



Laws of Exponents

Review From Gr. 9



Product of powers law: $a^m \cdot a^n = a^{m+n}$

$$2^3 \cdot 2^5 = 2^8$$

Quotient of powers law: $\frac{a^m}{a^n} = a^{m-n}$

$$\frac{3^5}{3^2} = 3^{5-2} = 3^3$$

Power of a power law: $(a^m)^n = a^{mn}$

$$\downarrow (4^2)^3 = 4^{2 \times 3} = 4^6$$

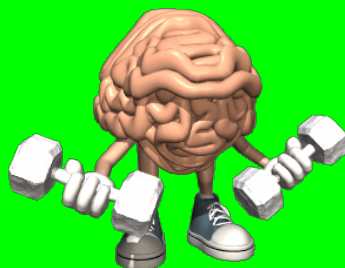
Write as a single power.

a) $3^2 \cdot 3^5 = 3^{2+5} = 3^7$

b) $(4^2)^5 = 4^{10}$

c) $(-5)^{10} \div (-5)^8 = (-5)^{10-8} = (-5)^2 = 25$





Warm Up

Name: _____

Period: _____

Simplify then evaluate

$$1) (2^4)^3$$

$$= 2^{4 \times 3}$$

$$= 2^{12} = 4096$$

$$2) [(-5)^2 \times 2]^3$$

$$(-5)^{2 \times 3} \times 2^3$$

$$(-5)^6 \times 2^3$$

$$15625 \times 8$$

$$125000$$

$$3) [(-1)^{11}]^3$$

$$(-1)^{11 \times 3}$$

$$(-1)^{33}$$

$$-1$$

Write each expression as a product or quotient of powers. Then evaluate.

$$1) [(-3) \times (5)]^2$$

$$(-3)^2 \times 5^2$$

$$9 \times 25$$

$$225$$

$$2) \left(\frac{6}{5}\right)^4$$

$$\frac{6^4}{5^4}$$

$$= \frac{1296}{625}$$

Math 10: Numbers, Functions & Relations

Name _____

😊 Laws of Exponents Review

Date _____

Simplify. Your answer should contain only positive exponents.

$$1) \left(\frac{2 \cdot 2^2}{2}\right)^3 = \left(2^{3-1}\right)^3 = (2^2)^3 = 2^6 = 64$$

$$2) \left(\frac{2^4}{2^3 \cdot 2^3}\right)^4 = \left(\frac{2^4}{2^6}\right)^4 = (2^{-2})^4 = 2^{-8}$$

$$3) \frac{2^2}{4^2} = \frac{2^2}{(2^2)^2} = \frac{2^2}{2^4} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

Handwritten notes: A red arrow points from the denominator 4^2 to $(2^2)^2$. Another red arrow points from 2^{-2} to $\frac{1}{2^2}$ with the note "move to bottom".

$$4) \frac{(2^3 \cdot 2^4)^4}{2} = \frac{(2^7)^4}{2^1} = (2^6)^4 = 2^{24} = 16777216$$

Handwritten notes: A red arrow points from 2^{-8} to $\frac{1}{2^8}$. Another red arrow points from 2^{-2} to $\frac{1}{2^2}$.

4.4 Fractional Exponents and Radicals

LESSON FOCUS

Relate rational exponents and radicals.

Make Connections

Coffee, tea, and hot chocolate contain caffeine. The expression $100(0.87)^{\frac{1}{2}}$ represents the percent of caffeine left in your body $\frac{1}{2}$ h after you drink a caffeine beverage.

Given that $0.87^1 = 0.87$ and $0.87^0 = 1$, how can you estimate a value for $0.87^{\frac{1}{2}}$?



★ Use a calculator to complete the table.

| Column 1 | Column 2 | Column 3 |
|----------|------------------------|----------|
| x | $x^{\frac{1}{2}}$ | |
| 1 | $1^{\frac{1}{2}} = 1$ | |
| 4 | $4^{\frac{1}{2}} = 2$ | |
| 9 | $9^{\frac{1}{2}} = 3$ | |
| 16 | $16^{\frac{1}{2}} = 4$ | |
| 25 | $25^{\frac{1}{2}} = 5$ | |
| | | |
| | | |
| | | |

a) What do you notice about the numbers in the first column?

$$a^{b/c} \\ 4^{1/2} = (1^{abc} 2)$$

b) Compare the numbers in the first and second columns. What conclusions can you make?

c) What do you think the exponent $\frac{1}{2}$ means?

$$\text{Exponent } \frac{1}{2} = \sqrt{\quad}$$

★ Use a calculator to complete the table.

