

Build Your Skills

1. a) Find the hypotenuse using the Pythagorean theorem.

$$t^2 = r^2 + s^2$$

$$t^2 = 2.5^2 + 7^2$$

$$t^2 = 6.25 + 49$$

$$t^2 = 55.25$$

$$t = \sqrt{55.25}$$

$$t \approx 7.43 \text{ cm}$$

Find R using the tangent function and its inverse.

$$\tan R = \frac{2.5}{7}$$

$$\tan R \approx 0.357142857$$

$$R = \tan^{-1} 0.357142857$$

$$R \approx 20^\circ$$

Or, a more direct solution is:

$$\tan^{-1} \frac{2.5}{7} \approx 20$$

$$S = 180^\circ - 90^\circ - 20^\circ$$

$$S \approx 70^\circ$$

b) $M = 90^\circ - 58^\circ$

$$M = 32^\circ$$

l and m can be found using the sine and cosine functions respectively using L.

$$\sin L = \frac{l}{n}$$

$$\sin 58^\circ = \frac{l}{6.3}$$

$$l = 6.3 \times \sin 58^\circ$$

Multiply both sides by 6.3.

$$l \approx 5.3 \text{ in}$$

$$\cos L = \frac{m}{n}$$

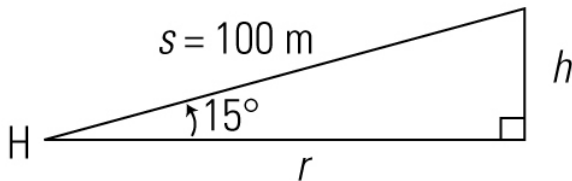
$$\cos 58^\circ = \frac{m}{6.3}$$

$$m = 6.3 \times \cos 58^\circ$$

Multiply both sides by 6.3.

$$m \approx 3.3 \text{ in}$$

2. a)



$$\sin H = \frac{h}{s}$$

$$\sin 15^\circ = \frac{h}{100}$$

$$h = 100 \sin 15^\circ$$

Multiply both sides by 100.

$$h \approx 25.9 \text{ m}$$

You would gain 25.9 m.

$$\text{b) } \cos H = \frac{r}{s}$$
$$\cos 15^\circ = \frac{r}{100}$$

$$r = 100 \cos 15^\circ$$

Multiply both sides by 100.

$$r \approx 96.6 \text{ m}$$

You would cover 96.6 m.

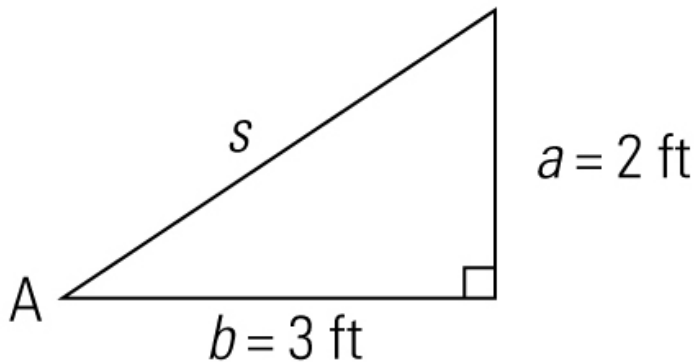
$$\text{c) } \text{grade} = \frac{\text{rise}}{\text{run}} \times 100\%$$

$$\text{grade} = \frac{25.9}{96.6} \times 100$$

$$\text{grade} \approx 26.8\%$$

The grade is 26.8%.

3.



a) Find A

$$A = \tan^{-1}\left(\frac{2}{3}\right)$$

$$A \approx 34^\circ$$

The angle of elevation is 34° .

b) Find the slant height of the roof using the Pythagorean theorem. The height of the triangle will be 12 ft (which is 2 times 6). The other leg is therefore 3 times 6, or 18 ft.

$$s^2 = 12^2 + 18^2$$

$$s^2 = 144 + 324$$

$$s^2 = 468$$

$$s^2 = \sqrt{468}$$

$$s \approx 21.6 \text{ ft}$$

Calculate the area of one side to be shingled.

$$l \times w = A$$

$$21.6 \times 20 = 432 \text{ ft}^2/\text{side}$$

Calculate the total area to be shingled.

$$432 \times 2 = 864 \text{ ft}^2$$

The total area to be shingled is 864 square feet.

