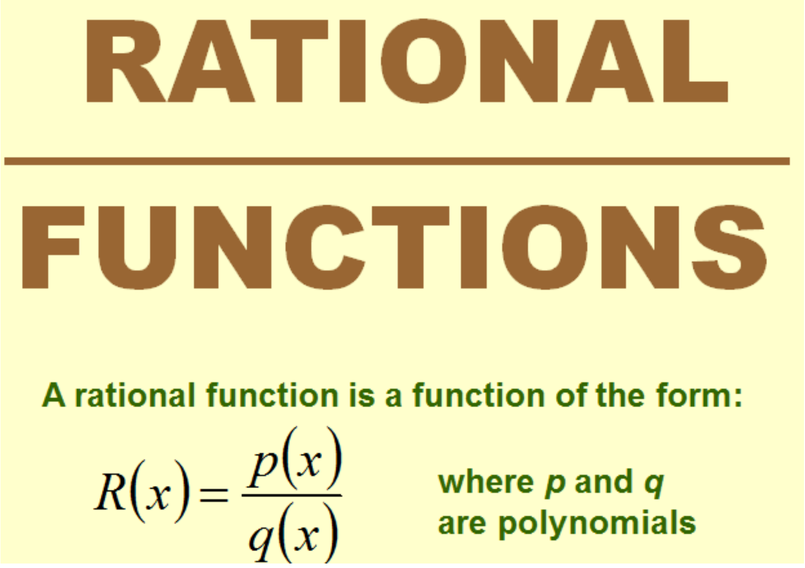


RATIONAL FUNCTIONS

A rational function is a function of the form:

