Section 8.1 The Pythagorean Theorem, Build Your Skills, p491-496
Student Resource p332-336

## Build Your Skills

1. a) $\mathrm{AB}^{2}=b^{2}+\mathrm{BC}^{2}$
$4.1^{2}=b^{2}+3.4^{2}$
$16.81=b^{2}+11.56$
$16.81-11.56=b^{2}$
$b^{2}=5.25$
$b=\sqrt{5.25}$
$b \approx 3.9$
AC is 2.3 m high.
b) $e=2.3-1.2$
$e=1.1$
$d^{2}=a^{2}+e^{2}$
$d^{2}=4.1^{2}+1.1^{2}$
$d^{2}=16.81+1.21$
$d^{2}=18.02$
$d=\sqrt{18.02}$
$d \approx 4.2$
AE is 4.2 m long.
2. a) Redraw the portion of the diagram that forms a right triangle, including the overhang, and label appropriately.


Since the base of the triangle is half the width of the garage, it is 3.75 m .
Solve for $c$.
$c^{2}=a^{2}+b^{2}$
$c^{2}=1.2^{2}+3.75^{2}$
$c^{2}=1.44+14.0625$
$c^{2}=15.5025$
$c=\sqrt{15.5025}$
$c \approx 3.9$
Calculate the length of rafter needed.
rafter length $=c+0.6$
rafter length $=3.9+0.6$
rafter length $=4.5 \mathrm{~m}$
Therefore, the rafter will have to be 4.5 m long.
b) This solution will be calculated the same as in part a), except that there will be two
diagrams, as below. In each case, the roof will be 1.2 m higher than the walls and will have an overhang of 0.6.

$c^{2}=b^{2}+a^{2}$
$c^{2}=1.2^{2}+3^{2}$
$c^{2}=1.44+9$
$c^{2}=10.44$
$c=\sqrt{10.44}$
$c \approx 3.23$

$c^{2}=a^{2}+b^{2}$
$c^{2}=1.2^{2}+4.5^{2}$
$c^{2}=1.44+20.25$
$c^{2}=21.69$
$c=\sqrt{21.69}$
$c \approx 4.66$
Calculate the length of the rafters.
left rafter length $=3.23+0.6$
left rafter length $=3.83 \mathrm{~m}$
right rafter length $=4.66+0.6$
right rafter length $=5.26 \mathrm{~m}$
The rafters will have to be 3.83 m long and 5.26 m long.
3. The top will be a rectangle that is 90 cm by 70 cm .

Find the hypotenuse to find the lengths of the two copper wires needed.
$c^{2}=$ length $^{2}+$ width $^{2}$
$c^{2}=90^{2}+70^{2}$
$c^{2}=8100+4900$
$c^{2}=13000$
$c=\sqrt{13000}$
$c \approx 114$

The top will need twice this length.
$2 \times 114=228 \mathrm{~cm}$ or 2.28 m
The front and the back will be 90 cm by 60 cm . There will be 4 diagonals.
$c^{2}=$ length $^{2}+$ width $^{2}$
$c^{2}=90^{2}+60^{2}$
$c^{2}=8100+3600$
$c^{2}=11700$
$c=\sqrt{11700}$
$c \approx 108$
The front and back will need a total of 4 times this length.
$4 \times 108=432 \mathrm{~cm}$ or 4.32 m of wire
The ends will be 70 cm by 60 cm and there will be a total of 4 diagonals.
$c^{2}=$ width $^{2}+$ height $^{2}$
$c^{2}=70^{2}+60^{2}$
$c^{2}=4900+3600$
$c^{2}=8500$
$c=\sqrt{8500}$
$c \approx 92$
$92 \times 4=368 \mathrm{~cm}$ or 3.68 m
The ends will need a total of 368 cm or 3.68 m of wire.
Total the lengths of the pieces of wire.
$228+432+368=1028 \mathrm{~cm}$
Suzanne will need 1028 cm or 10.28 m of wire.
Suzanne might want to buy a little more wire because most numbers in this case were rounded down, and if she wants it to go right to the corner she may be a bit short. On the other hand, the amounts rounded were insignificant (less than 0.2 cm ) and would probably not be noticeable.
4. a) $c^{2}=h^{2}+g^{2}$
$c^{2}=10^{2}+6^{2}$
$c^{2}=100+36$
$c^{2}=136$
$c=\sqrt{136}$
$c \approx 11.66$
$11.66 \mathrm{~m}=1166 \mathrm{~cm}$
Therefore, each guy wire will be 1166 cm long, so they will need double this plus the 153 cm needed for fastening.
$1166+1166+153=2485$
They need 2485 cm or 24.85 m of wire.
b) Guy wires are needed to secure a tall structure by giving it increased stability.
5.
a) The diameter will be the

