1) a) $\sqrt[3]{56} \longrightarrow \sqrt[3]{8 \times 7}$

look at perfect cubes

$\sqrt[3]{8} \cdot \sqrt[3]{7}$

$2\sqrt[3]{7}$

2) a) $3\sqrt[3]{5}$

$= \sqrt[3]{3^3 \cdot 5}$

$= \sqrt[3]{27 \times 5}$

$= \sqrt[3]{135}$

b) $2\sqrt[4]{6}$

$= \sqrt[4]{2^4 \cdot 6}$

$= \sqrt[4]{16 \cdot 6}$

$= \sqrt[4]{96}$

1b) $\sqrt{98}$ ← square root

$= \sqrt{49 \cdot 2}$

$= \sqrt{49}\sqrt{2}$

$= 7\sqrt{2}$

3 a) $3^{\frac{5}{2}}$ ← power (5); 2 ← root or index

$\sqrt{3^5}$

or

$\left(\sqrt{3}\right)^5$

b) $15^{-\frac{2}{3}}$

$= \left(\frac{1}{15}\right)^{\frac{2}{3}}$

$= \frac{1}{\left(\sqrt[3]{15}\right)^2}$

$\left(\frac{1}{\sqrt[3]{15^2}}\right)$

$\frac{1}{\sqrt[3]{15^2}}$

5 i) $\frac{2n^4}{(4m^4n^0)(m^1n)} = \frac{\overset{1}{\cancel{2}}n^4}{\overset{2}{\cancel{4}}m^5n^1} = \frac{1n^3}{2m^5}$

$\frac{2n^{4-1}}{4m^5}$

$\frac{1n^3}{2m^5}$

Perfect Squares

$(1)^2$ = 1 x 1 = **1**
$(2)^2$ = 2 x 2 = **4**
$(3)^2$= 3 x 3 = **9**
$(4)^2$= 4 x 4 = **16**
$(5)^2$= 5 x 5 = **25**
$(6)^2$= 6 x 6 = **36**
$(7)^2$= 7 x 7 = **49**
$(8)^2$= 8 x 8 = **64**
$(9)^2$= 9 x 9 = **81**
$(10)^2$= 10 x 10 = **100**
$(11)^2$= 11 x 11 = **121**
$(12)^2$= 12 x 12 = **144**
$(13)^2$= 13 x 13 = **169**
$(14)^2$= 14 x 14 = **196**
$(15)^2$= 15 x 15 = **225**
$(16)^2$= 16 x 16 = **256**
$(17)^2$= 17 x 17 = **289**
$(18)^2$= 18 x 18 = **324**
$(19)^2$= 19 x 19 = **361**
$(20)^2$= 20 x 20 = **400**
$(21)^2$= 21 x 21 = **441**
$(22)^2$= 22 x 22 = **484**
$(23)^2$= 23x 23 = **529**
$(24)^2$= 24 x 24 = **576**
$(25)^2$= 25 x 25 = **625**

Perfect Cubes

$(1)^3$ = 1 x 1 x 1 = **1**
$(2)^3$ = 2 x 2 x 2 = **8**
$(3)^3$ = 3 x 3 x 3 = **27**
$(4)^3$ = 4 x 4 x 4 = **64**
$(5)^3$ = 5 x 5 x5 = **125**
$(6)^3$ = 6 x 6 x 6= **216**
$(7)^3$ = 7 x 7 x 7 = **343**
$(8)^3$ = 8 x 8 x 8= **512**
$(9)^3$ = 9 x 9 x 9= **729**
$(10)^3$ = 10 x 10 x 10 = **1000**
$(11)^3$ = 11 x 11 x 11 = **1331**
$(12)^3$ = 12 x 12 x 12= **1728**
$(13)^3$ = 13 x 13 x 13 = **2197**
$(14)^3$ = 14 x 14 x 14= **2744**
$(15)^3$ = 15 x 15 x 15 = **3375**
$(16)^3$ = 16 x 16 x 16= **4096**
$(17)^3$ = 17 x 17 x 17 = **4913**
$(18)^3$ = 18 x 18 x18 = **5832**
$(19)^3$ = 19 x 19 x 19 = **6859**
$(20)^3$ = 20 x 20 x 20 = **8000**
$(21)^3$ = 21 x 21 x 21 = **9261**
$(22)^3$ = 22 x 22 x 22 = **10648**
$(23)^3$ = 23x 23 x 23 = **12167**
$(24)^3$ = 24 x 24 x 24 = **13824**
$(25)^3$ = 25 x 25 x 25 = **15625**

