polypracticeassign.doc

<u>MC</u> 1. D   5. B   9. B
      2. A   6. A   10. C
      3. D   7. B   11. B
      4. C   8. C   12. A

<u>Part B</u> :

1. $P(-1) = (-1)^3 + 2(-1)^2 + 3(-1) + 2$
    $= -1 + 2 - 3 + 2$
    $= 0$

2. (a) $x^4 - x^3 - 2x^2$
    $x^2(x^2 - x - 2)$
    $x^2(x - 2)(x + 1)$

(b) $x^3 + 5x^2 - 12$

$P(1) = -6$
$P(-1) = -8$
$P(2) = 16$
$P(-2) = 0 \checkmark$

$$
\begin{array}{r}
x^2 + 3x - 6 \\
x+2 \overline{\smash{\big)}\ x^3 + 5x^2 + 0x - 12} \\
\underline{x^3 + 2x^2} \\
3x^2 + 0x \\
\underline{3x^2 + 6x} \\
-6x - 12 \\
\underline{-6x - 12} \\
0
\end{array}
$$

$(x+2)(x^2 + 3x - 6)$

↑ this will not factor

(c) $15x^4 - 22x^2 + 8$ *Quad. Form

add −22
mult 120

$15x^4 - 12x^2 - 10x^2 + 8$
$3x^2(5x^2 - 4) - 2(5x^2 - 4)$
$(5x^2 - 4)(3x^2 - 2)$

Factoring Methods:
1. Common Factor
2. Simple Trinomial
3. Hard Trinomial
4. Difference of Squares
5. Sum / Diff of Cubes
6. Factor Theorem
7. Quadratic Form

d) $6x^4 + 13x^3 - 8x^2 - 17x + 6$

$P(1) = 0$

$$\begin{array}{r} 6x^3 + 19x^2 + 11x - 6 \\ x-1 \overline{)\, 6x^4 + 13x^3 - 8x^2 - 17x + 6} \\ \underline{6x^4 - 6x^3} \\ 19x^3 - 8x^2 \\ \underline{19x^3 - 19x^2} \\ 11x^2 - 17x \\ \underline{11x^2 - 11x} \\ -6x + 6 \\ \underline{-6x + 6} \\ 0 \end{array}$$

OR

$$\begin{array}{c|ccccc} -1 & 6 & 13 & -8 & -17 & 6 \\ & \downarrow & -6 & -19 & -11 & 6 \\ \hline & 6 & 19 & 11 & -6 & 0 \end{array}$$

$(x-1)(6x^3 + 19x^2 + 11x - 6)$

　　　　↑

$P(-2) = 0$

$$\begin{array}{r} 6x^2 + 7x - 3 \\ x+2 \overline{)\, 6x^3 + 19x^2 + 11x - 6} \\ \underline{6x^3 + 12x^2} \\ 7x^2 + 11x \\ \underline{7x^2 + 14x} \\ -3x - 6 \\ \underline{-3x - 6} \\ 0 \end{array}$$

Factors:

$(x-1)(x+2)(6x^2 + 7x - 3)$

　　　　　↓

$6x^2 + 9x - 2x - 3$

$3x(2x+3) - 1(2x+3)$

　　　$(2x+3)(3x-1)$

$(x+1)(x+2)(2x+3)(3x-1)$

(e) $x^3 - 17x^2 + 80x - 100$

$P(2) = 0$

$x - 2$ $(x-2)(x^2 - 15x + 50)$

$(x-2)(x-10)(x-5)$

$$\begin{array}{r|rrrr} -2 & 1 & -17 & 80 & -100 \\ & \downarrow & -2 & 30 & -100 \\ \hline & 1 & -15 & 50 & 0 \end{array}$$

(f) $27x^3 + 125y^3$

$(3x + 5y)(9x^2 - 15xy + 5y^2)$

* Sum of Cubes

(g) $15x^2 - 7xy - 4y^2$

$15x^2 - 12xy + 5xy - 4y^2$

$3x(5x - 4y) + y(5x - 4y)$

$(5x - 4y)(3x + y)$

(h) $x^8 - 256$ "diff. of squares"