diagonal of the square. Therefore, use the Pythagorean theorem.
$d^{2}=l^{2}+w^{2}$
$d^{2}=30^{2}+30^{2}$
$d^{2}=900+900$
$d^{2}=1800$
$c=\sqrt{1800}$
$c \approx 42.4 \mathrm{~mm}$
Therefore, he will need a round stock with a diameter of 45 mm . Make sure students notice that the question states that diameters are in multiples of 5. Otherwise, they may think the answer should be 43 mm .
b) Solve this as in a).
$d^{2}=l^{2}+w^{2}$
$d^{2}=0.75^{2}+0.75^{2}$
$d^{2}=0.5625+0.5625$
$d^{2}=1.125$
$c=\sqrt{1.125}$
$c \approx 1.1$ in
He will need stock that is $1 \frac{1}{4}$ " in diameter.
6. The shaft is one leg of the right triangle.
$h^{2}+g^{2}=s^{2}$
$h^{2}+75^{2}=94^{2}$
$h^{2}+5625=8836$
$h^{2}=8836-5625$
$h^{2}=3211$
$c=\sqrt{3211}$
$c \approx 57 \mathrm{~m}$
The shaft is 57 m long.
7. a) The legs of the right triangle will be equal in length, and the hypotenuse is 1.6 m .
$x^{2}+x^{2}=b^{2}$
$x^{2}+x^{2}=1.6^{2}$
$2 x^{2}=2.56$
$x^{2}=\frac{2.56}{2} \quad$ Divide both sides by 2 .
$x^{2}=1.28$
$x=\sqrt{1.28}$
$x \approx 1.13 \mathrm{~m}$
The length of the sloping roof pieces will be 1.13 m each.
b) The hypotenuse of the triangle will be a roof piece $(x)$ and one leg will be half the base, or The height of the doghouse will be 0.8 m .
$H^{2}+\left(\frac{b}{2}\right)^{2}=x^{2}$
$H^{2}+0.8^{2}=1.13^{2}$
$H^{2}+0.64=1.28$
$H^{2}=1.28-0.64$
$H^{2}=0.64$
$H=\sqrt{0.64}$
$H \approx 0.8 \mathrm{~m}$
The height of the doghouse will be 0.8 m .
c) Students will have to recall some of their geometry or use similarity. The hypotenuse will be half of the slant height and one leg will be half of 0.8 or 0.4 m .
$h^{2}+l^{2}=\left(\frac{x}{2}\right)^{2}$
$h^{2}+0.4^{2}=0.57^{2}$
$h^{2}+0.16=0.325$
$h^{2}=0.325-0.16$
$h^{2}=0.165$
$H=\sqrt{0.165}$
$H \approx 0.4$ m
The height will be 0.4 m .
d) No, this doghouse would not be suitable for a large dog as there would not be much room for it to move about.

## Extend Your Thinking

8. The old route is $n+e=3$ miles.

Draw a right triangle with the hypotenuse being the new route ( $d$ ) and the two legs being the north and east distances.


