Laws of Exponents

Review From Gr.9

Product of powers law: $a^m \cdot a^n = a^{m+n}$

Quotient of powers law: $\dfrac{a^m}{a^n} = a^{m-n}$

Power of a power law: $(a^m)^n = a^{mn}$

Write as a single power.

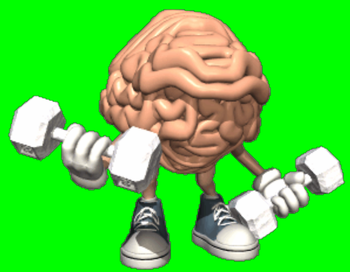
a) $3^2 \cdot 3^5 = 3^7$

b) $(4^2)^5 = 4^{10}$

c) $(-5)^{10} \div (-5)^8 = (-5)^2$

Homework Questions???

Page 218-219 #11j, 12b,d,f,h,i, 19(a,b,c), 20, 21, 22a, 23



Warm Up

Name:______________________ Period:_____

Simplify then evaluate

1) $(2^4)^3$

$= 2^{12}$

$= 4096$

2) $[(-5)^2 \times 2]^3$

$((-5)^2)^3 \times 2^3$

$\boxed{(-5)^6 \times 2^3}$

$+15625 \times 8$

125000

3) $[(-1)^{11}]^3$

$(-1)^{33}$

$= -1$

Write each expression as a product or quotient of powers. Then evaluate.

1) $[(-3) \times (5)]^2$

$(-15)^2$

$+225$

OR

$(-3)^2 \; (5)^2$

$9 \cdot 25$

225

2) $\left(\frac{6}{5}\right)^4$

$= \frac{6^4}{5^4}$

$= \frac{1296}{625}$

$= 2.0736$

Math 10: Numbers, Functions & Relations Name________________________________

Laws of Exponents Review

Date______________

Simplify. Your answer should contain only positive exponents.

multiply add exponents

1) $\left(\frac{2^1 \cdot 2^2}{2^1}\right)^3 = \left(\frac{2^3}{2^1}\right)^3$ divide like

$= (2^2)^3$

$= 2^6$

2) $\left(\frac{2^4}{2^3 \cdot 2^3}\right)^4 = \left(\frac{2^4}{2^6}\right)^4 = (2^{-2})^4$

$= 2^{-8}$

3) $\frac{2^2}{4^2} = \frac{2^2}{(2^2)^2}$

$= \frac{2^2}{2^4}$

$= 2^{-2}$

4) $\frac{(2^3 \cdot 2^4)^4}{2} \quad \frac{(2^7)^4}{2^1}$

$= \frac{2^{28}}{2^1}$

$= 2^{27}$

$$\left(\frac{2^4}{2^6}\right)^4$$

$$\left(\frac{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}}{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot 2\cdot 2}\right)^4$$

$$= \left(\frac{1}{2\cdot 2}\right)^4$$

$$= \left(\frac{1}{2^2}\right)^4$$

$$= \left(\frac{1^4}{2^8}\right)$$

$$= \frac{1}{2^8}$$

$$\left(\frac{2^1 \cdot 2^2}{2^1}\right)^3$$

$$\left(\frac{2 \cdot 2 \cdot \cancel{2}}{\cancel{2}}\right)^3$$

$$(2^2)^3$$

$$2^2 \cdot 2^2 \cdot 2^2$$

$$2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2$$

$$2^6$$

$2^1 \cdot 2^3$

$2 \cdot 2 \cdot 2 \cdot 2$

2^4

$2^1 \cdot 2^2$

$2 \quad 2 \cdot 2$

2^3

4.4 Fractional Exponents and Radicals

LESSON FOCUS Relate rational exponents and radicals.

Make Connections

Coffee, tea, and hot chocolate contain caffeine. The expression $100(0.87)^{\frac{1}{2}}$ represents the percent of caffeine left in your body $\frac{1}{2}$ h after you drink a caffeine beverage.

Given that $0.87^1 = 0.87$ and $0.87^0 = 1$, how can you estimate a value for $0.87^{\frac{1}{2}}$?

Use a calculator to complete the table.

a b/c

Column 1	Column 2
x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} = 1$
4	$4^{\frac{1}{2}} = 2$
9	$9^{\frac{1}{2}} = 3$
16	$16^{\frac{1}{2}} = 4$
25	$25^{\frac{1}{2}} = 5$
36	6
49	7

a) What do you notice about the numbers in the first column?

$\sqrt{\ } =$ exponent $\frac{1}{2}$

b) Compare the numbers in the first and second columns. What conclusions can you make?

$\sqrt{4} = 4^{\frac{1}{2}} = 2$

$\sqrt{9} = 9^{\frac{1}{2}} = 3$

c) What do you think the exponent $\frac{1}{2}$ means?