How are radicals that are rational numbers different from radicals that are not rational numbers?

Rational numbers terminate (end) or repeat

Irrational numbers do not terminate (end)

Which of these radicals are rational numbers?
Which are not rational numbers? How do you know?

$\sqrt{1.44}$	$\sqrt{\frac{64}{81}}$	$\sqrt[3]{-27}$	$\sqrt{\frac{4}{5}}$	$\sqrt{5}$
$= 1.2$	$= \frac{8}{9}$	$= 3$	$= \sqrt{0.8} = 0.8944...$	$= 2.236067....$

$$\frac{\sqrt{64}}{\sqrt{81}} = \frac{8}{9}$$

$$\frac{\sqrt{4}}{\sqrt{5}} = \frac{2}{\sqrt{5}}$$

Write 3 other radicals that are rational numbers. Why are they rational?

Write 3 other radicals that are not rational numbers. Why are they not rational?

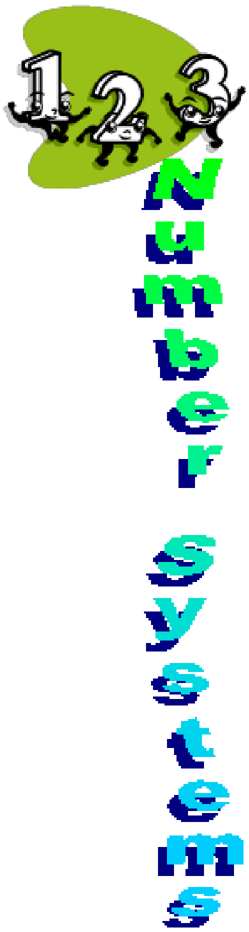
4.2 Irrational Numbers

When an irrational number is written as a radical, the radical is the exact value.

Examples: $\sqrt{2}$ $\sqrt[3]{-50}$ **exact**

When we use the square root or cube root key on our calculators we are obtaining approximate value of irrational numbers.

$\sqrt{2} \approx 1.4142$



Natural Numbers	N
Whole Numbers	W
Integers	I
Rational	Q
Irrational	$\overline{Q}$
Real	R

Natural Numbers : Ex. 1, 2, 3 etc

Whole Numbers: Counting numbers including zero.
Ex. 0, 1, 2, 3, etc

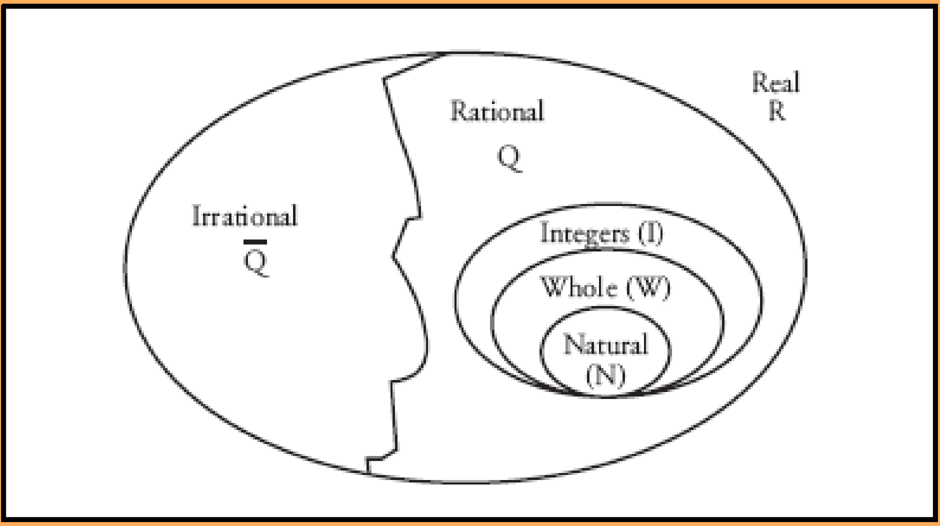
Integers: Are all positive and negative whole numbers.
(Remember zero is neither negative or positive)
Ex: ....3,2,1,0,-1-2,-3...

