the height of the cone.
7. a) Volume of two half-cone ends = volume of one cone
$$V_{\text{ends}} = \frac{1}{3}\pi r^2 h$$
$$V_{\text{ends}} = \frac{1}{3}\pi\left(\frac{51.4}{2}\right)^2 \times 18$$
$$V_{\text{ends}} \approx 12\ 450\ \text{m}^3$$
$$V_{\text{triangular prism}} = \frac{1}{2}(\text{width} \times \text{height}) \times \text{length}$$
$$V = \frac{1}{2}(51.4 \times 18) \times 50$$
$$V = 23\ 130\ \text{m}^3$$
Calculate the total volume.
12 450 + 23 130 = 35 580 m^3
The stockpile contains 35 580 m^3 of ore.
b) Let the angle of repose equal a.
$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$
$$\tan\alpha = \frac{\text{height}}{\text{half width}}$$
$$\tan\alpha = \frac{18}{25.7}$$
$$\alpha = \tan^{-1}\left(\frac{18}{25.7}\right)$$
$$\alpha \approx 35°$$
The angle of repose is 35°.
c) Volume of cone = $\frac{1}{3}\pi r^2 h$
Use the tangent ratio to find a relation between the radius and the height:
$\frac{h}{r}$ = tangent (angle of repose)
$h = r \times \tan 35°$
Substitute the expression for h into the volume formula.
Volume = $\frac{1}{3}\pi r^2 \times r \times \tan 35°$

Volume = $\frac{1}{3}\pi r^3 \times \tan 35°$

Use algebraic manipulation to solve for r^3.

$$r^3 = \frac{3(\text{volume of cone})}{\pi \times \tan 35°}$$

$$r^3 = \frac{3 \times 35\ 580}{\pi \times \tan 35°}$$

$r \approx 36.5$ m

$h = 36.5 \tan 35°$

$h \approx 25.6$ m

The height would be 25.6 m and the diameter would be 73 m.

Students should realize that the angle of repose remains the same.