1. (a) $f(x) = \frac{2x^2}{(3-x)(3+x)}$; $f(x) = \frac{2x^2}{9-x^2}$ VA: $x=3$, $x=-3$ HA $y=-2$

(b) $f(x) = \frac{8}{(2x+5)^3}$; $f(x) = \frac{8}{8x^3+60x^2+150x+125}$ VA $x = -5/3$ HA $y = 0/8 = 0$

2. (a) $f(x) = 2x^3 + x^2 - 20x + 1$

$f'(x) = 6x^2 + 2x - 20$

$2(3x^2 + x - 10) = 0$

$2(3x-5)(x+2) = 0$

$x = 5/3, -2$

$3x^2 + x - 10$
$3x^2 + 6x - 5x - 10$
$3x(x+2) - 5(x+2)$
$(x+2)(3x-5)$

into orig.

$x = -2$ $y = 29$ max

$x = 5/3$ $y = -20.3$ min

$(-2, 29)$ $(5/3, -20.3)$

	$3(3x-5)(x+2)$			y'
$(-\infty, -2)$	+	−	−	+ Inc
$(-2, 5/3)$	+	−	+	− Dec
$(5/3, \infty)$	+	+	+	+ Inc

(b) $f(x) = (x-2)^3(x+1)^4$

$f'(x) = (x-2)^3 4(x+1)^3 + (x+1)^4 3(x-2)^2$

$4(x-2)^3(x+1)^3 + 3(x-2)^2(x+1)^4 = 0$

$(x-2)^2(x+1)^3\left(4(x-2)+3(x+1)\right) = 0$

$(x-2)^2(x+1)^3(4x-8+3x+3) = 0$

$(x-2)^2(x+1)^3(7x-5) = 0$

$x = 2, -1, 5/7$

$x = -1$ $y = (-3)^3(0)^4 = 0$ $(-1, 0)$ may

$x = 5/7$ $y = -18.35$ $(5/7, -18.35)$ min

$x = 2$ $y = 0^3(3)^4 = 0$ $(2, 0)$ may

	$(x-2)^2$	$(x+1)^3$	$(7x-5)$	y'
$(-\infty, -1)$	+	−	−	+ Inc
$(-1, 5/7)$	+	+	−	− Dec
$(5/7, 2)$	+	+	+	+ Inc
$(2, \infty)$	+	+	+	+ Inc

