



Review :

$$x^{\frac{y}{z}} = \sqrt[z]{x^y}$$

a)  $\sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{3}{5}$

b)  $\sqrt[5]{7776} = 6$

c)  $\sqrt[3]{0.125} = \frac{\sqrt[3]{125}}{\sqrt[3]{1000}} = \frac{5}{10} = \frac{1}{2} = 0.5$

d)  $\sqrt{2.25} = \frac{\sqrt{225}}{\sqrt{100}} = \frac{15}{10} = \frac{3}{2} = 1.5$

2. Order the following radicals from least to greatest

$\sqrt{40}$ ,  $\sqrt{98}$ ,  $\sqrt[3]{98}$ ,  $\sqrt{75}$ ,  $\sqrt[3]{300}$

$\sqrt{36} = 6$   
 $\sqrt{49} = 7$   
 $\approx 6.3$

$\sqrt{81} = 9$   
 $\sqrt{100} = 10$   
 $\approx 9.9$

$\sqrt[3]{64} = 4$   
 $\sqrt[3]{125} = 5$   
 $\approx 4.8$

$\sqrt[3]{512} = 8$   
 $\sqrt[3]{583} \approx 8.6$

$\sqrt[3]{216} = 6$   
 $\sqrt[3]{343} \approx 7$   
 $\approx 6.7$

$\sqrt[3]{98}$ ,  $\sqrt{40}$ ,  $\sqrt[3]{300}$ ,  $\sqrt{75}$ ,  $\sqrt{98}$

# Homework

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Questions 1-6

Page 206 (1-6) questions

$\sqrt[n]{\text{radicand}}$   
 ↑  
 product

a)  $\sqrt{16}$ ,  $\sqrt[3]{27}$ ,  $\sqrt[4]{81}$ ,  $\sqrt[5]{243}$

b) index: 2, radicand: 16; index: 3, radicand: 27; index: 4, radicand: 81; index: 5, radicand: 243

what # do I multiply by itself 2 times to get 16  
 by itself 2 times to get 16

c) You are taking the "n" root of the radicand.  
 what number multiplied by itself n times will give you the radicand

2a)  $\sqrt{36} = \sqrt{6^2} = 6$   
 b)  $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$   
 c)  $\sqrt[4]{10000} = \sqrt[4]{10^4} = 10$   
 d)  $\sqrt[5]{-32} = \sqrt[5]{-2^5} = -2$   
 e)  $\sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{\sqrt[3]{3^3}}{\sqrt[3]{5^3}} = \frac{3}{5}$   
 f)  $\sqrt{2.25} \Rightarrow \sqrt{225} = 15$   
 $\frac{\sqrt{225}}{\sqrt{100}} = \frac{15}{10} \approx 1.5$   
 g)  $\sqrt[3]{0.125} = \sqrt[3]{125}$   
 $\frac{\sqrt{125}}{\sqrt{1000}} = \frac{5}{10} = \frac{1}{2} = 0.5$   
 $\frac{5}{10} = 0.5 \therefore \sqrt[3]{0.125} = 0.5$   
 h)  $\sqrt[4]{625} = \sqrt[4]{5^4} = 5$

3)  $\sqrt{8}$  closer

$$\begin{array}{cc} \sqrt{4} & \sqrt{9} \\ \downarrow & \downarrow \\ 2 & 3 \end{array}$$

$\approx 2.8$

b)  $\sqrt[3]{9}$

$$\begin{array}{cc} \sqrt[3]{8} & \sqrt[3]{27} \\ \downarrow & \downarrow \\ 2 & 3 \end{array}$$

$\approx 2.1$

c)  $\sqrt[4]{10}$  closer

$$\begin{array}{cc} \sqrt[4]{1} & \sqrt[4]{16} \\ \downarrow & \downarrow \\ 1 & 2 \end{array}$$

$\approx 1.7$

d)  $\sqrt{13}$  closer

$$\begin{array}{cc} \sqrt{9} & \sqrt{16} \\ \downarrow & \downarrow \\ 3 & 4 \end{array}$$

$\approx 3.5$

e)  $\sqrt[3]{15}$  closer

$$\begin{array}{cc} \sqrt[3]{8} & \sqrt[3]{27} \\ \downarrow & \downarrow \\ 2 & 3 \end{array}$$

$\approx 2.4$

f)  $\sqrt[4]{17}$

$$\begin{array}{cc} \sqrt[4]{16} & \sqrt[4]{81} \\ \downarrow & \downarrow \\ 2 & 3 \end{array}$$

$\approx 2.1$

g)  $\sqrt{19}$

$$\begin{array}{cc} \sqrt{16} & \sqrt{25} \\ 4 & 5 \end{array}$$

$\approx 4.3$

h)  $\sqrt[3]{20}$

$$\begin{array}{cc} \sqrt[3]{8} & \sqrt[3]{27} \\ \downarrow & \downarrow \\ 2 & 3 \end{array}$$

$\approx 2.7$

4)  $\sqrt{-4} = \text{DNE}$

$a \times a = (-)$   
Must be the same #'s

only way you can multiply two #'s to get a neg  
 $(-)(+)$   
Not the same #'s

b) any even index

$\sqrt{-16} = \text{DNE}$

c) i) any odd index  
ii) any even index

$\sqrt[\text{odd}]{(-)} = \text{work } (-)$   
 $\sqrt[\text{even}]{(-)} = \text{DNE}$