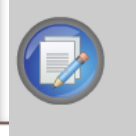
| Column 1 | Column 2 | Column 3 |
|----------|---------------------------------------|----------|
| x | $x^{\frac{1}{3}}$ | |
| 1 | $1^{\frac{1}{3}} = 1$ | |
| 8 | $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$ | |
| 27 | $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$ | |
| 64 | $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$ | |
| 125 | | |
| | | |
| | | |
| | | |

same as $\sqrt[3]{\quad}$

a) What do you notice about the numbers in the first column?

b) Compare the numbers in the first and second columns. What conclusions can you make?

c) What do you think the exponent $\frac{1}{3}$ means?



★ What do you think $a^{\frac{1}{4}}$ and $a^{\frac{1}{5}}$ mean?

$$a^{\frac{1}{4}} \text{ is same as } \sqrt[4]{a}$$

$$a^{\frac{1}{5}} \text{ is } \sqrt[5]{a}$$

What does $a^{\frac{1}{n}}$ mean? Explain your reasoning

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Find n^{th}
root of
"a"

Rational Exponents and Radicals ☆

Let's examine radicals...

$$\sqrt{5} \times \sqrt{5} = \sqrt{5^2} = 5$$

How would this play out with exponent laws?

$$5^? \times 5^? = 5^1$$

$$5^0 \times 5^1$$

$$5^{\frac{1}{2}} \times 5^{\frac{1}{2}} = 5^1$$

*
 RULE: $\sqrt{x} = x^{\frac{1}{2}}$

What about other rational exponents and radicals?

$$8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} =$$

$$\text{Rule: } \sqrt[3]{x} = x^{\frac{1}{3}}$$

In general... $(\sqrt[n]{x})^m$ or $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Rational Exponents

- To evaluate exponents that are rational (fractions), the denominator of the fraction indicates which root to take and the numerator indicates which power the entire base is to be raised.

Example

$$16^{\frac{1}{4}}$$

$$125^{\frac{1}{3}}$$

$$125^{\frac{2}{3}}$$

**Example 1**Evaluating Powers of the Form $a^{\frac{1}{n}} = \sqrt[n]{a}$

Evaluate each power without using a calculator.

a) $27^{\frac{1}{3}}$

$$\sqrt[3]{27}$$

$$= 3$$

b) $0.49^{\frac{1}{2}}$

$$= \sqrt{0.49}$$

$$= \frac{\sqrt{49}}{\sqrt{100}}$$

$$= \frac{7}{10}$$

$$= 0.70$$

c) $(-64)^{\frac{1}{3}}$

$$\sqrt[3]{-64}$$

$$= -4$$

d) $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

$$= \frac{\sqrt{4}}{\sqrt{9}}$$

$$= \frac{2}{3}$$

$$= 0.\bar{6}$$



CHECK YOUR UNDERSTANDING



✕

A fraction can be written as a terminating or repeating decimal, so we can interpret powers with decimal exponents; for example, $0.2 = \frac{1}{5}$, so $32^{0.2} = 32^{\frac{1}{5}}$.

We can evaluate $32^{\frac{1}{5}}$ and $32^{0.2}$ on a calculator to show that both expressions have the same value.

$$32^{(1/5)}$$

2

$$32^{0.2}$$

2



Examples: Express each exponential in radical form, then evaluate.



$$1. 8^{\frac{2}{3}} = (8^{\frac{1}{3}})^2$$

$$= \left[\sqrt[3]{8} \right]^2$$

$$= (2)^2$$

$$= 4$$

$$2. -125^{\frac{1}{3}} =$$

$$\sqrt[3]{-125}$$

$$= -5$$

$$3. 32^{\frac{7}{5}} = \left(\sqrt[5]{32} \right)^7$$

$$= (2)^7$$

$$= 128$$

$$4. \frac{3}{9^{\frac{3}{2}}} = \frac{3}{(\sqrt{9})^3}$$

$$= \frac{3}{(3)^3}$$

$$= \frac{3}{27} \text{ Reduce}$$

$$= \frac{1}{9}$$



$$\left(\sqrt[n]{x}\right)^n = x^{\frac{n}{n}}$$

Express as a exponent:

