



Laws of Exponents

Review From Gr. 9



Product of powers law: $a^m \cdot a^n = a^{m+n}$

Quotient of powers law: $\frac{a^m}{a^n} = a^{m-n}$

Power of a power law: $(a^m)^n = a^{mn}$

Write as a single power.

a) $3^2 \cdot 3^5 = 3^{2+5} = 3^7$

b) $(4^2)^5 = 4^{2 \cdot 5} = 4^{10}$

c) $(-5)^{10} \div (-5)^8 = \frac{(-5)^{10}}{(-5)^8}$



$$= (-5)^{10-8}$$

$$= (-5)^2$$

evaluate 25

Simplify and evaluate

$$(2^2 \cdot 2^3)^5$$

$$2^5 \cdot 2^{15}$$

$$32 \cdot 2^{15}$$

4.4 Fractional Exponents and Radicals

$$\left(\frac{3}{1}\right)^{-2} = \left(\frac{1}{3}\right)^2$$

Homework Questions???

Page 218-219 #11j, 12b,d,f,h,i, 19(a,b,c), 20, 21, 22a, 23

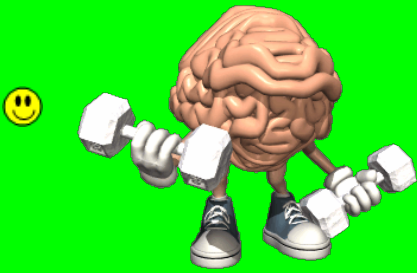
11 h) $\sqrt[3]{100}$ not \div by ⁶⁴27, 8
 so already simplified

j) $\sqrt[3]{375}$

$$\begin{aligned} &\sqrt[3]{125 \cdot 3} \\ &\sqrt[3]{125} \sqrt[3]{3} \\ &5 \sqrt[3]{3} \end{aligned}$$

12 a) $3\sqrt{2}$

$$\begin{aligned} &\sqrt{3^2 \times 2} \\ &\sqrt{9 \times 2} \\ &\sqrt{18} \end{aligned}$$



Warm Up

Name: _____

Period: _____

Simplify then evaluate

$$1) (2^4)^3 = 2^{12} \\ = 4096$$

$$2) [(-5)^2 \times 2]^3 \\ (-5)^2 \times 2^3 \\ 15625 \times 8 \\ 125000$$

$$3) [(-1)^{11}]^3 = (-1)^{33} = -1$$

Write each expression as a product or quotient of powers. Then evaluate.

$$1) [(-3) \times (5)]^2$$

power law

$$(-3)^2 \times (5)^2$$

product power

$$2) \left(\frac{6}{5}\right)^4 = \frac{6^4}{5^4} = \frac{1296}{625}$$

$$= 9 \times 25 \\ = 225$$

4.4 Fractional Exponents and Radicals



LESSON FOCUS

Relate rational exponents and radicals.

Make Connections

Coffee, tea, and hot chocolate contain caffeine. The expression $100(0.87)^{\frac{1}{2}}$ represents the percent of caffeine left in your body $\frac{1}{2}$ h after you drink a caffeine beverage.

Given that $0.87^1 = 0.87$ and $0.87^0 = 1$, how can you estimate a value for $0.87^{\frac{1}{2}}$?



★ Use a calculator to complete the table.



Column 1	Column 2
x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} =$
4	$4^{\frac{1}{2}} = 2$
9	$9^{\frac{1}{2}} = 3$
16	$16^{\frac{1}{2}} = 4$
25	$25^{\frac{1}{2}} = 5$

a) What do you notice about the numbers in the first column?

$$4 \square \wedge \square \neq \square 2$$

b) Compare the numbers in the first and second columns. What conclusions can you make?

$$\sqrt{x} = x^{\frac{1}{2}}$$

c) What do you think the exponent $\frac{1}{2}$ means?



★ Use a calculator to complete the table.

Column 1 Column 2 Column 3

x	$x^{\frac{1}{3}}$
1	$1^{\frac{1}{3}} = 1$
8	$8^{\frac{1}{3}} = 2$
27	$27^{\frac{1}{3}} = 3$
64	
125	


a) What do you notice about the numbers in the first column?


b) Compare the numbers in the first and second columns. What conclusions can you make?

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

c) What do you think the exponent $\frac{1}{3}$ means?

