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whole area not just rectangle

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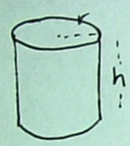
UNB 2002

(8) 8. A cylindrical container with no top is to be constructed to hold 45 m^3 of liquid. The cost of the material used for the bottom is \$5 per m^2 and the cost of the material used for the curved face is \$3 per m^2. Use calculus to find the dimensions (radius and height) of the least expensive container. Justify why this is the least expensive.

2002



optimize cost

no top

Constraint:

$V = 45$

$\pi r^2 h = 45$

$h = \frac{45}{\pi r^2}$

SA = bottom + side

$SA = \pi r^2 + 2\pi r h$

$Cost = 5\pi r^2 + 3(2\pi r h)$

$C = 5\pi r^2 + 6\pi r h$

$C = 5\pi r^2 + 6\pi r\left(\frac{45}{\pi r^2}\right)$

$C = 5\pi r^2 + \frac{270}{r}$

$C = 5\pi r^2 + 270 r^{-1}$

$C' = 10\pi r - 270 r^{-2}$

$10\pi r - 270 r^{-2} = 0$

$10 r^{-2}(\pi r^3 - 27) = 0$

$\pi r^3 - 27 = 0$

$\pi r^3 = 27$

$r^3 = 27/\pi$ OR $\sqrt[3]{\frac{27}{\pi}}$

$r = 3/\sqrt[3]{\pi}$

as decimals

$r = 2.05$

$h = 3.41$

$h = \frac{45}{\pi r^2}$

$h = \frac{45}{\pi\left(\frac{3}{\sqrt[3]{\pi}}\right)^2}$

$h = \frac{45}{\pi \frac{9}{(\pi^{1/3})^2}}$

$= \frac{45}{\pi \cdot \frac{9}{\pi^{2/3}}}$

$= \frac{45}{9\pi^{1/3}}$

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12.  Constraint

$V = 1\,L$

$\pi r^2 h = 1000$

$h = \frac{1000}{\pi r^2}$

$SA = 2\pi r^2 + 2\pi r h$

$SA = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$SA = 2\pi r^2 + \frac{2000}{r}$

$SA = 2\pi r^2 + 2000 r^{-1}$

$SA' = 4\pi r - 2000 r^{-2}$

$4r^{-2}(\pi r^3 - 500) = 0$

$\pi r^3 - 500 = 0$

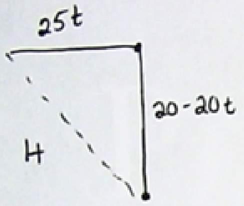
$\pi r^3 = 500$

$r^3 = \frac{500}{\pi}$

$r = \sqrt[3]{\frac{500}{\pi}}$

decimal $r \cong 12.62c$

17.



$$H^2 = (25t)^2 + (20-20t)^2$$

$$H^2 = 625t^2 + 400 - 800t + 400t^2$$

$$H^2 = 1025t^2 - 800t + 400$$

$$H = \sqrt{1025t^2 - 800t + 400}$$

$$H' = \frac{1}{2}(1025t^2 - 800t + 400)^{-1/2}(2050t - 800)$$

undefined
-exp

$$2050t - 800 = 0$$

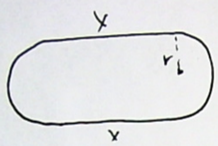
$$2050t = 800$$

$$t = \frac{800}{2050}$$

$$t = \frac{16}{41} = 0.39$$

hours

19.



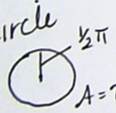
$P = 1\text{ km}$

$2\pi r + 2x = 1$

$2x = 1 - 2\pi r$

$x = \frac{1}{2} - \pi r$

circle



$A = \pi\left(\frac{1}{2\pi}\right)^2$

$= 15.5\text{cm}^2$

Largest track is a circle

Max Area:

$\text{Area} = \pi r^2 + x(2r)$

$A = \pi r^2 + 2xr$

$A = \pi r^2 + 2\left(\frac{1}{2} - \pi r\right) r$

$A = \pi r^2 + 2r\left(\frac{1}{2} - \pi r\right)$

$A = \pi r^2 + r - 2\pi r^2$

$A = r - \pi r^2$

$A' = 1 - 2\pi r$

$1 - 2\pi r = 0$

$-2\pi r = 1$

$r = \frac{1}{2\pi}\text{ km}$

$x = \frac{1}{2} - \pi r$

$= \frac{1}{2} - \pi\left(\frac{1}{2\pi}\right)$

$= 0$