Rational Numbers: All whole numbers, fractions, mixed numbers, decimals and their negatives
The decimal must repeat or terminate also.
Ex: 1/3, 4, 3/4

Irrational Numbers: Decimals that never terminate or repeat.
Ex: $\sqrt{2}$

Real Numbers: All rational and irrational numbers are real numbers
Ex: All possible numbers

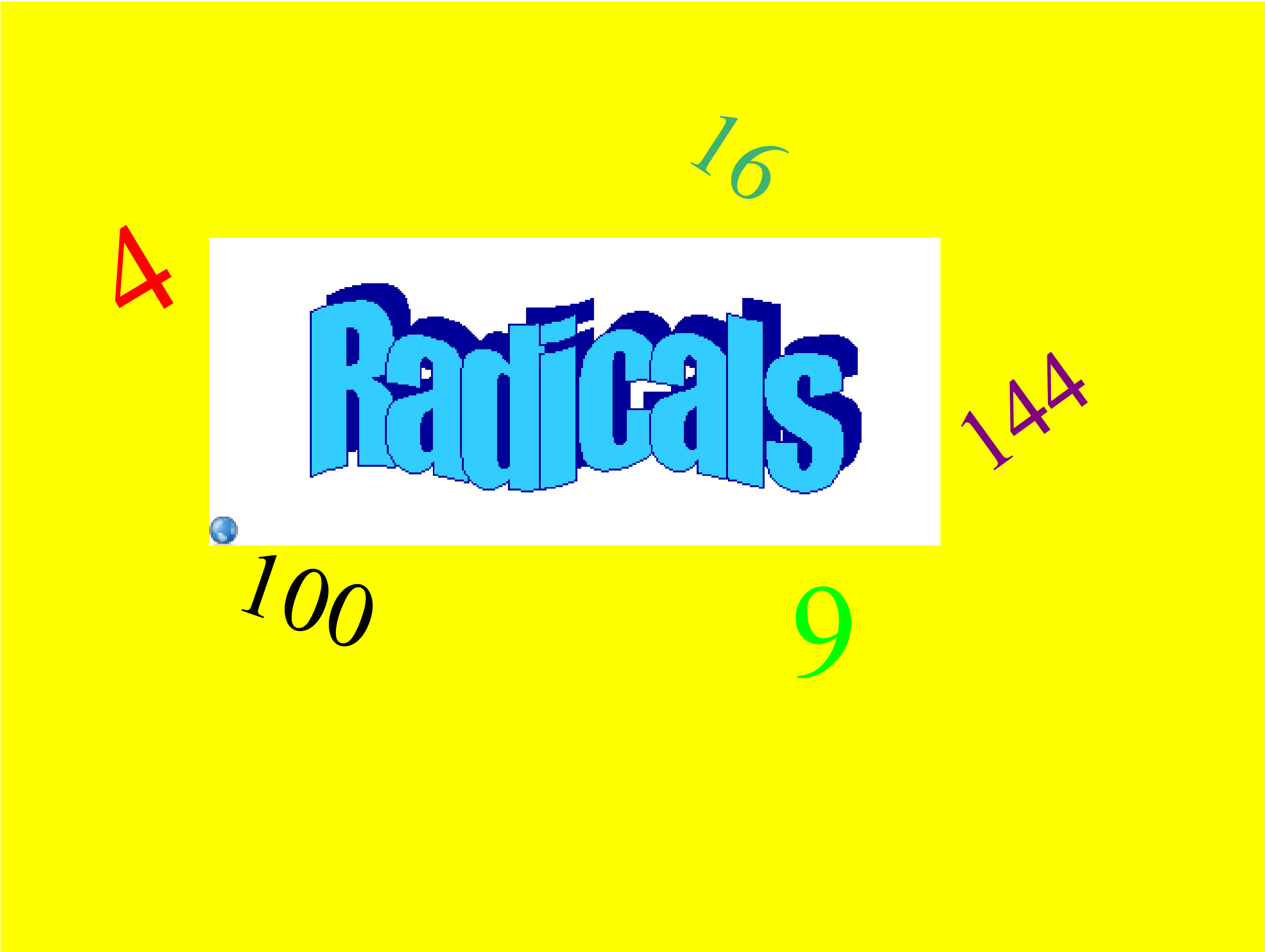
Review of Types of Number Systems



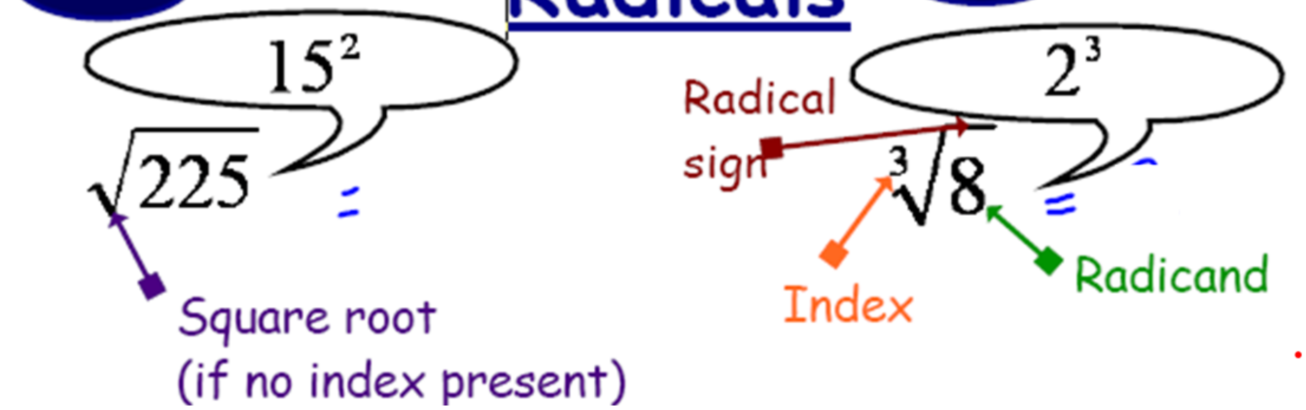
Exercise

Complete the table

	N	W	I	Q	$\overline{Q}$	R
5						
-2						
$\frac{3}{4}$						
-1.3						
$\sqrt{7}$						
$\sqrt{95}$						


16
4
Radicals
144
100
9

Radicals


15^2
$\sqrt{225}$ =
Square root
(if no index present)
2^3
Radical
sign
$\sqrt[3]{8}$ =
Index
Radicand

Radicals

Write a fraction that is equivalent to:

$$\frac{3}{4}$$

Just as with fractions, Radicals expressions have equivalent expressions:

$$\sqrt{16 \cdot 9} = \sqrt{16} \cdot \sqrt{9}$$
$$= 4 \cdot 3$$
$$= 12$$

or

$$\sqrt{16 \cdot 9} = \sqrt{144}$$
$$= 12$$

Same works if we change the "index":

$$\sqrt[3]{8 \cdot 27} = \sqrt[3]{8} \cdot \sqrt[3]{27}$$
$$= 2 \cdot 3$$
$$= 6$$

or

$$\sqrt[3]{8 \cdot 27} = \sqrt[3]{216}$$
$$= 6$$

Reducing Radicals

Multiplication Property of Radicals

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$,

where n is a natural number, and a and b are real numbers

Radicals

Mixed Radical - has a coefficient in front of the radical sign.