★ What do you think $a^{\frac{1}{4}}$ and $a^{\frac{1}{5}}$ mean?


$$\sqrt[4]{a}$$


$$\sqrt[5]{a}$$

index
is the
bottom
→ root

What does $a^{\frac{1}{n}}$ mean? Explain your reasoning.

$$\sqrt[n]{a}$$



Rational Exponents and Radicals ☆

Let's examine radicals...

$$\sqrt{5} \times \sqrt{5} = (\sqrt{5})^2 = 5$$

$$= \sqrt{25}$$

$$= 5$$

How would this play out with exponent laws?

$$5^{\frac{1}{2}} \times 5^{\frac{1}{2}} = 5^1$$

$$\sqrt{5} \times \sqrt{5} = 5$$

RULE: $\sqrt{x} = x^{\frac{1}{2}}$

radical form exponent form

What about other rational exponents and radicals?

$$8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 8^{\frac{3}{3}} = 8^1 = 8$$

$$(\sqrt[3]{8})^3 = 8$$

Rad exponent

Rule: $\sqrt[3]{x} = x^{\frac{1}{3}}$

Study

In general... $(\sqrt[n]{x})^m$ or $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

exponent (m) index (root) (n)

Rational Exponents

- To evaluate exponents that are rational (fractions), the denominator of the fraction indicates which root to take and the numerator indicates which power the entire base is to be raised.

Example

Exponent form Radical

$$16^{\frac{1}{4}} = \sqrt[4]{16}$$

$$= 2 \quad \leftarrow \text{evaluate}$$

$$125^{\frac{1}{3}} = \sqrt[3]{125}$$

$$= 5$$

index

$$125^{\frac{2}{3}}$$

exponent (2) index (3)

$$= (\sqrt[3]{125})^2$$

$$= (5)^2$$

$$= 25$$



Example 1

Evaluating Powers of the Form $a^{\frac{1}{n}} = \sqrt[n]{a}$



Evaluate each power without using a calculator.

a) $27^{\frac{1}{3}}$

$\sqrt[3]{27}$
use list
 $= 3$

b) $0.49^{\frac{1}{2}}$

$\sqrt{0.49}$
 $= 0.7$

c) $(-64)^{\frac{1}{3}}$

$\sqrt[3]{-64}$
 $= -4$

d) $(\frac{4}{9})^{\frac{1}{2}}$

$\frac{4^{\frac{1}{2}}}{9^{\frac{1}{2}}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$



CHECK YOUR UNDERSTANDING



Exponential form

Radical form

$$X^{\frac{m}{n}} = \left(\sqrt[n]{X} \right)^m$$

Power exponent (pointing to $\frac{m}{n}$)
Root index (pointing to n)

Write the following in radical form and evaluate.

$$\text{Ex) } 8^{\frac{2}{3}} = \left(\sqrt[3]{8} \right)^2 = (2)^2 = 4$$

exponent (pointing to $\frac{2}{3}$)

If Exponent is a decimal, then rewrite exponent as a fraction (Reduce Fraction)

$$32^{0.2} = 32^{\frac{2}{10}} \text{ Reduce}$$

$$= 32^{\frac{1}{5}}$$

$$= \sqrt[5]{32}$$

$$= 2$$

Examples: Express each exponential in radical form, then evaluate.



$$1. 8^{\frac{2}{3}} =$$

$$2. 125^{\frac{1}{3}} =$$

$$3. 32^{\frac{7}{5}} =$$

Handwritten annotations:
An arrow points from the word "exponent" to the fraction $\frac{7}{5}$.
An arrow points from the word "index" to the denominator 5.

$$= \left(\sqrt[5]{32} \right)^7$$

$$= \left(2 \right)^7$$

Handwritten note: A red wavy line under the radical symbol in the previous step is followed by the text "look in perfect 5th list".

$$= 128$$

☺ $x^{\frac{\text{num}}{\text{den}}}$ $(\sqrt[n]{x})^m$ ← numerator

denominator →

Express as a exponent:

a) $\sqrt[5]{32}$

$32^{\frac{1}{5}}$

b) $\sqrt[3]{-64}$

$-64^{\frac{1}{3}}$

c) $(\sqrt{144})^3$

$(144)^{\frac{3}{2}}$

Express as a Radical:

a) $8^{\frac{5}{3}}$

$(\sqrt[3]{8})^5$

b) $49^{\frac{3}{2}}$

c) $(-125)^{\frac{3}{2}}$

Homework

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$$\begin{aligned}
 &4c) \\
 &100^{0.5} \\
 &100^{\frac{5}{10}} \\
 &100^{\frac{1}{2}} \\
 &\sqrt{100} \\
 &= 10
 \end{aligned}$$

4.4 Fractional Exponents and Radicals

Exercises

A

3 4 5 6 7

B

8 9 10 11 12 13 14 15

16 17 18 19 20 21

C

22

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#3 a d f
 #4 a c d
 #5 a b c
 #6 a c
 #8 a b c
 #9
 #12 a b f
~~#15~~

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