$(x^4 - 16)(x^4 + 16)$

$(x^2 - 4)(x^2 + 4)(x^2 + 16)$

$(x - 2)(x + 2)(x^2 + 16)$

(2) $8x^2 + 13x - 6$

$8x^2 + 16x - 3x - 6$

$8x(x + 2) - 3(x + 2)$

$(x + 2)(8x - 3)$

3. $f(x) = 2x^4 + 3x^3 - x^2 - 3x - 1$

   $f(x) = (x+1)(x-1)^2(2x+1)$

   $x\text{-int:} \ -1, 1, -\frac{1}{2}$

4. (a) $(x-3)^2(x+4)(x+6) \gtreqless 0$

just look at

$>$

$(x-3)(x-3)(x+4)(x+6) > 0$

$LC = 1x^4$ positivè, even ↑.   ↑   Rough sketch

x-int 3 , -4, -6
just
touches

y-int +216

216

-6 -5 -4 -3 -2 -1 1 2 3 4

It is positive :
$(-\infty, -6)$ $(-4, 3)$ $(3, \infty)$

4. Solve:
   (a) $(x-3)^2(x+4)(x+6) \geq 0$
   chart

Zeros: 3, -4, -6

Sol'n
$(-\infty, -6]$ $[-4, 3]$ $[3, \infty)$

	$(x-3)^2$	$(x+4)$	$(x+6)$	≥ 0
$(-\infty, -6]$	+	−	−	+
$[-6, -4]$	+	−	+	−
$[-4, 3]$	+	+	+	+
$(3, \infty)$	+	+	+	+

4. (b) $2x^3 + 7x^2 - 10x - 24 < 0$

$P(2) = 0$

$$\begin{array}{r} 2x^2 + 11x + 12 \\ x-2 \overline{)\, 2x^3 + 7x^2 - 10x - 24} \\ \underline{2x^3 - 4x^2} \\ 11x^2 - 10x \\ \underline{11x^2 - 22x} \\ 12x - 24 \\ \underline{12x - 24} \\ 0 \end{array}$$

Hard Trinomial

$2x^2 + 11x + 12$

$2x^2 + 8x + 3x + 12$

$2x(x+4) + 3(x+4)$

$(x+4)(2x+3)$

$(x-2)(2x^2 + 11x + 12) < 0$

$(x-2)(x+4)(2x+3) < 0$

LC $2x^3$ positive odd ↓ ↑

x-int 2, -4, $-\frac{3}{2}$

y-int -24

It is neg.

$(-\infty, -4)$

$(-\frac{3}{2}, 2)$

4. (b) $2x^3 + 7x^2 - 10x - 24 < 0$

factor: $(x-2)(x+4)(2x+3) < 0$

$x = 2, -4, -\frac{3}{2}$

	$(x-2)$	$(x+4)$	$(2x+3)$	< 0
$(-\infty, -4)$	−	−	−	−
$(-4, -\frac{3}{2})$	−	+	−	+
$(-\frac{3}{2}, 2)$	−	+	+	−
$(2, 0)$	+	+	+	+

Sol'h:

$(-\infty, -4)$ $(-\frac{3}{2}, 2)$

5. $y = (x+4)^2 (x-1)^2 (x+1)$

(a) Degree 5

(b) 4 possible turn. points

(c) lead term: x^5 odd positive

$x \to -\infty$	$x \to +\infty$
$y \to -\infty$	$y \to +\infty$
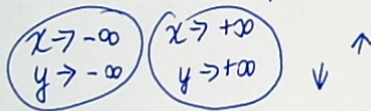
(d) zeros: $-4, 1, -1$

	$(x+4)^2$	$(x-1)^2$	$(x+1)$	
$(-\infty, -4)$	$+$	$+$	$-$	$-$
$(-4, -1)$	$+$	$+$	$-$	$-$
$(-1, 1)$	$+$	$+$	$+$	$+$
$(1, \infty)$	$+$	$+$	$+$	$+$