ex: $3\sqrt{5}$ OR $\frac{2\sqrt{26}}{3}$ OR $-3\sqrt[3]{3}$

Entire Radical - has a coefficient of 1 or -1 in front of the radical sign. Everything is entirely under the radical sign

ex: $\sqrt{12}$ OR $-\sqrt{45}$

$\sqrt[3]{216}$ or $-1(\sqrt[4]{72})$

Reducing Radicals

To reduce $\sqrt{125}$
you must find the largest square number that will divide into 125 evenly!

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Greatest perfect n^{th}

$\sqrt{125} = \sqrt{25 \cdot 5}$
$= \sqrt{25} \cdot \sqrt{5}$
$= 5\sqrt{5}$

Your Life Line

4
9
16
25
36
49
64
81
100
121

Try these:

a) $\sqrt{12}$

$\sqrt{4 \cdot 3}$

$\sqrt{4} \cdot \sqrt{3}$

$2\sqrt{3}$

b) $\sqrt{72}$

$\sqrt{36 \cdot 2}$

$\sqrt{36} \cdot \sqrt{2}$

$6\sqrt{2}$

c) $\sqrt{54}$

$\sqrt{9 \cdot 6}$

$\sqrt{9}\sqrt{6}$

$3\sqrt{6}$

$\sqrt{12} = 2\sqrt{3}$

We can also use prime factorization to simplify a radical.

Example 1 Simplifying Radicals Using Prime Factorization

Simplify each radical.

a) $\sqrt{80}$ b) $\sqrt[3]{144}$ c) $\sqrt[4]{162}$

SOLUTION

CHECK YOUR UNDERSTANDING

a) $\sqrt{80} = \sqrt{16 \cdot 5} = \sqrt{16} \times \sqrt{5}$

$= 4\sqrt{5}$

b) $\sqrt[3]{144} = \sqrt[3]{8 \cdot 18} = \sqrt[3]{8} \cdot \sqrt[3]{18}$

$= 2\sqrt[3]{18}$

c) $\sqrt[4]{162} = \sqrt[4]{81 \cdot 2} = \sqrt[4]{81}\ \sqrt[4]{2}$

$= 3\sqrt[4]{2}$

Entire → Mixed

$\sqrt[3]{144}$

$= \sqrt[3]{8 \cdot 18}$

$= \sqrt[3]{8} \; \sqrt[3]{18}$

$= 2 \sqrt[3]{18}$

1^3	1
2^3	8
3^3	27
4^3	64
5^3	125
6^3	216
7^3	334
8^3	512

Entire Radicals
(mixed ⇒ entire)

mixed	entire
$a\sqrt[n]{b}$ →	$\sqrt{(a^n) \cdot b}$

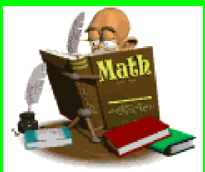
$7 \times 7 = 49$

$7^2 = 49$

$\sqrt[2]{49} = 7$

Express as an entire radical.

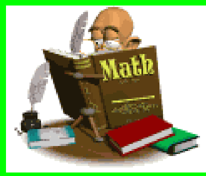
$2\sqrt[4]{7}$



MATH 10

Radicals

WARMUP



1) Change the following to mixed radicals in simplest form

a) $\sqrt{486} = \sqrt{81 * 6}$

$= \sqrt{81} * \sqrt{6}$

$= 9\sqrt{6}$

Just Erase

2) Write the following as entire radicals

a) $2\sqrt{11} = \sqrt{(2)^2 * 11}$

$= \sqrt{4 * 11}$

$= \sqrt{44}$

Just Erase

18. Write each mixed radical as an entire radical.

a) $6\sqrt[4]{3}$ b) $7\sqrt[4]{2}$

c) $3\sqrt[5]{4}$ d) $4\sqrt[5]{3}$

18. a) $\sqrt[4]{3888}$ b) $\sqrt[4]{4802}$

c) $\sqrt[5]{972}$ d) $\sqrt[5]{3072}$

4.4 Fractional Exponents and Radicals

LESSON FOCUS Relate rational exponents and radicals.