3. (a) $f(x) = x^3 - 2x^2 + x + 1$

$f'(x) = 3x^2 - 4x + 1$

$f''(x) = 6x - 4$

$6x - 4 = 0$

$2(3x - 2) = 0$

$x = 2/3$

	$2(3x-2)$	y'	
$(-\infty, 2/3)$	+ −	−	CD
$(2/3, \infty)$	+ +	+	CU

$x = 2/3$

$y = 29/27$

$(2/3, 29/27)$ point of Inf

or $(2/3, 1.07)$

(b) $f(x) = x^2 - \frac{27}{x^2}$

$f(x) = x^2 - 27x^{-2}$

$f'(x) = 2x + 54x^{-3}$

$f''(x) = 2 - 162x^{-4}$

$2 - 162x^{-4} = 0$

$2x^{-4}(x^4 - 81) = 0$

$2x^{-4}(x^2 - 9)(x^2 + 9) = 0$

$2x^{-4}(x-3)(x+3)(x^2+9) = 0$

$x = 0, x = 3, x = -3$

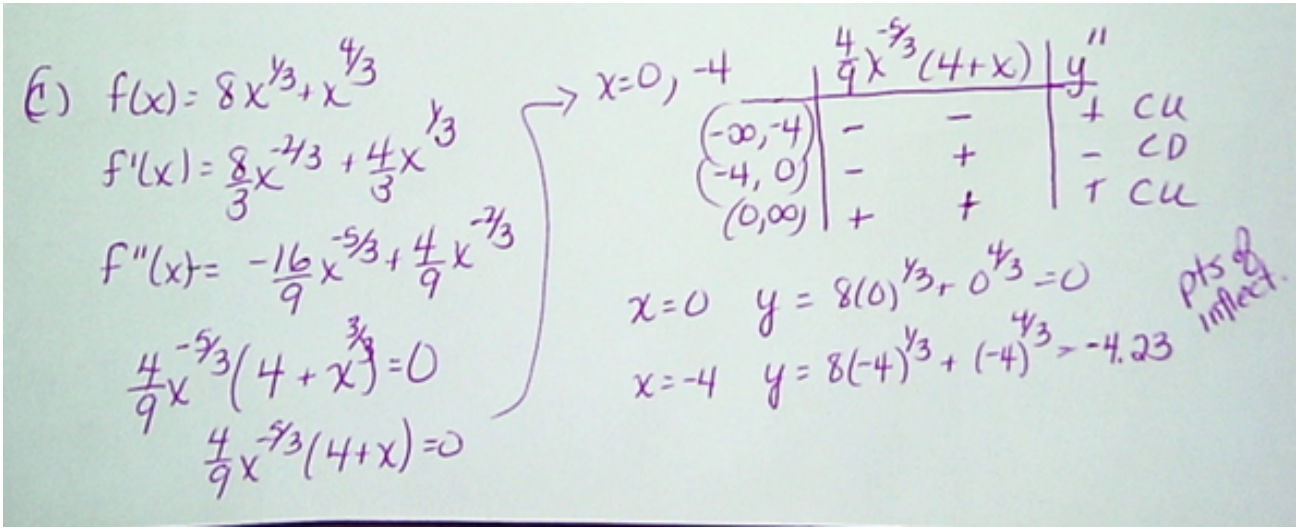
VA

	$\frac{2}{x^4}$	$(x-3)$	$(x+3)$	(x^2+9)	y''
$(-\infty, -3)$	+	−	−	+	+
$(-3, 0)$	+	−	+	+	−
$(0, 3)$	+	−	+	+	−
$(3, \infty)$	+	+	+	+	+

$x = -3 \quad y = 9 - 3 = 6 \quad (-3, 6)$

$x = 3 \quad y = 9 - 3 = 6 \quad (3, 6)$

points of inflection


(1) $f(x) = 8x^{1/3} + x^{4/3}$
$f'(x) = \frac{8}{3}x^{-2/3} + \frac{4}{3}x^{1/3}$
$f''(x) = -\frac{16}{9}x^{-5/3} + \frac{4}{9}x^{-2/3}$
$\frac{4}{9}x^{-5/3}(4 + x^{3/3}) = 0$
$\frac{4}{9}x^{-5/3}(4+x) = 0$
$x = 0, -4$
$\frac{4}{9}x^{-5/3}(4+x)$ | y''
$(-\infty, -4)$ | $-$ | $-$ | $+$ CU
$(-4, 0)$ | $-$ | $+$ | $-$ CD
$(0, \infty)$ | $+$ | $+$ | $+$ CU
$x = 0$ $y = 8(0)^{1/3} + 0^{4/3} = 0$
$x = -4$ $y = 8(-4)^{1/3} + (-4)^{4/3} = -4.23$
pts of inflect.

4. $f(x) = x^2 - \frac{16}{x}$

$f(x) = x^2 - 16x^{-1}$

$f'(x) = 2x + 16x^{-2}$

$f''(x) = 2 - 32x^{-3}$

$x = -2$ is a critical #

Is it a max or min?

sub $x = -2$ into $f''(x) = 2 - 32x^{-3}$

$f''(-2) = 2 - \frac{32}{(-2)^3}$

$= 2 - \frac{32}{-8}$

$= 2 + 4$

$= 6$

Since $f''(-2)$ is positive it means that $x = -2$ represents a minimum by the second derivative test

5. (a) $f(x) = x^4 - x^2$; $f'(x) = 4x^3 - 2x$; $f''(x) = 12x^2 - 2$

$x^2(x^2-1) = 0$ $\qquad 2x(2x^2-1) = 0$ $\qquad 2(6x^2-1)$

$x^2(x-1)(x+1) = 0$

~~Inc/Dec~~ Max/Min

A. Domain $x \in R$

B. Intercepts: x-int $= 0, 1, -1$
y-int $= 0$

C. Asymptotes: —

D Inc/Dec $2x(2x^2-1) = 0$
$x = 0$ $x = -0.71, +0.71$
quad. form

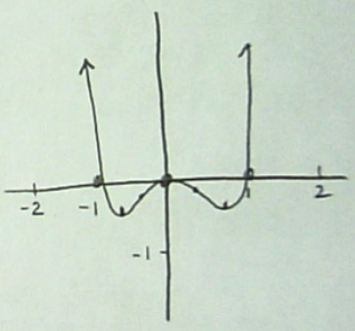
	$2x(2x^2-1)$	y'	
$(-\infty, -0.71)$	−	+	− dec
$(-0.71, 0)$	−	−	+ inc
$(0, 0.71)$	+	−	− dec
$(0.71, \infty)$	+	+	+ inc

E. $x = 0$ $y = 0$
$x = -0.71$ $y = -0.25$
$x = 0.71$ $y = -0.25$

F. Conc. $2(6x^2-1) = 0$
$x = -0.41$
$x = +0.41$

	$2(6x^2-1)$	y''	
$(-\infty, -0.41)$	+	+	+ CU
$(-0.41, 0.41)$	+	−	− CD
$(0.41, \infty)$	+	+	+ CU