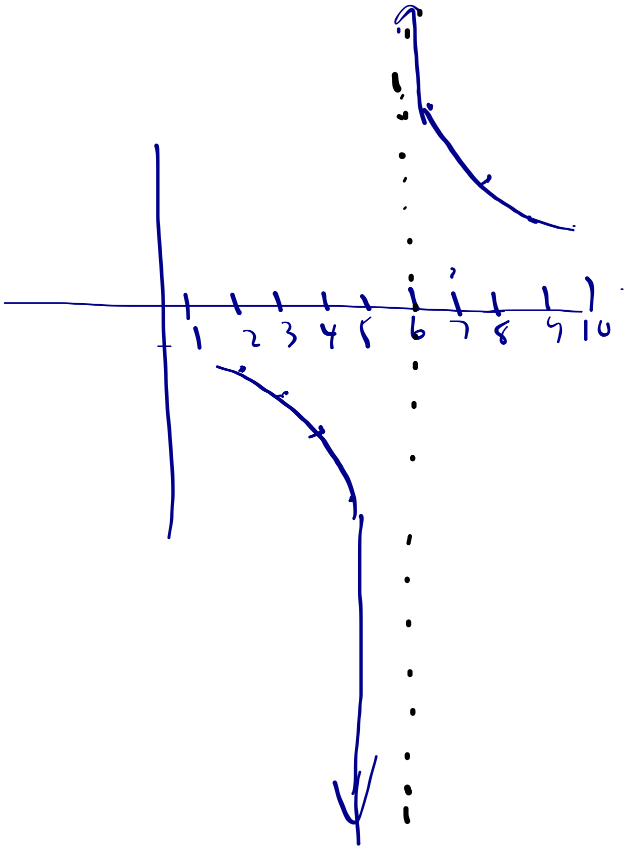
$$R(x)=\frac{p(x)}{q(x)}$$

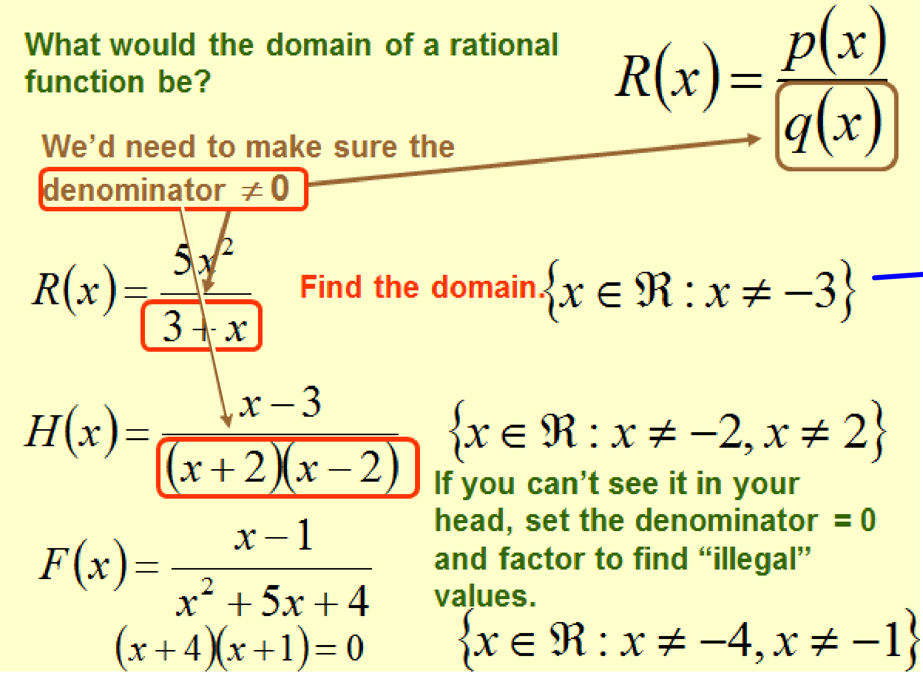
where *p* and *q* are polynomials

example: $\frac{x^3-6x^2+7x+8}{x^2-16}$

$y=\frac{x+4}{x-6}$

x	
5.9	$9.9/-0.1=-99$
5.99	$\frac{9.99}{-0.01}=-999$
6.1	$\frac{10.1}{0.1}=101$
6.01	$\frac{10.01}{0.01}=1001$





What would the domain of a rational function be?

$$R(x)=\frac{p(x)}{\boxed{q(x)}}$$

We'd need to make sure the $\boxed{\text{denominator } \neq 0}$

$$R(x)=\frac{5x^2}{\boxed{3+x}}$$

Find the domain. $\{x \in \Re : x \neq -3\}$

$$H(x)=\frac{x-3}{\boxed{(x+2)(x-2)}}$$

$\{x \in \Re : x \neq -2, x \neq 2\}$

If you can't see it in your head, set the denominator = 0 and factor to find "illegal" values.

$$F(x)=\frac{x-1}{x^2+5x+4}$$

$(x+4)(x+1)=0$

$\{x \in \Re : x \neq -4, x \neq -1\}$

restricted domain
denominator ≠ 0

NOTE: numerator can equal 0

NOTE: denominator cannot equal 0

Example: State the domain

$$g(x) = \frac{x+8}{(x-11)(9-x)} \longrightarrow x\neq 11, x\neq 9$$

$$m(x) = \frac{x^2-4}{3x^2-10x+8} \xrightarrow{\text{factor}} m(x) = \frac{(x-\cancel{2})(x+2)}{(3x-4)(x-\cancel{2})} \quad x\neq \frac{4}{3}, x\neq 2$$

Issue with the 2!

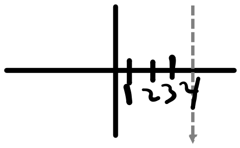
✓ Rational functions are quotients of polynomials: $y=\frac{f(x)}{g(x)}$

✓ Rational functions often have vertical asymptotes, these are vertical lines that the graph will approach but never cross.

✓ Rational functions often have horizontal asymptotes that the graph approaches ......