$$4. \text{ a) } 13^2 + 8^2 = x^2$$

$$169 + 64 = x^2$$

$$233 = x^2$$

$$\sqrt{233} = x$$

$$15.3 \text{ m} = x$$

The extension cylinder must be 15.3 m long.

$$\text{b) } A = \tan^{-1}\left(\frac{13}{8}\right) \approx 58^\circ$$

The angle of elevation is 58° .

$$5. \text{ a) } A = \sin^{-1}\left(\frac{12}{15}\right)$$

$$A \approx 53^\circ$$

The angle of elevation is 53° .

b) Use the Pythagorean theorem for accuracy.

$$h^2 + l^2 = c^2$$

$$12^2 + b^2 = 15^2$$

$$144 + b^2 = 225$$

$$b^2 = 225 - 144$$

$$b^2 = 81$$

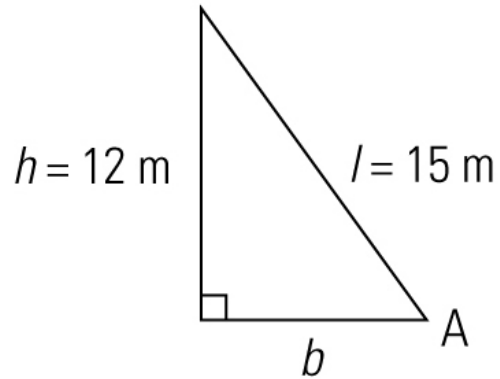
$$b^2 = \sqrt{81}$$

$$b = 9 \text{ m}$$

The ladder is 9 m from the base of the apartment.

Alternative Solution

Since 15 equals 3 times 5, and 12 equals 3 times 4, this is a 3-4-5 Pythagorean triple, so the other leg is 3 times 3 or 9 m.



6. The angle of depression will be the same as the angle of elevation from the top of the theodolite to the top of the cliff.

Subtract the theodolite's height from the height of the cliff.

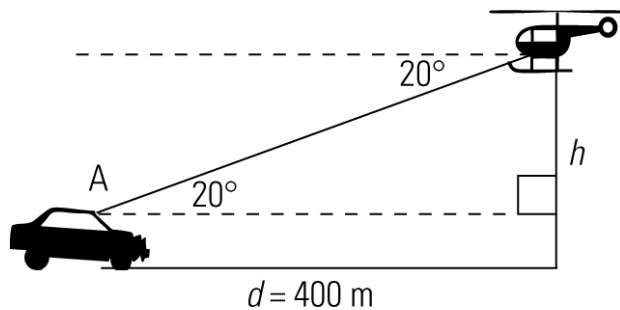
$$7.2 - 1.8 = 5.4 \text{ m}$$

Use the tangent ratio.

$$\tan^{-1}\left(\frac{5.4}{15.9}\right) \approx 19^\circ$$

The angle of depression is 19° .

7. a)



Find h , the height above the ground when the angle of depression is 20° .

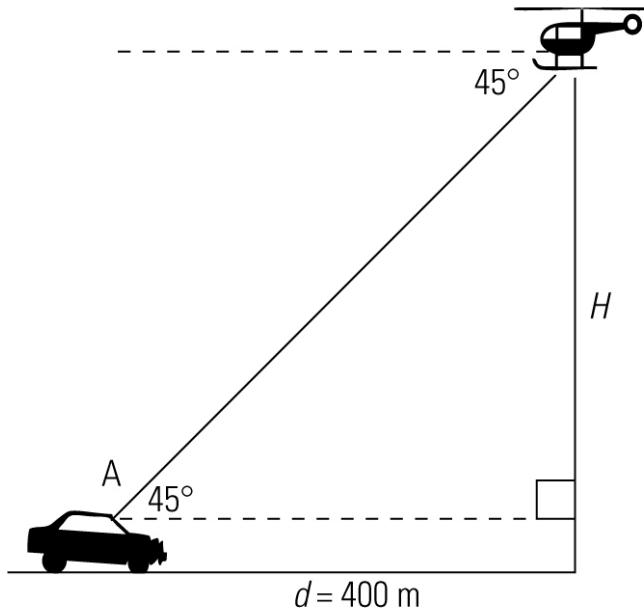
$$\tan A = \frac{h}{d}$$

$$\tan 20^\circ = \frac{h}{400}$$

$$h = 400 \tan 20^\circ$$

Multiply both sides by 400.

