1. (a) $f(x) = [\tan(x^2+3)]^4 - \sec(x^2+3)^4$

$f'(x) = 4[\tan(x^2+3)]^3 \sec^2(x^2+3)\cdot 2x - \sec(x^2+3)^4 \tan(x^2+3)^4 4(x^2+3)^3 \cdot 2x$

(b) $y = \cos(\cot(x^2-3x+2))$

$y' = -\sin(\cot(x^2-3x+2))(-\csc(x^2-3x+2)\cot(x^2-3x+2)(2x-3))$

(c) $f(x) = \cos(2x-x^{-5}) - 5\csc 2x^{-2} - \tan\sqrt{x}$

$f'(x) = -\sin(2x-x^{-5})(2+5x^{-6}) + 5\csc 2x^{-2}\cot(2x^{-2})\cdot(-4x^{-3}) - \sec^2\sqrt{x}\cdot\frac{1}{2}x^{-1/2}$

(d) $y = (\tan(5-2x)^4 - (\sin x)^{1/2})^{1/2}$

$y' = \frac{1}{2}(\tan(5-2x)^4 - (\sin x)^{1/2})^{1/2}(\sec^2(5-2x)^4 4(5-2x)^3(-2) - \frac{1}{2}(\sin x)^{-1/2}\cos x)$

(e) $f(x) = \dfrac{(\cos x)^4 - x}{\sec x}$

$f'(x) = \dfrac{\sec x(4(\cos x)^3(-\sin x) - 1) - ((\cos x)^4 - 1)\sec x \tan x}{\sec^2 x}$

(f) $y = \sin x^2 \sec x^3$

$y' = \sin x^2 \cdot \sec x^3 \tan x^3 \cdot 3x^2 + \sec x^3 \cdot \cos x^2 \cdot 2x$

(g) $y = (\tan(x-2)^{1/2} + (x^2-2)^3)^{-3}$

$y' = -3(\tan(x-2)^{1/2} + (x^2-2)^3)^{-4}(\sec^2(x-2)^{1/2}\cdot\frac{1}{2}(x-2)^{-1/2} + 3(x^2-2)^2\cdot 2x)$

2. $f(x) = \sec x \tan x$

$f'(x) = \sec x \cdot \sec^2 x + \tan x \cdot \sec x \tan x$

$f'(x) = \sec^3 x + \sec x \tan^2 x$

$f'(x) = (\sec x)^3 + \sec x (\tan x)^2$

$f''(x) = 3(\sec x)^2 \sec x \tan x + \sec x \cdot 2(\tan x) \sec^2 x + (\tan x)^2 \sec x \tan x$

$f''(x) = 3(\sec^3 x)^3 \tan x + 2(\sec x)^3 \tan x + \sec x (\tan x)^3$

$f''(x) = 5\sec^3 x \tan x + \sec x \tan^3 x$

$y = 12 \sin x \cos x$

$y' = 12 \sin x \cdot (-\sin x) + \cos x \cdot 12 \cos x$

$y' = -12 \sin^2 x + 12 \cos^2 x$

$y' = -12(\sin x)^2 + 12(\cos x)^2$

$y'' = -24(\sin x) \cdot \cos x + 24(\cos x)(-\sin x)$

PR. Rule

3. (a) $\frac{1}{2}\sin(3y) + \sec(x^2+2) = \tan x \cdot \cos y$

$\frac{1}{2}\cos 3y \cdot 3\frac{dy}{dx} + \sec(x^2+2)\tan(x^2+2)2x = \tan x\left(\sin y \frac{dy}{dx}\right) + \cos y \sec^2 x$

$\frac{3}{2}\cos 3y \frac{dy}{dx} + 2x\sec(x^2+2)\tan(x^2+2) = -\tan x \sin y \frac{dy}{dx} + \cos y \sec^2 x$

$\frac{3}{2}\cos 3y \frac{dy}{dx} + \tan x \sin y \frac{dy}{dx} = \cos y \sec^2 x - 2x\sec(x^2+2)\tan(x^2+2)$

$\frac{dy}{dx}\left(\frac{3}{2}\cos 3y + \tan x \sin y\right) = \cos y \sec^2 x - 2x\sec(x^2+2)\tan(x^2+2)$

$$\frac{dy}{dx} = \frac{\cos y \sec^2 x - 2x\sec(x^2+2)\tan(x^2+2)}{\frac{3}{2}\cos 3y + \tan x \sin y}$$