The vertical asymptote(s) (VA's) are the values of x that are undefined, thus, they are the values that make the denominator 0

Examples:



$f(x) = \frac{-3}{x-4}$ **Vertical Asymptote: x=4**

$g(x) = \frac{5x}{(x-6)^2}$ **Vertical Asymptote: x=6**

$y = \frac{(x+3)}{(x-3)(x+5)}$ **VA: x=3 , x= -5**

$y = \frac{2x-3}{(5x+1)(x-6)(3x-7)}$ **VA:** $x = -\frac{1}{5}$ **,** $x = 6$ **,** $x = \frac{7}{3}$

$y = \frac{(x-5)(x+8)}{(x+1)^2(x-9)}$ **VA: x= -1 , x= 9**

If the denominator is not in factored form then it must be factored to determine the vertical asymptotes.

$$y = \frac{2x+3}{x^2+9x-22} \qquad y = \frac{2x+3}{(x+11)(x-2)} \qquad \text{VA } x=-11,\ x=2$$

factor simple trinomial

factor simple trinomial

$$y = \frac{x^2+4x+5}{3x^2-14x+16} \qquad y = \frac{(x+5)(x+1)}{(3x-8)(x-2)} \qquad \text{VA } x=\frac{8}{3},\ x=2$$

factor hard trinomial

$3x^2-14x+16$

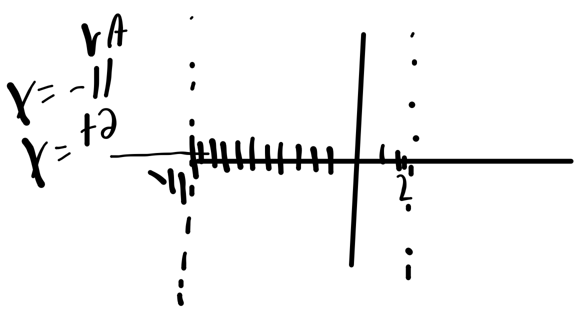
$3x^2-8x-6x+16$

$x(3x-8)-2(3x-8)$

$(3x-8)(x-2)$

$$y = \frac{2x+3}{x^2+9x-22} \ ; \quad y = \frac{2x+3}{(x+11)(x-2)} \leftarrow$$

zeros of denominator are vertical asymptotes



as $x \to -11$ the function $\to$ + or - ∞

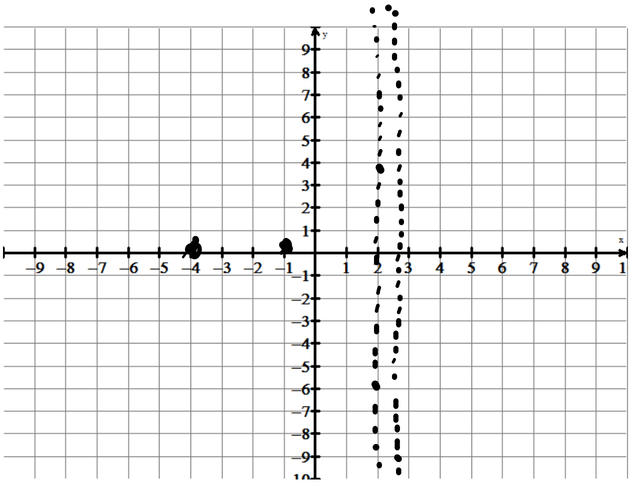
↑ approaches

So Far: the zeros of the denominator
are the vertical asymptotes

Now... what are the zeros off the numerator??
Answer... the zeros of the numerator are
the x-intercepts because y=0
when the numerator is 0

$y = \frac{x^2+5x+4}{3x^2-14x+16}$ $y = \frac{(x+4)(x+1)}{(3x-8)(x-2)}$ x-intercepts: x= -4, x= -1