Make Connections

Coffee, tea, and hot chocolate contain caffeine. The expression $100(0.87)^{\frac{1}{2}}$ represents the percent of caffeine left in your body $\frac{1}{2}$ h after you drink a caffeine beverage.

Given that $0.87^1 = 0.87$ and $0.87^0 = 1$, how can you estimate a value for $0.87^{\frac{1}{2}}$?

Rational Exponents and Radicals

Let's examine radicals...

$$\sqrt{5} \times \sqrt{5} =$$

How would this play out with exponent laws?

$$5^{?} \times 5^{?} = 5^{1}$$

$$\text{RULE: } \sqrt{x} = x^{\frac{1}{2}}$$

What about other rational exponents and radicals?

$$8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} =$$

$$\text{Rule: } \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\text{In general... } \left(\sqrt[n]{x}\right)^{m} \text{ or } \sqrt[n]{x^{m}} = x^{\frac{m}{n}}$$

Rational Exponents

- ***To evaluate exponents that are rational (fractions), the denominator of the fraction indicates which root to take and the numerator indicates which power the entire base is to be raised.***

Example

$16^{1/4}$

$125^{1/3}$

$125^{2/3}$

Example 1 Evaluating Powers of the Form $a^{\frac{1}{n}}$

Evaluate each power without using a calculator.

a) $27^{\frac{1}{3}}$ b) $0.49^{\frac{1}{2}}$ c) $(-64)^{\frac{1}{3}}$ d) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

$x^{\frac{1}{n}}$ ← power (the 1) ← index or root (the n)

SOLUTION

a) $27^{\frac{1}{3}}$

$\sqrt[3]{27}$

$= 3$

b) $0.49^{\frac{1}{2}}$

$\sqrt{0.49}$

CHECK YOUR UNDERSTANDING

Examples: Express each exponential in radical form , then evaluate.

1. $8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2$

$= (2)^2$

$= 4$

(power ← 2, index ← 3)

2. $125^{-\frac{1}{3}} = \left(\frac{1}{125}\right)^{\frac{1}{3}} = \frac{1^{\frac{1}{3}}}{125^{\frac{1}{3}}}$

$= \frac{\sqrt[3]{1}}{\sqrt[3]{125}}$

$= \frac{1}{5}$

3. $32^{-\frac{7}{5}} =$

$= \left(\frac{1}{32}\right)^{\frac{7}{5}}$

$= \frac{1^{\frac{7}{5}}}{32^{\frac{7}{5}}}$

$= \frac{1}{\left(\sqrt[5]{32}\right)^7}$

$= \frac{1}{(2)^7}$

$= \frac{1}{128}$

4. $\frac{3}{9^{-\frac{3}{2}}} =$

What do you think $a^{\frac{1}{4}}$ and $a^{\frac{1}{5}}$ mean?

What does $a^{\frac{1}{n}}$ mean? Explain your reasoning.

Express as a exponent:

a) $\sqrt[5]{32}$ b) $\sqrt[3]{-64}$ c) $(\sqrt{144})^3$

Express as a Radical:

a) $8^{\frac{5}{3}}$ b) $49^{\frac{3}{2}}$ c) $(-125)^{\frac{2}{3}}$

Example 2 Rewriting Powers in Radical and Exponent Form

a) Write $40^{\frac{2}{3}}$ in radical form in 2 ways.

b) Write $\sqrt{3^5}$ and $(\sqrt[3]{25})^2$ in exponent form.

SOLUTION

CHECK YOUR UNDERSTANDING