

$$2. (a) y = (x^5 + 2x)^3$$

$$y' = 3(x^5 + 2x)^2(5x^4 + 2)$$

$$y'' = 3(x^5 + 2x)^2 \cdot (20x^3) + (5x^4 + 2)6(x^5 + 2x)(5x^4 + 2)$$

$$y'' = 60x^3(x^5 + 2x)^2 + 6(5x^4 + 2)^2(x^5 + 2x)$$

$$(b) y = 4 \sin(5x)$$

$$y' = 4 \cos(5x) \cdot 5$$

$$y' = 20 \cos(5x)$$

$$y'' = -20 \sin(5x) \cdot 5$$

$$y'' = -100 \sin(5x)$$

$$3. (a) 5y^2 - 6x + \frac{7}{3}y^6 = 3y + 4x^5 + 10$$

$$-10y^3 \frac{dy}{dx} - 6 + \frac{42}{3}y^5 \frac{dy}{dx} = 3 \frac{dy}{dx} + 20x^4$$

$$-10y^3 \frac{dy}{dx} + 14y^5 \frac{dy}{dx} - 3 \frac{dy}{dx} = 20x^4 + 6$$

$$\frac{dy}{dx}(-10y^3 + 14y^5 - 3) = 20x^4 + 6$$

$$\frac{dy}{dx} = \frac{20x^4 + 6}{-10y^3 + 14y^5 - 3}$$

$$(b) x^4 + 5x^2y^3 + 8y^2 = 3y - 4$$

$$4x^3 + 5x^2 \cdot 3y^2 \frac{dy}{dx} + y^3 \cdot 10x + 16y \frac{dy}{dx} = 3 \frac{dy}{dx}$$

$$4x^3 + 15x^2y^2 \frac{dy}{dx} + 10xy^3 + 16y \frac{dy}{dx} = 3 \frac{dy}{dx}$$

$$15x^2y^2 \frac{dy}{dx} + 16y \frac{dy}{dx} - 3 \frac{dy}{dx} = -4x^3 - 10xy^3$$

$$\frac{dy}{dx}(15x^2y^2 + 16y - 3) = -4x^3 - 10xy^3$$

$$\frac{dy}{dx} = \frac{-4x^3 - 10xy^3}{15x^2y^2 + 16y - 3}$$

$$4. y = 40x^{\frac{1}{2}} - 96x^{\frac{1}{4}}$$

$$y' = 20x^{-\frac{1}{2}} - 24x^{-\frac{3}{4}}$$

$$m_{x=16} = 20(16)^{-\frac{1}{2}} - 24(16)^{-\frac{3}{4}}$$

$$\frac{20}{16^{\frac{1}{2}}} - \frac{24}{16^{\frac{3}{4}}}$$

$$\frac{20}{4} - \frac{24}{8}$$

$$5 - 3$$

$$= 2$$

$$5. y = \frac{x^2}{(2x-9)^2}$$

$$m = 4 \quad (6, 4)$$

$$y - 4 = 4(x - 6)$$

$$y - 4 = -4x + 24$$

$$y = -4x + 28$$

$$y' = \frac{(2x-9)^2 \cdot 2x - x^2 \cdot 2(2x-9) \cdot 2}{(2x-9)^4}$$

$$m = \frac{2x(2x-9)^2 - 4x^2(2x-9)}{(2x-9)^4}$$

$$m_{x=6} = \frac{2 \cdot 6(3)^2 - 4 \cdot 6^2(3)}{3^4}$$

$$\frac{108 - 432}{81}$$

$$= \frac{-324}{81} = -4$$

6.  $f(x) = 2x^3 + 7x^2 - 8x + 9$  tangent is  $\perp$  to  $y = \frac{1}{12}x + 10$   
 $f'(x) = 6x^2 + 14x - 8$   
 $6x^2 + 14x - 8 = -12$   
 $6x^2 + 14x + 4 = 0$   
 $2(3x^2 + 7x + 2) = 0$   
 $\downarrow$  decomposition  
 $2(3x + 1)(x + 2)$   
 $x = -\frac{1}{3}, -2$

tangent is  $\perp$  to  $y = \frac{1}{12}x + 10$   
 $m = \frac{1}{12}$   
 $\perp m = -12$

Sub into original  
 $y = 2x^3 + 7x^2 - 8x + 9$   
 $x = -\frac{1}{3} \quad y = \frac{334}{27} \approx 12.4$   
 $x = -2 \quad y = 37$   
 $(-\frac{1}{3}, 12.4) \quad (-2, 37)$

7.  $y = \frac{9}{2x-3}$   
 $y = 9(2x-3)^{-1}$   
 $y' = -9(2x-3)^{-2} \cdot 2$   
 $y' = \frac{-18}{(2x-3)^2} \text{ set } = -2$   
 $\frac{-18}{(2x-3)^2} = -2$   
 $-18 = -2(2x-3)^2$   
 $-18 = -2(4x^2 - 12x + 9)$   
 $-18 = -8x^2 + 24x - 18$   
 $8x^2 - 24x = 0$   
 $8x(x-3) = 0$   
 $x = 0, +3$

parallel to  $2x + y + 11 = 0$   
 $y = -2x - 11$   
 $m = -2$

$y = \frac{9}{2x-3}$   
 $x = 0 \quad y = \frac{9}{-3} = -3$   
 $x = 3 \quad y = \frac{9}{3} = 3$   
 $(0, -3) \quad (3, 3)$

8.  $12x^3 - 5xy + 12y = -6 - y^2$  at  $(2, -3)$   
 $36x^2 - 5x \cdot 2y \frac{dy}{dx} + y^2(-5) + 12 \frac{dy}{dx} = -2y \frac{dy}{dx}$   
 $36x^2 - 10xy \frac{dy}{dx} - 5y^2 + 12 \frac{dy}{dx} = -2y \frac{dy}{dx}$   
 $-10xy \frac{dy}{dx} + 12 \frac{dy}{dx} + 2y \frac{dy}{dx} = -36x^2 + 5y^2$   
 $\frac{dy}{dx}(-10xy + 12 + 2y) = -36x^2 + 5y^2$   
 $\frac{dy}{dx} = \frac{-36x^2 + 5y^2}{-10xy + 12 + 2y}$

$M_{(2,-3)} =$   
 $\frac{-36(2)^2 + 5(-3)^2}{-10(2)(-3) + 12 + 2(-3)}$   
 $\frac{-144 + 45}{60 + 12 - 6}$   
 $\frac{-99}{66} = -\frac{3}{2}$   
 $y + 3 = -\frac{3}{2}(x - 2)$   
 $y + 3 = -\frac{3}{2}x + 3$   
 $y = -\frac{3}{2}x$

9.  $y = 8 \cos^3 x$   
 $y = 8(\cos x)^3$   
 $y' = 24(\cos x)^2(-\sin x)$   
 $y' = -24(\cos x)^2 \sin x$   
 $m_{x=\frac{\pi}{6}} = -24(\cos \frac{\pi}{6})^2 \sin \frac{\pi}{6}$   
 $-24(\frac{\sqrt{3}}{2})^2 \cdot \frac{1}{2}$   
 $-24(\frac{3}{4}) \cdot \frac{1}{2}$   
 $-\frac{72}{8}$   
 $-9$