G. Point(s) Inf.
$x = -0.41$ $y = -0.14$
$x = 0.41$ $y = -0.14$



5(b) $f(x) = x^3 - 3x + 2$; $f'(x) = 3x^2 - 3$; $f''(x) = 6x$

$P(1) = 0$
long division $(x-1)^2(x+2)$; $f'(x) = 3(x^2-1)$
$3(x-1)(x+1)$

A. Domain $x \in R$

B. Intercepts x-int: 1, -2
y-int: 2

C. Asymptotes —

D. Inc/Dec $3(x-1)(x+1) = 0$
$x = 1, -1$

	$3(x-1)(x+1)$	y'
$(-\infty, -1)$	+	+ Inc
$(-1, 1)$	−	− Dec
$(1, \infty)$	+	+ Inc

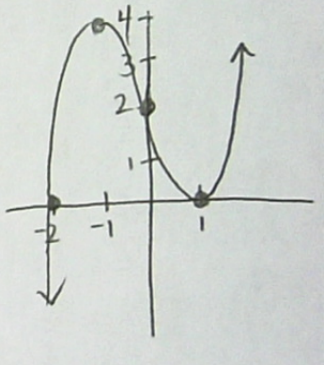
E. Max/min

$x = 1$	$y = 0$
$x = -1$	$y = 4$
$(1, 0)$	$(-1, 4)$

F. Conc:
$6x = 0$
$x = 0$

	$6x$	y''
$(-\infty, 0)$	−	− CD
$(0, \infty)$	+	+ CU

G Point of Inf
$x = 0, y = 2$
$(0, 2)$



(c) $y = \frac{2x+12}{x-4}$; $y' = \frac{-20}{(x-4)^2}$; $y'' = \frac{40}{(x-4)^3}$

A. Domain $x \neq 4$

B. Intercepts x-int: -6
y-int -3

C. Asymptote: VA $x=4$
HA $y=2$

D. Inc/Dec $\frac{-20}{(x-4)^2} = 0$
$x=4$

	$-20/(x-4)^2$	y'
$(-\infty,4)$	$-/+$	$-$ Dec
$(4,\infty)$	$-/+$	$-$ Dec

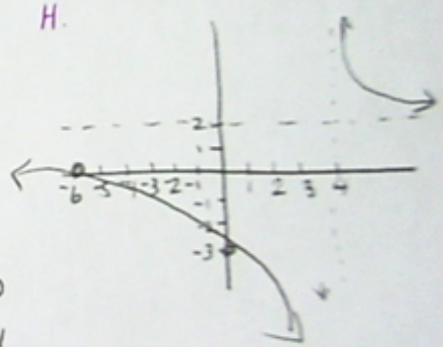
E. Max/min
no max/min
$x=-4$ is VA

F. Concavity $\frac{40}{(x-4)^3} = 0$
$x=4$

	$40/(x-4)^3$	y''
$(-\infty,4)$	$+/-$	$-$ CD
$(4,\infty)$	$+/+$	$+$ CU

G. Point Inf no pt. of inf
$x=4$ is VA

H.



*Behavior $\frac{2x+12}{x-4}$

3.9	4.1
$+/-$	$+/+$
$-\infty$	$+\infty$

(d) $f(x) = \frac{3x^2-48}{x^2-9}$ $f'(x) = \frac{42x}{(x^2-9)^2}$ $f''(x) = \frac{-42x^2-378}{(x^2-9)^3}$

$f(x) = \frac{3(x-4)(x+4)}{(x-3)(x+3)}$ $f''(x) = \frac{-42(x^2+9)}{(x^2-9)^3}$

A. Domain $x \neq 3, -3$

B. Intercepts x-int 4, -4
y-int $48/9 = 5.\bar{3}$

C. Asymptotes VA $x = 3, -3$
HA $y = 3$

D Inc/Dec $\frac{42x}{(x^2-9)^2} = 0$

$x = 0, 3, -3$

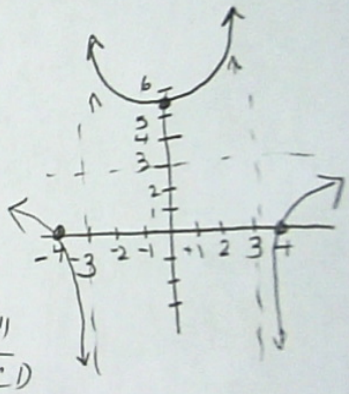
	$42x/(x^2-9)^2$	y'
$(-\infty, -3)$	$-/+$	$-$ dec
$(-3, 0)$	$-/+$	$-$ dec
$(0, 3)$	$+/+$	$+$ inc
$(3, \infty)$	$+/+$	$+$ inc