For the new route,
$n^{2}+e^{2}=d^{2}$
$1^{2}+2^{2}=d^{2}$
$1+4=d^{2}$
$5=d^{2}$
$d=\sqrt{5}$
$d \approx 2.24 \mathrm{~m}$
The new route is 2.24 miles.
The difference is 3 minus 2.24 which equals 0.76 .
The new route is 0.76 miles shorter.
9. a) Sara incorrectly took the square root of each term in the Pythagorean theorem. She cannot do this. She must take the square root of each side. If Sara had used the Pythagorean theorem correctly she would have:
$h^{2}=a^{2}+b^{2}$
$h^{2}=9^{2}+12^{2}$
$h^{2}=81+144$
$h^{2}=225$
$h=\sqrt{225}$
$h \approx 15 \mathrm{~cm}$
b) $15 \mathrm{~cm} \times 100$ rectangles $=1500 \mathrm{~cm}$

She would need 1500 cm or 15 m of ribbon.

## Alternative Solution

Some students may have recognized this as a 3-4-5 Pythagorean triple multiplied by 3.
10. If the sides angled outward, he would have to be sure that they had enough support and wouldn't fall outward. If they angled inward, he would have the same type of problem - they couldn't angle inward too much in case of falling inward. He would also have to ensure that the opening was big enough so that the toys would fit in.
11. Find the length of the hypotenuse.
$c^{2}=w^{2}+l^{2}$
$c^{2}=4.5^{2}+3.6^{2}$
$c^{2}=20.25+12.96$
$c^{2}=33.21$
$c=\sqrt{33.21}$
$c \approx 5.8$
Add the segments.
$4.5+3.6+5.8=13.9 \mathrm{~m}$ or 1390 cm .
The perimeter will be approximately 13.9 m or 1390 cm .
$1390 \div 30=47$ paving blocks.
Each brick is 30 cm . Therefore, Rutger will need 47 paving blocks.

## Alternative Solution

Find the number of paving blocks per side.
$450 \div 30=15$
$360 \div 30=12$
$580 \div 30=20$
Total the paving blocks for all 3 sides.
$15+12+20=47$
12. a) For each triangle, the vertical height represents one of the legs of the right triangle. Labelling each triangle as below, the vertical height is $a$. Therefore, use the Pythagorean theorem to solve the problem.

i) $a^{2}+b^{2}=c^{2}$
$a^{2}+125^{2}=200^{2}$

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\(a^{2}+15625=40000\)
\(a^{2}=40000-15625\)
\(a^{2}=24375\)
\(a=\sqrt{24375}\)
\(a \approx 156\)
ii) \(a^{2}+b^{2}=c^{2}\)
\(a^{2}+75^{2}=190^{2}\)
\(a^{2}+5625=36100\)
\(a^{2}=36100-5625\)
\(a^{2}=30475\)
\(a=\sqrt{30475}\)
\(a \approx 175\)
iii) \(a^{2}+b^{2}=c^{2}\)
\(a^{2}+90^{2}=125^{2}\)
\(a^{2}+8100=15625\)
\(a^{2}=15625-8100\)
\(a^{2}=7525\)
\(a=\sqrt{7525}\)
\(a \approx 87\)
iv) \(a^{2}+b^{2}=c^{2}\)
\(a^{2}+90^{2}=100^{2}\)
\(a^{2}+8100=10000\)
\(a^{2}=10000-8100\)
\(a^{2}=1900\)
\(a=\sqrt{1900}\)
\(a \approx 44\)
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Add the segments.
$156+175+87+44=462 \mathrm{~m}$

The total height is approximately 462 m .
b) John would not likely be able to measure the horizontal distance in one measurement, so he would have to break the hill up into parts.
Also, he would get a different result if he measured along the hill because the hill is steeper at different sections. However, if he were able to measure the straight distance (see diagram) from the bottom to the top,
 along with the horizontal distance, he would get the same result.
13. No. Fermat's Last Theorem states that there are no such numbers. He initially stated this in 1637, but the theorem remained unproven, although there were many tries to verify it, until 1995. You can find out more about Fermat's Last Theorem on the Mathworld website (http://mathworld.wolfram.com/FermatsLastTheorem.html).