VA: $x=\frac{8}{3}$, x=2



$k(x) = \dfrac{4x^3}{x^3-4x^2+9x-36}$, $k(x) = \dfrac{4x^3}{(x-4)(x^2+9)}$

x-int : $x = 0$

VA: $x = 4$

factor using
factor theorem

this will not factor
and does not have
any zeros

$p(4)=0$

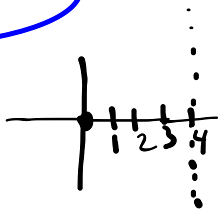
x^2+9

$x-4 \overline{) x^3-4x^2+9x-36}$

x^3-4x^2

$9x-36$

$9x-36$



$y = \dfrac{2x^3-50x}{x^3-64}$, $y = \dfrac{2x(x-5)(x+5)}{(x-4)(x^2+4x+16)}$

x-int : $x = 0$, 5, -5

VA: $x = 4$

common factor
diff of sq

diff of cubes

x^3-64

$(x-4)(x^2+4x+16)$

no zeros check with quad formula

$2x^3-50x$

$2x(x^2-25)$

$2x(x-5)(x+5)$

The y-intercept of any function is found by letting x=0

$y = \dfrac{x^2+x-12}{2x^2+x-15}$, $y = \dfrac{(x+4)(x-3)}{(2x-5)(x+3)}$

find VA, x-int (s) and y-int

VA $x = 9/2$, $x = {}^{-}3$

x-int -4, $+3$

y-int $-12/-15 = 4/5$

$$0 = \frac{x^2+x-12}{2x^2+x-15}$$

$$0 = x^2+x-12$$

y-int { or

$$y = \frac{0^2+(0)-12}{2(0)^2-0-15} = \frac{-12}{15} = \frac{-4}{5}$$

they are the same no matter if you substitute into either form

$$y = \frac{(0+4)(0-3)}{(2(0)-5)(0+3)} = \frac{(4)(-3)}{(-5)(+3)} = \frac{-12}{15}$$

RECAP(for now): Rational Functions!!!!

1. x-intercepts: zeros of the numerator
   y-intercepts: let x=0 and solve for y
2. VA: zeros of the denominator

Examples:

$f(x) = \dfrac{x^2-8x+12}{x^3+6x^2+5x-12}$, $f(x)=\dfrac{(x-6)(x-2)}{(x-1)(x+3)(x+4)}$

x-int: 6, 2
y-int: $\dfrac{12}{-12} = -1$
VA: x=1, x=-3, x=-4

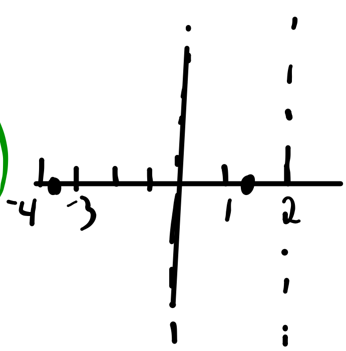
$y = \dfrac{2x^2+5x-10}{6x^2-12x}$, $y = \dfrac{2x^2+5x-10}{6x(x-2)}$

