



# Warm Up

Sept. 8

Review :

$$x^{\frac{1}{y}} = \sqrt[y]{x}$$

a)  $\sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{3}{5}$

b)  $\sqrt[5]{7776} = 6$

c)  $\sqrt[3]{0.125} = \frac{\sqrt[3]{125}}{\sqrt[3]{1000}} = \frac{5}{10} = \frac{1}{2} = 0.5$  (Note: "Reduce" written above)

d)  $\sqrt{2.25} = \frac{\sqrt{225}}{\sqrt{100}} = \frac{15}{10} = 1.5$

2. Order the following radicals from least to greatest (Show work)

$\sqrt[5]{40}$	$\sqrt[5]{98}$	$\sqrt[5]{100}$	$\sqrt[4]{98}$	$\sqrt[5]{75}$	$\sqrt[6]{300}$	$\sqrt[7]{343}$
$\sqrt[3]{40}$	$\sqrt[3]{98}$	$\sqrt[3]{98}$	$\sqrt[3]{75}$	$\sqrt[3]{300}$		

$\approx 6.3$  ✓

$\approx 9.9$

$\approx 4.6$  ✓

$\approx 8.6$

$\approx 6.6$  ✓

$\sqrt[3]{28}, \sqrt{40}, \sqrt[3]{300}, \sqrt{75}, \sqrt{98}$

$\sqrt[8]{64} = 2$   
 $\sqrt[9]{81} = 2$

$\sqrt[6]{216} = 3$   
 $\sqrt[7]{343} = 7$

# Homework

## Page 206

### Questions 1-6

Page 206 (1-6) questions

1a)  $\sqrt{16}$ ,  $\sqrt[3]{27}$ ,  $\sqrt[4]{81}$ ,  $\sqrt[5]{243}$

b) index: 2, radicand: 16      index: 3, radicand: 27      index: 4, radicand: 81      index: 5, radicand: 243

c) You are taking the "n<sup>th</sup>" root of the radicand.  
What number multiplied by itself n times will give you the radicand

2a)  $\sqrt{36} = \sqrt{6^2} = 6$

b)  $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$

$\sqrt[4]{10000} = \sqrt[4]{10^4} = 10$

d)  $\sqrt[5]{-32} = \sqrt[5]{-2^5} = -2$

e)  $\sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{\sqrt[3]{3^3}}{\sqrt[3]{5^3}} = \frac{3}{5}$

f)  $\sqrt{2.25} \Rightarrow \sqrt{225} = 15$   
 $\frac{\sqrt{225}}{\sqrt{100}} = \frac{15}{10} \approx 1.5$

g)  $\sqrt[3]{0.125} = \sqrt[3]{125}$   
 $\frac{\sqrt{125}}{\sqrt{1000}} = \frac{5}{10} = \frac{1}{2} = \frac{1}{2} = \frac{1}{2} = 0.5$   
 $\therefore \sqrt[3]{0.125} = 0.5$

h)  $\sqrt[4]{625} = \sqrt[4]{5^4} = 5$

3)  $\sqrt{8}$  closer

$$\begin{array}{cc} \sqrt{4} & \sqrt{9} \\ \downarrow & \downarrow \\ 2 & 3 \end{array}$$

$\approx 2.8$

b)  $\sqrt[3]{9}$

$$\begin{array}{cc} \sqrt[3]{8} & \sqrt[3]{27} \\ \downarrow & \downarrow \\ 2 & 3 \end{array}$$

$\approx 2.1$

c)  $\sqrt[4]{10}$  closer

$$\begin{array}{cc} \sqrt[4]{1} & \sqrt[4]{16} \\ \downarrow & \downarrow \\ 1 & 2 \end{array}$$

$\approx 1.7$

d)  $\sqrt{13}$  closer

$$\begin{array}{cc} \sqrt{9} & \sqrt{16} \\ \downarrow & \downarrow \\ 3 & 4 \end{array}$$

$\approx 3.5$

e)  $\sqrt[3]{15}$  closer

$$\begin{array}{cc} \sqrt[3]{8} & \sqrt[3]{27} \\ \downarrow & \downarrow \\ 2 & 3 \end{array}$$

$\approx 2.4$

f)  $\sqrt[4]{17}$

$$\begin{array}{cc} \sqrt[4]{16} & \sqrt[4]{81} \\ \downarrow & \downarrow \\ 2 & 3 \end{array}$$

$\approx 2.1$

g)  $\sqrt{19}$

$$\begin{array}{cc} \sqrt{16} & \sqrt{25} \\ 4 & 5 \end{array}$$

$\approx 4.3$

h)  $\sqrt[3]{20}$

$$\begin{array}{cc} \sqrt[3]{8} & \sqrt[3]{27} \\ \downarrow & \downarrow \\ 2 & 3 \end{array}$$