E. Max/Min
$x = 0$ $y = \frac{-48}{-9} = 5.\bar{3}$
$(0, 5.\bar{3})$

F. $\frac{-42(x^2+9)}{(x^2-9)^3} = 0$

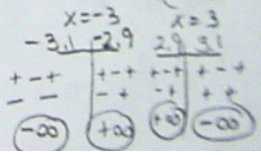
$x = 3, x = -3$

	$\frac{-42(x^2+9)}{(x^2-9)^3}$	y''
$(-\infty, -3)$	$-+/+$	$-$ CD
$(-3, 3)$	$-+/-$	$+$ CU
$(3, \infty)$	$-+/+$	$-$ CD



G. P of I None
$x = 3, x = -3$ are VA

Behaviour $\frac{3(x-4)(x+4)}{(x-3)(x+3)}$



(F). $f(x) = \frac{(x-1)^2}{(x-2)^2}$; $f'(x) = \frac{-2x+2}{(x-2)^3}$; $f''(x) = \frac{4x-2}{(x-2)^4}$

A. Domain $x \neq 2$

B. Intercepts x-int 1, y-int 1/4

C. Asymptotes VA $x=2$, HA $y=1$

$f(x) = \frac{x^2-2x+1}{x^2-4x+4}$

D. Inc/Dec $\frac{-2x+2}{(x-2)^3} = 0$

$\frac{-2(x-1)}{(x-2)^3} = 0$

$x = 1, 2$

	$\frac{-2(x-1)}{(x-2)^3}$	y'
$(-\infty, 1)$	$-/-$	- dec
$(1, 2)$	$-/+$	+ inc
$(2, \infty)$	$-/+$	- dec

E. Max/Min

$x = 1$ $y = 0$

F. Conc $\frac{4x-2}{(x-2)^4} = 0$

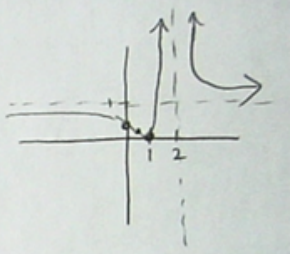
$x = 1/2, x = 2$

	$\frac{4x-2}{(x-2)^4}$	y''
$(-\infty, 1/2)$	$-/+$	- CD
$(1/2, 2)$	$+/+$	+ CU
$(2, \infty)$	$+/+$	+ CU

G. PoI

$x = 1/2$ $y = \frac{(1/2-1)^2}{(1/2-2)^2} = 1/9$

$(1/2, 1/9)$



Behav: $\frac{(x-1)^2}{(x-2)^2}$

$x = 2$

1.9	2.1
$+/+$	$+/+$
$+\infty$	$+\infty$

(g) $f(x) = \frac{2x^2-4x+2}{x^2+1}$; $f'(x) = \frac{4x^2-4}{(x^2+1)^2}$; $f''(x) = \frac{24x-8x^3}{(x^2+1)^3}$

$f(x) = \frac{2(x^2-2x+1)}{x^2+1}$ $f'(x) = \frac{4(x-1)(x+1)}{(x^2+1)^2}$ $f''(x) = \frac{8x(3-x^2)}{(x^2+1)^3}$

$= \frac{2(x-1)(x-1)}{(x^2+1)}$

A. Domain $x \in R$

B. Intercepts: x-int: 1

y-int 2

C. Asymptotes: VA —

HA $y=2$

D. Inc/Dec $\frac{4(x-1)(x+1)}{(x^2+1)^2} = 0$

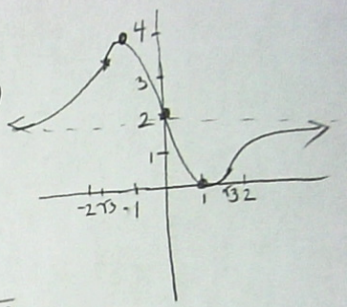
$x = 1, -1$

	$\frac{4(x-1)(x+1)}{(x^2+1)^2}$	y'
$(-\infty, -1)$	$+ - -/+$	+ inc
$(-1, 1)$	$+ - +/+$	- dec
$(1, \infty)$	$+ + +/+$	+ inc

E. Max/Min

$x=-1$ $y=4$ $(-1,4)$

$x=1$ $y=0$ $(1,0)$



F. Conc: $\frac{8x(3-x^2)}{(x^2+1)^3}$

$x=0$, $x=\sqrt{3}$, $x=-\sqrt{3}$

	$\frac{8x(3-x^2)}{(x^2+1)^3}$	y''
$(-\infty, -\sqrt{3})$	$- -/+$	+ CU
$(-\sqrt{3}, 0)$	$- +/+$	- CD
$(0, \sqrt{3})$	$+ +/+$	+ CU
$(\sqrt{3}, \infty)$	$+ -/+$	- CD

G. Inf: $x=0$ $y=2$

$x=-\sqrt{3}$ $y=3.73$

$x=\sqrt{3}$ $y=0.268$