3(b). $(\cos 2x^3)^2 + 5\tan y^4 = \cot(\sin y)$

$2(\cos 2x^3)(-\sin 2x^3 \cdot 6x^2) + 5\sec^2 y^4 \cdot 4y^3 \frac{dy}{dx} = -\csc(\sin y)\cot(\sin y)\cdot \cos y \frac{dy}{dx}$

$-12x^2 \cos 2x^3 \sin 2x^3 + 20y^3 \sec^2 y^4 \frac{dy}{dx} = -\csc(\sin y)\cot(\sin y)\cos y \frac{dy}{dx}$

$20y^3 \sec^2 y^4 \frac{dy}{dx} + \csc(\sin y)\cot(\sin y)\cos y \frac{dy}{dx} = 12x^2 \cos 2x^3 \sin 2x^3$

$\frac{dy}{dx}\left(20y^3 \sec^2 y^4 + \csc(\sin y)\cot(\sin y)\cos y\right) = 12x^2 \cos 2x^3 \sin 2x^3$

$$\frac{dy}{dx} = \frac{12x^2 \cos 2x^3 \sin 2x^3}{20y^3 \sec^2 y^4 + \csc(\sin y)\cot(\sin y)\cos y}$$

4. $y = 4\sin^2 x - \cos^2 x$

$y = 4(\sin x)^2 - (\cos x)^2$

$y' = 8(\sin x)\cdot\cos x - 2(\cos x)(-\sin x)$

$y' = 8\sin x\cos x + 2\sin x\cos x$

$y' = 10\sin x\cos x$

$m_{x=\frac{2\pi}{3}} = 10\sin\frac{2\pi}{3}\cos\frac{2\pi}{3}$

$= 10\cdot\frac{\sqrt{3}}{2}\cdot\left(-\frac{1}{2}\right) = \frac{-10\sqrt{3}}{4}$

$x = \frac{2\pi}{3}$

$8\sin\frac{2\pi}{3}\cos\frac{2\pi}{3} + 2\sin\frac{2\pi}{3}\cos\frac{2\pi}{3}$

$8\cdot\frac{\sqrt{3}}{2}\left(-\frac{1}{2}\right) + 2\frac{\sqrt{3}}{2}\left(-\frac{1}{2}\right)$

$-\frac{8\sqrt{3}}{4} - \frac{2\sqrt{3}}{4}$

$-\frac{10\sqrt{3}}{4}$

5. $y = 12(\sec x)^3 - \cos 4x$

$y' = 36(\sec x)^2 \sec x \tan x + \sin 4x \cdot 4$

$y' = 36\sec^3 x \tan x + 4\sin 4x$

$x = \pi/6 \Rightarrow 36\left(\sec\frac{\pi}{6}\right)^3 \tan\frac{\pi}{6} + 4\sin\frac{4\pi}{6}$

$36\left(\frac{2}{\sqrt{3}}\right)^3 \cdot \left(\frac{1}{\sqrt{3}}\right) + 4\frac{\sqrt{3}}{2}$

$36\left(\frac{8}{\sqrt{3^3}}\right)\frac{1}{\sqrt{3}} + \frac{4\sqrt{3}}{2}$

$\frac{288}{\sqrt{81}} + 2\sqrt{3}$

$\frac{288}{9} + 2\sqrt{3}$

$32 + 2\sqrt{3}$

6. $y = 3\sec x + \tan 2x$

$y' = 3\sec x \tan x + \sec^2 2x \cdot 2$

$m_{x=\pi/6} = 3\sec\frac{\pi}{6}\tan\frac{\pi}{6} + 2\left(\sec 2\cdot\frac{\pi}{6}\right)^2$

$= 3\cdot\frac{2}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}} + 2\left(\sec\frac{\pi}{3}\right)^2$

$\frac{6}{3} + 2(2)^2$

$2 + 8$

$= 10$

$\left(\frac{\pi}{6}, 3\sqrt{3}\right)\quad m = 10$

$y - 3\sqrt{3} = 10\left(x - \frac{\pi}{6}\right)$

$y - 3\sqrt{3} = 10x - \frac{10\pi}{6}$

$y = 10x - \frac{5\pi}{3} + 3\sqrt{3}$