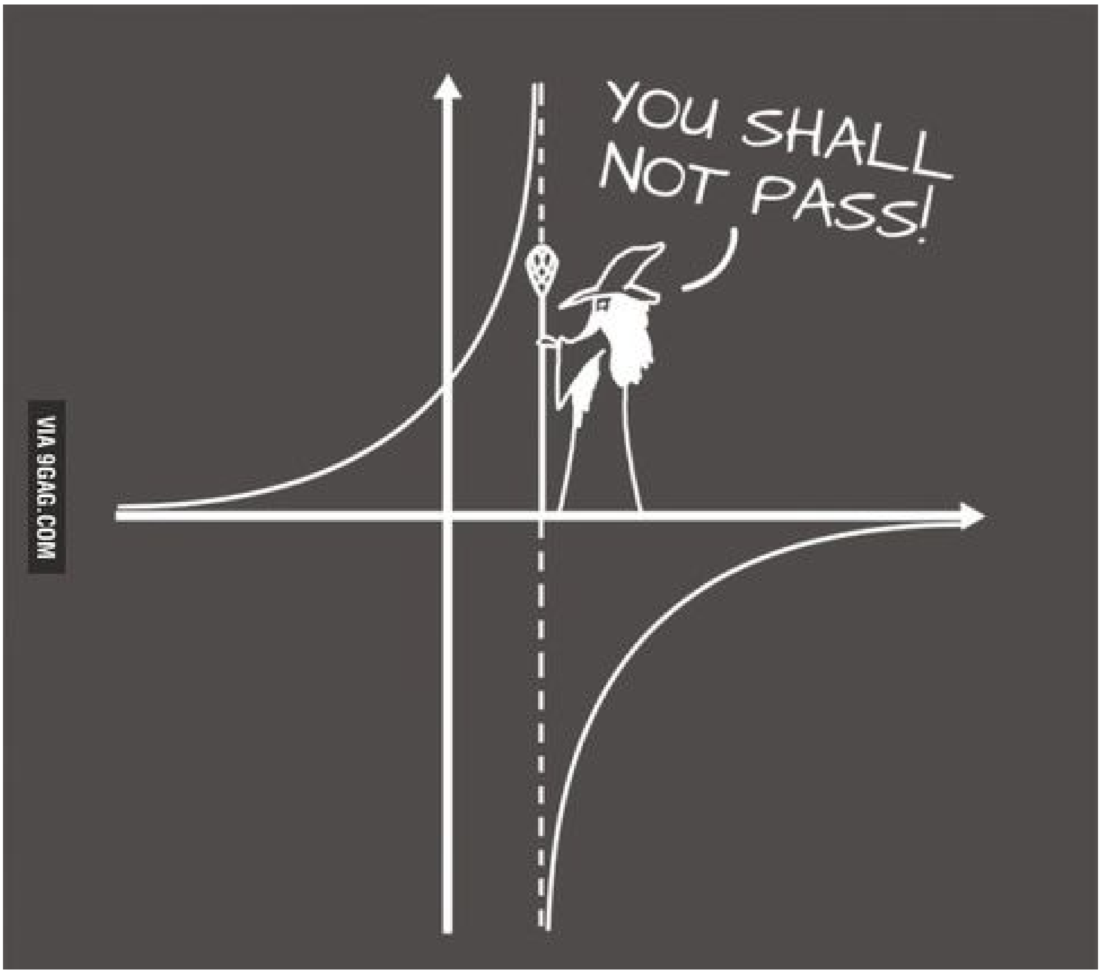
$\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

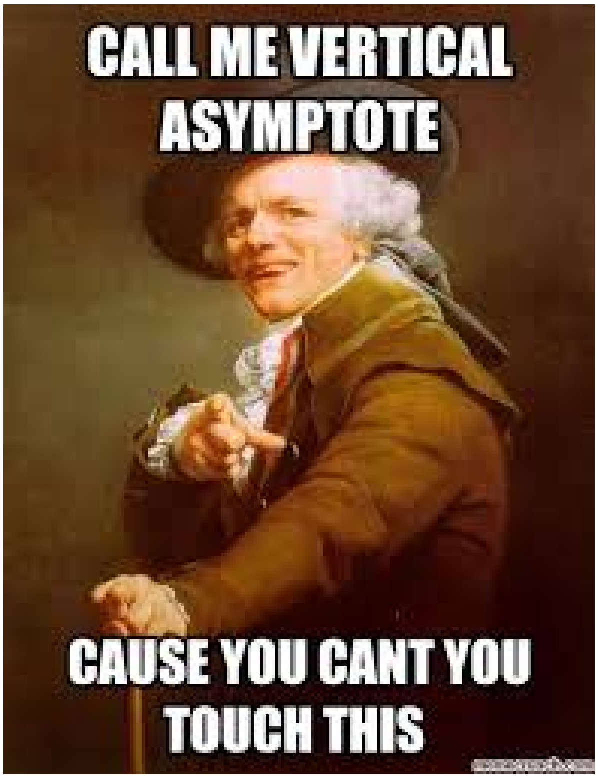
$\dfrac{-5\pm\sqrt{5^2-4(2)(-10)}}{2(2)}$

$\dfrac{-5\pm\sqrt{105}}{4}$

x-int: x= 1.3 , -3.8
y-int: -10/0 = undefined
therefore no y-int
VA: x=0, x=2




YOU SHALL NOT PASS!
VIA 9GAG.COM


CALL ME VERTICAL ASYMPTOTE
CAUSE YOU CANT YOU TOUCH THIS