$\approx 2.7$

4)  $\sqrt{-4} = \text{DNE}$

$a \times a = (-)$   
Must be the same #'s

only way you can multiply two #'s to get a neg  
(-)(+)

Not the same #'s

b) any even index

$\sqrt[4]{-16} = \text{DNE}$

c) i) any odd index  
ii) any even index

6 i) square roots

a)  $\sqrt{4} = 2$

b)  $\sqrt{9} = 3$

c)  $\sqrt{16} = 4$

d)  $\sqrt{100} = 10$

e)  $\sqrt{0.81} = 0.9$

f)  $\sqrt{0.04} = 0.2$

ii) cube roots

a)  $\sqrt[3]{8} = 2$

b)  $\sqrt[3]{27} = 3$

c)  $\sqrt[3]{81} = 4$

d)  $\sqrt[3]{1000} = 10$

e)  $\sqrt[3]{0.729} = 0.9$

f)  $\sqrt[3]{0.008} = 0.2$

iii) Fourth root

a)  $\sqrt[4]{16} = 2$

b)  $\sqrt[4]{81} = 3$

c)  $\sqrt[4]{256} = 4$

d)  $\sqrt[4]{10000} = 10$

e)  $\sqrt[4]{0.6561} = 0.9$

f)  $\sqrt[4]{0.0016} = 0.2$

6) a)  $\sqrt[4]{81} = 3$

b)  $\sqrt[3]{-125} = -5$

c)  $\sqrt{49} = 7$

d)  $\sqrt[3]{18}$



# List

Evaluate each radical. Justify you answer

$$\begin{aligned} \text{a) } \sqrt{49} \\ = \sqrt{7^2} \\ = 7 \end{aligned}$$

$$\begin{aligned} \text{b) } \sqrt[4]{1024} \\ \swarrow \searrow \\ \sqrt[4]{625} \quad \sqrt[4]{1296} \\ 5 \qquad \qquad 6 \\ \approx 5.7 \end{aligned}$$

$$\begin{aligned} \text{c) } \sqrt[3]{729} = 9 \\ \sqrt[3]{9^3} \end{aligned}$$

Estimate to one decimal (Show Work)

$$\begin{aligned} \text{a) } \sqrt[4]{78} \\ \swarrow \searrow \\ \sqrt[4]{16} \quad \sqrt[4]{81} \\ \downarrow \quad \downarrow \\ 2 \qquad \quad 3 \\ \approx 2.8 \end{aligned}$$

$$\begin{aligned} \text{b) } \sqrt[3]{576} \\ \swarrow \searrow \\ \sqrt[3]{512} \quad \sqrt[3]{729} \\ \downarrow \quad \downarrow \\ 8 \qquad \quad 9 \\ \approx 8.3 \end{aligned}$$

Remember

**Rational numbers** are numbers that can be written as a fraction or is a decimal that repeats or terminates. Ex)  $\sqrt[4]{\frac{1296}{10000}}$  Ex)  $\sqrt[3]{\frac{8}{27}}$

**Irrational numbers** are numbers that cannot be written as a fraction and its decimal neither terminates or repeats.  $\sqrt{28}$

# Radicals

**Mixed Radical** - has a coefficient in front of the radical sign.

ex:  $3\sqrt{5}$  OR  $\frac{2\sqrt{26}}{3}$  OR  $-3\sqrt[3]{3}$  .

**Entire Radical** - has a coefficient of 1 or -1 in front of the radical sign. Everything is entirely under the radical sign

ex:  $\sqrt{12}$  OR  $-\sqrt{45}$

$$\sqrt[3]{216}$$

$$= 6$$

Rational

or

$$-1 \left( \sqrt[4]{72} \right)$$

$$(-1) (2.9\dots)$$

-2.9...  
can't predict  
next #  
Irrational

Have from last day

# Reducing Radicals

## Multiplication Property of Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b},$$

where  $n$  is a natural number, and  $a$  and  $b$  are real numbers

Same works if we change the "index":

$$\sqrt[3]{ab}$$

$$\begin{aligned}\sqrt[3]{8 \cdot 27} &= \sqrt[3]{8} \cdot \sqrt[3]{27} \\ &= 2 \cdot 3 \\ &= 6\end{aligned}$$


or

$$\begin{aligned}\sqrt[3]{8 \cdot 27} &= \sqrt[3]{216} \\ &= 6\end{aligned}$$



# Reducing Radicals

To reduce a radical, you must find the largest "n<sup>th</sup>" number that will divide into the radicand

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$


Greatest perfect n<sup>th</sup>

# Entire to Mixed

## Reducing Radicals

- 1
- 4
- 9
- 16
- 25
- 36
- 49
- 64
- 81
- 100

To reduce  $\sqrt{125}$   
 you must find the **largest** square number  
 that will divide into 125 evenly!