4.4 Fractional Exponents and Radicals

Use a calculator to complete the table.

Column 1 Column 2 Column 3

x	$x^{\frac{1}{3}}$
1	
8	
27	
64	
125	

a) What do you notice about the numbers in the first column?

b) Compare the numbers in the first and second columns. What conclusions can you make?

c) What do you think the exponent $\frac{1}{3}$ means?

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

What do you think $a^{\frac{1}{4}}$ and $a^{\frac{1}{5}}$ mean?

$$a^{\frac{1}{4}} = \sqrt[4]{a}$$

$$a^{\frac{1}{5}} = \sqrt[5]{a}$$

What does $a^{\frac{1}{n}}$ mean? Explain your reasoning.

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Rational Exponents and Radicals ☆

Let's examine radicals...

grade 8 material $\sqrt{5}\times\sqrt{5} = \sqrt{5\times 5} = \sqrt{25} = 5$

$(\sqrt{5})^2 = 5$

How would this play out with exponent laws?

$$5^{?}\times 5^{?} = 5^{1}$$

0.5 $\frac{1}{2}$ 0.5 $\frac{1}{2}$

$$\boxed{\text{RULE: } \sqrt{x} = x^{\frac{1}{2}}}$$

What about other rational exponents and radicals?

$$8^{\frac{1}{3}}\times 8^{\frac{1}{3}}\times 8^{\frac{1}{3}} = 8^{\frac{3}{3}} = 8^{1} = 8$$

$$\text{Rule: } \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\text{In general... } \left(\sqrt[n]{x}\right)^{m} \text{ or } \sqrt[n]{x^{m}} = x^{\frac{m}{n}}$$

Rational Exponents

- ***To evaluate exponents that are rational (fractions), the denominator of the fraction indicates which root to take and the numerator indicates which power the entire base is to be raised.***

Example

$16^{1/4}$

$125^{1/3}$

$125^{2/3}$

Example 1 Evaluating Powers of the Form $a^{\frac{1}{n}} = \sqrt[n]{a}$

Evaluate each power without using a calculator.

a) $27^{\frac{1}{3}}$

$= \sqrt[3]{27}$

$= 3$

b) $0.49^{\frac{1}{2}}$

$\sqrt{0.49}$

$\frac{\sqrt{49}}{\sqrt{100}}$

$= \frac{7}{10}$

$= 0.7$

c) $(-64)^{\frac{1}{3}}$

$\sqrt[3]{-64}$

$= -4$

look in cube list

d) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

$= \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3} = 0.\overline{6}$

CHECK YOUR UNDERSTANDING

4.4 Fractional Exponents and Radicals

Examples:

Express each exponential in radical form , then evaluate.

exponent

Index

1. $8^{\frac{2}{3}} = \left(\underbrace{\sqrt[3]{8}}\right)^2$

base

$= (2)^2$

$= 4$

2. $-125^{\frac{1}{3}} = \sqrt[3]{-125}$

$= -5$

ok not to know yet

3. $32^{\frac{7}{5}} = \left(\underbrace{\sqrt[5]{32}}\right)^7$

$= (2)^7$

$= 128$

4. $\dfrac{3}{9^{\frac{3}{2}}} = \dfrac{3}{\left(\sqrt{9}\right)^3}$

$\dfrac{3^1}{(3)^{-3}}$

$3^{1-(-3)}$ add opp

3^4 :)

Express as a exponent:

a) $\sqrt[5]{32}$

$32^{\frac{1}{5}}$

b) $\sqrt[3]{-64}$

$(-64)^{\frac{1}{3}}$

c) $\left(\sqrt{144}\right)^3$

$144^{\frac{3}{2}}$

denom Index, exponent (numerator)

$$\left(\sqrt[\text{Index}]{\text{Base}}\right)^{\text{exponent}} = (\text{Base})^{\frac{\text{exp}}{\text{Index}}}$$

Express as a Radical:

a) $8^{\frac{5}{3}}$

$\left(\sqrt[3]{8}\right)^5$

b) $49^{\frac{3}{2}}$

$\left(\sqrt{49}\right)^3$

c) $(-125)^{\frac{2}{3}}$

$\left(\sqrt[3]{-125}\right)^2$

These examples illustrate that the numerator of a fractional exponent represents a power and the denominator represents a root. The root and power can be evaluated in any order.

?

Power

$$x^{\frac{m}{n}}$$

Root

Homework

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4.4 Fractional Exponents and Radicals

Exercises

A

3 4 5 6 7

B

8 9 10 11 12 13 14 15

16 17 18 19 20 21

C

22

#3 adf
~~#4 acd~~
#5 abc
#6 ac
#8 abc
#9
#12 abf

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