a) $\sqrt[5]{32}$
 denomin
 $32^{\frac{1}{5}}$

b) $\sqrt[3]{-64}$
 $(-64)^{\frac{1}{3}}$

c) $(\sqrt{144})^3$
 $144^{\frac{3}{2}}$

Express as a Radical:

a) $8^{\frac{5}{3}}$
 $= \left(\sqrt[3]{8}\right)^5$
 \downarrow
 $(2)^5$
 32

b) $49^{\frac{3}{2}}$
 $(\sqrt{49})^3$
 \downarrow
 $(7)^3$
 343

c) $(-125)^{\frac{2}{3}}$
 $(\sqrt[3]{-125})^2$
 \downarrow
 $(-5)^2$
 25

~~These~~ ^{These} examples illustrate that the numerator of a fractional exponent represents a power and the denominator represents a root. The root and power can be evaluated in any order.

?
COPY

$$x^{\frac{m}{n}} = \sqrt[n]{x^m}$$

Power (points to m in the numerator)

Root (points to n in the denominator)

☆

Example 2 Rewriting Powers in Radical and Exponent Form

a) Write $40^{\frac{2}{3}}$ in radical form in 2 ways.

a) Use $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ or $\sqrt[n]{a^m}$.

$$40^{\frac{2}{3}} = (\sqrt[3]{40})^2 \text{ or } \sqrt[3]{40^2}$$

b) Write $\sqrt{3^5}$ and $(\sqrt[3]{25})^2$ in exponent form.



CHECK YOUR UNDERSTANDING

$$(\sqrt[3]{25})^2 = 25^{\frac{2}{3}}$$

Example 3**Evaluating Powers with Rational Exponents and Rational Bases**

Evaluate.

a) $0.04^{\frac{3}{2}}$ b) $27^{\frac{4}{3}}$ c) $(-32)^{0.4}$ d) $1.8^{1.4}$

SOLUTION

$$\begin{aligned} \text{a) } 0.04^{\frac{3}{2}} &= \left(0.04^{\frac{1}{2}}\right)^3 \\ &= \left(\sqrt{0.04}\right)^3 \\ &= 0.2^3 \\ &= 0.008 \end{aligned}$$

$$\begin{aligned} \text{b) } 27^{\frac{4}{3}} &= \left(27^{\frac{1}{3}}\right)^4 \\ &= \left(\sqrt[3]{27}\right)^4 \\ &= 3^4 \\ &= 81 \end{aligned}$$

(Solution continues.)



Example 3**Evaluating Powers with Rational Exponents and Rational Bases**

c) The exponent $0.4 = \frac{4}{10}$ or $\frac{2}{5}$

$$\begin{aligned}\text{So, } (-32)^{0.4} &= (-32)^{\frac{2}{5}} \\ &= \left[(-32)^{\frac{1}{5}}\right]^2 \\ &= (\sqrt[5]{-32})^2 \\ &= (-2)^2 \\ &= 4\end{aligned}$$

d) $1.8^{1.4}$

Use a calculator.



1.8^1.4
2.277096874

$$1.8^{1.4} = 2.2770\dots$$



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Homework

Page 227-228

4.4 Fractional Exponents and Radicals

Exercises

A

3 4 5 6 7

B

8 9 10 11 12 13 14 15

16 17 18 19 20 21

C

22

pg 227

#3 a d f

#4 a c d

#5 a b c

#6 a c

#8 a b c

#9

#12 a b f

#15

$$5a) \quad 36^{\frac{1}{3}} = \sqrt[3]{36}$$

b)

4 a)

$$100^{0.5} \leftarrow \text{fraction}$$

$$100^{\frac{5}{10}} \leftarrow \text{Reduce}$$

$$100^{\frac{1}{2}}$$

$$\sqrt{100}$$
$$\sqrt{\quad}$$
$$10$$

d) $1024^{0.2}$

$$1024^{\frac{2}{10}}$$

$$1024^{\frac{1}{5}}$$

$$\sqrt[5]{1024}$$

$$= 4$$

Page 218_219 Solutions.notebook