

$$1. (a) f(x) = \frac{2x^3}{2x^3 + 5x^2 - 12x} ; f(x) = \frac{2x^3}{x(2x-3)(x+4)}$$

$$VA: x=0, x=\frac{3}{2}, x=-4$$

$$HA: y = \frac{2}{2} = 1$$

$$(b) f(x) = \frac{-x^2}{(2x-1)(x+5)} ; f(x) = \frac{-x^2}{2x^2 + 9x - 5}$$

$$VA: x = \frac{1}{2}, x = -5$$

$$HA: y = -\frac{1}{2}$$

(a) $f(x) = x^3 - 27x + 30$
 $f'(x) = 3x^2 - 27$
 $3(x^2 - 9) = 0$
 $3(x-3)(x+3) = 0$
 $3, -3$

	$3(x-3)(x+3)$			y'	
$(-\infty, -3)$	+	-	-	+	inc
$(-3, 3)$	+	-	+	-	dec
$(3, \infty)$	+	+	+	+	inc

$x = -3 \quad y = 84 \text{ max}$
 $x = 3 \quad y = -24 \text{ min}$

(b) $y = (x-2)^8(x+1)^4$
 $y' = (x-2)^8 \cdot 4(x+1)^3 + (x+1)^4 \cdot 8(x-2)^7$
 $y' = 4(x-2)^8(x+1)^3 + 8(x-2)^7(x+1)^4$
 $4(x-2)^7(x+1)^3((x-2) + 2(x+1))$
 $4(x-2)^7(x+1)^3(x-2+2x+2)$
 $y' = 4(x-2)^7(x+1)^3(3x)$
 $y' = 12x(x-2)^7(x+1)^3$
 $x = 0, 2, -1$

	$12x(x-2)^7(x+1)^3$			y'	
$(-\infty, -1)$	-	-	-	-	dec
$(-1, 0)$	-	-	+	+	inc
$(0, 2)$	+	-	+	-	dec
$(2, \infty)$	+	+	+	+	inc

$x = -1 \quad y = 0 \text{ min}$
 $x = 0 \quad y = 256 \text{ max}$
 $x = 2 \quad y = 0 \text{ min}$

$$3.(a) f(x) = x^4 - 8x^3 - 192x^2 + 90x + 150$$

$$f'(x) = 4x^3 - 24x^2 - 384x + 90$$

$$f''(x) = 12x^2 - 48x - 384$$

$$12(x^2 - 4x - 32) = 0$$

$$12(x-8)(x+4) = 0$$

$$x = 8, -4$$

	$12(x-8)(x+4)$			y''
$(-\infty, -4)$	+	-	-	+ CU
$(-4, 8)$	+	-	+	- CD
$(8, \infty)$	+	+	+	+ CU

Points of Inf:

$$\begin{cases} x = -4 & y = -2514 & (-4, -2514) \\ x = 8 & y = -11418 & (8, -11418) \end{cases}$$

$$3(b) \quad y = \frac{1}{2}x^3 - \frac{24}{x}$$

$$y = \frac{1}{2}x^3 - 24x^{-1}$$

$$y' = \frac{3}{2}x^2 + 24x^{-2}$$

$$y'' = 3x - 48x^{-3}$$

$$3x^{-3}(x^4 - 16) = 0$$

$$3x^{-3}(x^2 - 4)(x^2 + 4) = 0$$

$$3x^{-3}(x-2)(x+2)(x^2+4) = 0$$

$$x = 0, 2, -2$$

	$\frac{3}{x^3}(x-2)(x+2)(x^2+4)$	y''
$(-\infty, -2)$	- - - +	-CD
$(-2, 0)$	- - + +	+CU
$(0, 2)$	+ - + +	-CD
$(2, \infty)$	+ + + +	+CU

Pts of Inf

$$x = -2 \quad y = 8 \quad (-2, 8)$$

$$x = 0 \quad \text{undef.} \quad (2, -8)$$

$$x = 2 \quad y = -8$$

4. (a) $y = x^3 - 6x^2 + 16$; $y' = 3x^2 - 12x$; $y'' = 6x - 12$

A. $x \in \mathbb{R}$

B. Intercepts

(x) $(x-2)(x^2 - 4x - 8)$ (y) $y = 16$

\downarrow \downarrow

2 $\frac{4 \pm \sqrt{48}}{2}$

$= -1.46, 5.46$

F. Concavity $6x - 12 = 0$
 $6(x-2) = 0$
 $x = 2$

	$6(x-2)$	y''	
$(-\infty, 2)$	+	-	\downarrow CD
$(2, \infty)$	+	+	+ CU

C. Asymptotes —

Point(s) of Inf:

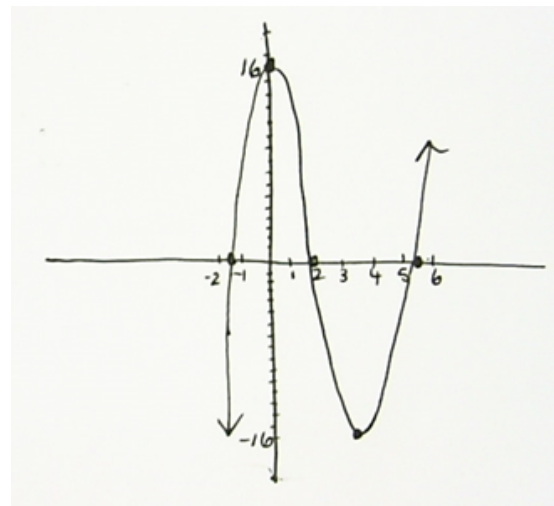
$x = 2$ $y = 0$ $(2, 0)$

D. Inc/Dec $3x^2 - 12x = 0$
 $3x(x-4) = 0$
 $0, 4$

	$3x(x-4)$	y'	
$(-\infty, 0)$	-	-	+ Inc
$(0, 4)$	+	-	- Dec
$(4, \infty)$	+	+	+ Inc

E. max/min

$x = 0$ $y = 16$ max
 $x = 4$ $y = -16$ min



(b) $y = \frac{2x^2 - 8}{x^2 - 1}$; $y' = \frac{12x}{(x^2 - 1)^2}$; $y'' = \frac{-36x^2 - 12}{(x^2 - 1)^3}$

A) $x \neq -1, 1$

B) Intercepts

$y = \frac{2(x-2)(x+2)}{(x^2-1)}$; $y = -\frac{8}{-1} = 8$

$x = 2, x = -2$

(c) Asymptotes: VA $x = 1, -1$
 HA $y = 2$

D) Inc/Dec $\frac{12x}{(x^2-1)^2} = 0$

$x = 0, 1, -1$

	$\frac{12x}{(x^2-1)^2}$	y'
$(-\infty, -1)$	-/+	- dec
$(-1, 0)$	-/+	- dec
$(0, 1)$	+/+	+ inc
$(1, \infty)$	+/+	+ inc

$y = \frac{2x^2 - 8}{x^2 - 1}$
 Behaviour

-1.1	-0.9	0.9	1.1
-∞	-∞	∞	∞

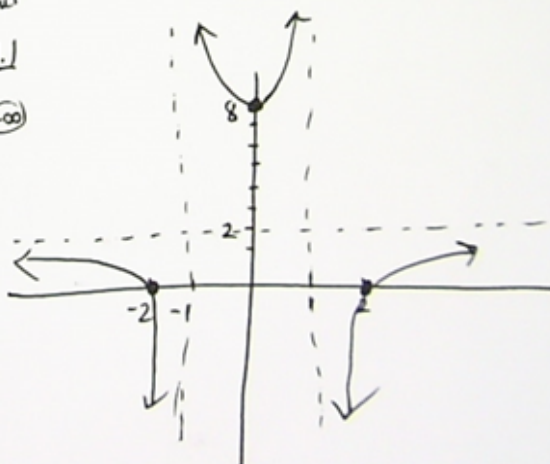
E) max/min $x = 0, y = 8 (0, 8)$

$x = -1$
 $x = 1$ } undefined

F) Concavity $\frac{-36x^2 - 12}{(x^2 - 1)^3} = 0$ ← no zeros

	$\frac{-36x^2 - 12}{(x^2 - 1)^3}$	y''
$(-\infty, -1)$	-/+	- CD
$(-1, 1)$	-/-	+ CU
$(1, \infty)$	-/+	- CD

No points of inflection



c) $y = \frac{4-8x}{1+x}$, $y' = \frac{-12}{(1+x)^2}$, $y'' = \frac{24}{(1+x)^3}$

A) $x \neq -1$

B) $x\text{-int } x = \frac{1}{2}$, $y\text{-int } y = 4$

C) Asymptote VA $x = -1$
HA $y = -8$

D) Inc/Dec $\frac{-12}{(1+x)^2} = 0$ (no zero) $\rightarrow x = -1$

	$\frac{-12}{(1+x)^2}$	
$(-\infty, -1)$	- / +	- dec
$(-1, \infty)$	- / +	- dec

E) No max/min

F) Concavity $\frac{24}{(1+x)^3} = 0$ (no zero) $\rightarrow -$

	$\frac{24}{(1+x)^3}$	y''
$(-\infty, -1)$	+ / -	- CD
$(-1, \infty)$	+ / +	+ CU

No pt. of Inflection $y = \frac{4-8x}{1+x}$