(e) x-int: $-4, 1, -1$

y-int. 16
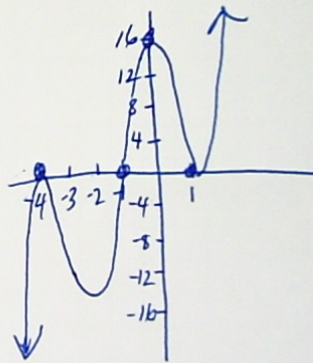
6.　Given $y = -x^4 + 2x^3 + 3x^2 - 4x - 4$ find　　　$\angle C: -x^4$

　　a.　The degree
　　b.　The number of turning points
　　c.　End behavior
　　d.　State the intervals where the polynomial is positive and negative
　　e.　Sketch and clearly show the x and y-intercepts

$$y = -x^4 + 2x^3 + 3x^2 - 4x - 4 \longrightarrow P(-1) =$$

$$y = -1\left(x^4 - 2x^3 - 3x^2 + 4x + 4\right)$$

6. $y = -x^4 + 2x^3 + 3x^2 - 4x - 4$

$y = -1(x^4 - 2x^3 - 3x^2 + 4x + 4)$

$y = -1(x+1)^2(x-2)^2$

a) Degree: 4

b) possible turn pts: 3

c) End Beh: lead term $-x^4$ neg even

$\begin{array}{c} x \to -\infty \\ y \to -\infty \end{array}$ $\begin{array}{c} x \to +\infty \\ y \to -\infty \end{array}$ $\downarrow$ $\downarrow$

$P(-1) = 0$

$$
\begin{array}{r}
x^3 - 3x^2 + 4 \\
x+1 \overline{\smash{)}\ x^4 - 2x^3 - 3x^2 + 4x + 4} \\
\underline{x^4 + x^3} \\
-3x^3 - 3x^2 \\
\underline{-3x^3 - 3x^2} \\
4x + 4 \\
\underline{4x + 4} \\
0
\end{array}
$$

$(x+1)(x^3 - 3x^2 + 0x + 4)$

$\uparrow P(-1) = 0$

$$
\begin{array}{r}
x^2 - 4x + 4 \\
x+1 \overline{\smash{)}\ x^3 - 3x^2 + 0x + 4} \\
\underline{x^3 + x^2} \\
-4x^2 + 0x \\
\underline{-4x^2 - 4x} \\
4x + 4 \\
\underline{4x + 4} \\
0
\end{array}
$$

$(x+1)(x+1)(x^2 - 4x + 4)$

$(x+1)(x+1)(x-2)(x-2)$

(e).
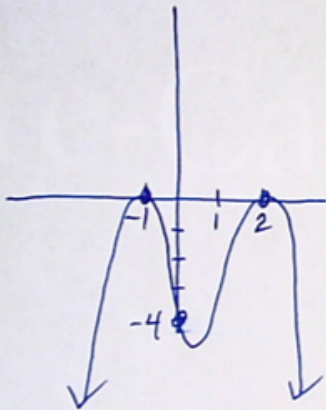
x-int -1, 2

-1 Just Touch   2 Just Touch

y-int: -4

(d) $(-\infty, -1)$ $(-1, 2)$ $(2, \infty)$ it is neg.

it is never positive

7.

(a) $y = a(x+3)(x-1)(x-7)$

$30 = a(2+3)(2-1)(2-7)$

$30 = a(5)(1)(-5)$

$30 = -25a$

$\dfrac{30}{-25} = a$

$-6/5 = a$

OR -1.2

$y = -\dfrac{6}{5}(x+3)(x-1)(x-7)$

(b) $y = a(x+5)(x+1)^2(x-2)(x-4)$

$20 = a(0+5)(0+1)^2(0-2)(0-4)$

$20 = a(5)(1)^2(-2)(-4)$

$20 = 40a$

$\dfrac{20}{40} = a$

$a = \dfrac{1}{2}$

$y = \dfrac{1}{2}(x+5)(x+1)^2(x-2)(x-4)$

Test tomorrow:

Review from book:

Page 153: 1-11 (omit 4)

Page 155-156: 1-4, 6-8

Page 148: 3

Attachments

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