121  
144 ← 25  
169  
196  
225  
Check  
Perfect  
Square  
List

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\begin{aligned} \sqrt{125} &= \sqrt{(25) \cdot (5)} \\ &= \sqrt{25} \cdot \sqrt{5} \\ &= 5\sqrt{5} \end{aligned}$$

Greatest perfect  $n^{\text{th}}$

Must Know list of perfect  $n^{\text{th}}$

$$\begin{aligned} \sqrt{48} &= \sqrt{16 \cdot 3} \\ &= \sqrt{16} \cdot \sqrt{3} \\ &= 4\sqrt{3} \end{aligned}$$

Separate

Perfect Square List

- 1
- 4
- 9
- 16
- 25
- 36 ← 48
- 49

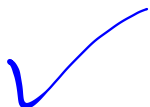
## Entire to Mixed

continued

# Reducing Radicals

Prime Factorization

To reduce  $\sqrt{125}$   
you must find the **largest** square number  
that will divide into 125 evenly!



$$a) \sqrt{125}$$

$$\begin{aligned} b) \sqrt{54} &= \sqrt{9 \times 6} \\ &= \sqrt{9} \times \sqrt{6} \\ &= 3\sqrt{6} \end{aligned}$$



4  
9  
16  
25  
36  
49  
64  
81  
100  
121

Try these:

a)  $\sqrt{12}$   
 $= \sqrt{4 \times 3}$   
 $= \sqrt{4} \times \sqrt{3}$   
2  $\sqrt{3}$

d)  $\sqrt{81}$   
 $= 9$

b)  $\sqrt{72}$

c)  $\sqrt{54}$

e)  $7\sqrt{128}$



Prime # 2, 3, 5, 7, 11, 13, 17...

Remember Prime Factorization of square roots:

ex)  $\sqrt{162}$

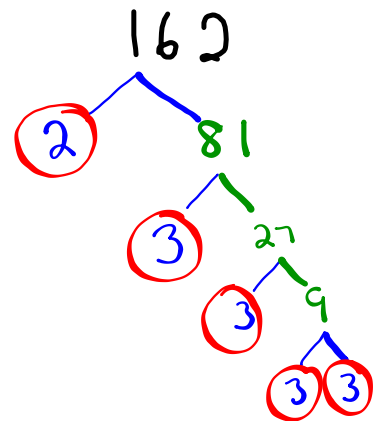
$$\sqrt{(3 \times 3) \times (3 \times 3) \times 2}$$

$$= \sqrt{3 \times 3} \cdot \sqrt{3 \times 3} \cdot \sqrt{2}$$

$$3 \cdot 3 \cdot \sqrt{2}$$

$$9\sqrt{2}$$

Factor tree



ex)  $\sqrt{128}$

ex)  $\sqrt[3]{256}$

We can also use prime factorization to simplify a radical.

### Example 1 Simplifying Radicals Using Prime Factorization

Simplify each radical.

a)  $\sqrt{80}$

b)  $\sqrt[3]{144}$

c)  $\sqrt[4]{162}$



**SOLUTION**

a)  $\sqrt{80}$

b)  $\sqrt[3]{144}$

c)  $\sqrt[4]{162}$



CHECK YOUR UNDERSTANDING

We can also use prime factorization to simplify a radical.

### Example 1 Simplifying Radicals Using Prime Factorization

Simplify each radical.

a)  $\sqrt{80}$       b)  $\sqrt[3]{144}$       c)  $\sqrt[4]{162}$



**SOLUTION**

Same questions using largest perfect nth factors

$$\begin{aligned} \text{a) } \sqrt{80} &= \sqrt{16 \cdot 5} = \sqrt{16} \times \sqrt{5} \\ &= 4\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{b) } \sqrt[3]{144} &= \sqrt[3]{8 \cdot 18} = \sqrt[3]{8} \cdot \sqrt[3]{18} \\ &= 2\sqrt[3]{18} \end{aligned}$$

4.3 Mixed and Entire Radicals



CHECK YOUR UNDERSTANDING

$$\begin{aligned} \text{c) } \sqrt[4]{162} &= \sqrt[4]{81 \cdot 2} = \sqrt[4]{81} \sqrt[4]{2} \\ &= 3\sqrt[4]{2} \end{aligned}$$

**Example 2** Writing Radicals in Simplest Form

Write each radical in simplest form, if possible.

a)  $\sqrt[3]{40}$     b)  $\sqrt{26}$     c)  $\sqrt[4]{32}$

either use prime factorization  
or largest nth factor

 **SOLUTION**



CHECK YOUR UNDERSTANDING



# Homework

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# 4 a c d

# 9

# 10 a b c

# 11 a b

→ look in perfect cube list.

#4<sup>(abc<sup>2</sup>)</sup>, #7a, #8a, #9, #10<sup>abcd</sup>, #11<sup>abcd</sup>

a)  $c^2 = a^2 + b^2$

$$c = \sqrt{a^2 + b^2}$$

b)  $\sqrt{45} = \sqrt{(\quad) \cdot (\quad)}$