$$h \approx 145.6 \text{ m}$$



Find H , the height above the ground when the angle of depression is 45° .

$$\tan A = \frac{H}{d}$$

$$\tan 45^\circ = \frac{H}{400}$$

$$H = 400 \tan 45^\circ$$

Multiply both sides by 400.

$$H \approx 400 \text{ m}$$

Subtract the two to find how much the helicopter rose in 3 minutes.

$$400 - 145.6 = 254.4 \text{ m}$$

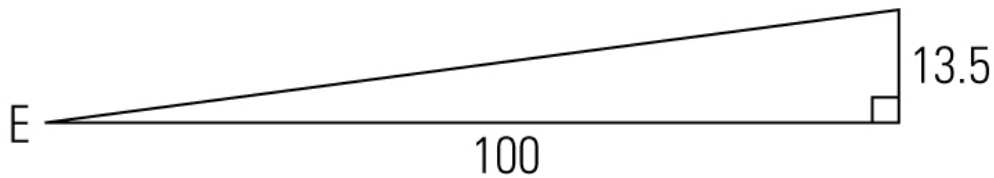
The helicopter rose 254.4 m in 3 minutes.

b) Divide 254.4 by 3 to determine the speed per minute.

$$254.4 \div 3 = 84.8$$

The helicopter's speed was approximately 84.8 m/min, or 1.4 m/sec.

8.



A grade of 13.5% means there is a rise of 13.5 units per 100 units of run.

$$E = \tan^{-1} \left(\frac{\text{rise}}{\text{run}} \right)$$

$$E = \tan^{-1} \left(\frac{13.5}{100} \right)$$

$$E \approx 8^\circ$$

The angle of elevation is 8° .

Extend Your Thinking

9. a) Use the tangent function to find the run for each of the angled pipes.

Left-hand side:

$$\tan x = \frac{\text{opp}}{\text{adj}}$$

$$\tan 60^\circ = \frac{1.8}{r}$$

$$r \tan 60^\circ = 1.8$$

$$\frac{r \tan 60^\circ}{\tan 60^\circ} = \frac{1.8}{\tan 60^\circ}$$

$$r \approx 1.0$$

Right-hand side:

$$\tan x = \frac{\text{opp}}{\text{adj}}$$

$$\tan 45^\circ = \frac{1.8}{m}$$

$$m \tan 45^\circ = 1.8$$

$$\frac{m \tan 45^\circ}{\tan 45^\circ} = \frac{1.8}{\tan 45^\circ}$$

$$h \approx 2.1$$

Multiply both sides by r .

Divide both sides by $\tan 60^\circ$.

Multiply both sides by m .

Divide both sides by $\tan 60^\circ$.

Alternatively, many students may have realized that if the elbow was 45° , the run would be equal to the offset.

Calculate to find the total distance between the pipes.

$$1.0 + 2.6 + 1.8 = 5.4 \text{ m}$$

The pipes are approximately 5.4 m apart.

b) Use the sine function to determine the length of each travel pipe.

$$\sin X = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 60^\circ = \frac{1.8}{h}$$

$$h \sin 60^\circ = 1.8$$

$$\frac{h \sin 60^\circ}{\sin 60^\circ} = \frac{1.8}{\sin 60^\circ}$$

$$m \approx 1.8$$

The left travel pipe is about 2.1 m long.

$$\sin X = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 45^\circ = \frac{1.8}{n}$$

$$n \sin 45^\circ = 1.8$$

$$\frac{n \sin 45^\circ}{\sin 45^\circ} = \frac{1.8}{\sin 45^\circ}$$

$$n = \frac{1.8}{\sin 45^\circ}$$

$$n \approx 2.5$$

Multiply both sides by h .

Divide both sides by $\sin 60^\circ$.

Multiply both sides by n .

Divide both sides by $\sin 45^\circ$.

The right travel pipe is about 2.5 m long.

Add to find the total length of the pipe needed.

$$2.1 + 2.6 + 2.5 = 7.2$$

The pipefitter will need about 7.2 m of pipe to get around the obstruction.

c) One tenth of a metre is 10 cm, which is a big discrepancy in length for pipe fitting. He would probably round to the nearest cm, and the elbow could probably accommodate the difference.