6 i) square roots

- a)  $\sqrt{4} = 2$
- b)  $\sqrt{9} = 3$
- c)  $\sqrt{16} = 4$
- d)  $\sqrt{100} = 10$
- e)  $\sqrt{0.81} = 0.9$
- f)  $\sqrt{0.04} = 0.2$

ii) cube roots

- a)  $\sqrt[3]{8} = 2$
- b)  $\sqrt[3]{27} = 3$
- c)  $\sqrt[3]{81} = 4$
- d)  $\sqrt[3]{1000} = 10$
- e)  $\sqrt[3]{0.729} = 0.9$
- f)  $\sqrt[3]{0.008} = 0.2$

iii) Fourth root

- a)  $\sqrt[4]{16} = 2$  2x2x2x2
- b)  $\sqrt[4]{81} = 3$  3x3x3x3
- c)  $\sqrt[4]{256} = 4$  4x4x4x4
- d)  $\sqrt[4]{10000} = 10$
- e)  $\sqrt[4]{0.6561} = 0.9$
- f)  $\sqrt[4]{0.0016} = 0.2$

6) a)  $\sqrt[4]{81} = 3$   
 $\sqrt{81} = 9$   
 $\sqrt{36} = 6$

c)  $\sqrt{49} = 7$

Ex 2)  $\frac{\sqrt{36}}{\sqrt{25}}$   
 $= \frac{6}{5}$

odd      neg

b)  $\sqrt[3]{-125} = -5$

d)  $\sqrt[3]{18}$   
 $\sqrt{45}$



# List

Evaluate each radical. Justify your answer

a)  $\sqrt{49} = 7$   
 $= \sqrt{7^2}$  ← rewrite as perfect square  
 $= 7$

b)  $\sqrt[4]{1296}$  ← rewrite as power of 4  
 $= \sqrt[4]{6^4}$   
 $= 6$

c)  $\sqrt[3]{729}$   
 $=$   
 $=$

Estimate to one decimal

$\sqrt[3]{9}$

$\sqrt[3]{8} \approx 2$

$\sqrt[3]{27} \approx 3$

closer = 3

$\approx 2.1$

b)  $\sqrt[5]{1562}$

Remember

**Rational numbers** are numbers that can be written as a fraction or is a decimal that repeats or terminates. Ex)  $\sqrt[4]{\frac{1296}{10000}}$  Ex)  $\sqrt[3]{\frac{8}{27}}$

$$= \frac{\sqrt[4]{1296}}{\sqrt[4]{10000}} = \frac{6}{10} = 0.60$$

**Irrational numbers** are numbers that cannot be written as a fraction and its decimal neither terminates or repeats.  $\sqrt{28}$

# Radicals

**Mixed Radical** - has a coefficient in front of the radical sign.

ex:  $3\sqrt{5}$  OR  $\frac{2\sqrt{26}}{3}$  OR  $-3\sqrt[3]{3}$  .

**Entire Radical** - has a coefficient of 1 or -1 in front of the radical sign. Everything is entirely under the radical sign

ex:  $\sqrt{12}$  OR  $-\sqrt{45}$

$\sqrt[3]{216}$  OR  $-1\left(\sqrt[4]{72}\right)$

# Reducing Radicals

## Multiplication Property of Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b},$$

where  $n$  is a natural number, and  $a$  and  $b$  are real numbers

Same works if we change the "index":

$$\begin{aligned}\sqrt[3]{8 \cdot 27} &= \sqrt[3]{8} \cdot \sqrt[3]{27} \\ &= 2 \cdot 3 \\ &= 6\end{aligned}$$

or

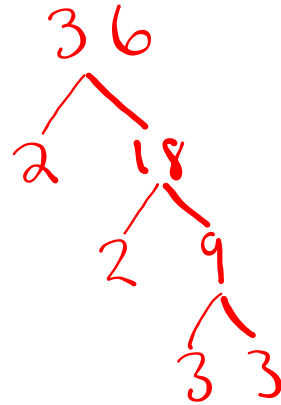
$$\begin{aligned}\sqrt[3]{8 \cdot 27} &= \sqrt[3]{216} \\ \sqrt[2]{8} \times \sqrt[3]{27} &= 6 \\ 2 \times 3 &= 6\end{aligned}$$



$$\begin{aligned} & \sqrt{36} \\ &= \sqrt{9 \times 4} \\ &= \sqrt{9} \times \sqrt{4} \\ & \quad 3 \times 2 \\ & \quad 6 \end{aligned}$$


← product of perfect squares

$$\begin{aligned} & \sqrt{36} \\ &= \sqrt{(2 \times 2) \times (3 \times 3)} \\ &= \sqrt{2 \times 2} \times \sqrt{3 \times 3} \\ &= \sqrt{4} \times 3 \\ &= 2 \times 3 \\ &= 6 \end{aligned}$$



# Reducing Radicals

To reduce a radical, you must find the largest "n<sup>th</sup>" number that will divide into the radicand

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$


Greatest perfect n<sup>th</sup>

# Entire to Mixed

## Reducing Radicals

To reduce  $\sqrt{125}$   
 you must find the **largest** square number  
 that will divide into 125 evenly!

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

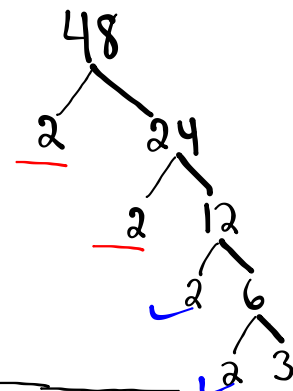
$\sqrt{125}$   
 $\sqrt{(25) \times (5)}$   
 Separate  
 $\sqrt{25} \times \sqrt{5}$   
 5  $\times$   $\sqrt{5}$  can't do leave it

Greatest perfect  $n^{\text{th}}$

Must Know list of perfect nth

$$\sqrt{48}$$

go with  $\sqrt{16} \times \sqrt{3}$   
 $4 \sqrt{3}$



$$\sqrt{(2 \times 2) \times (2 \times 2) \times 3}$$

$$\sqrt{2 \times 2} \times \sqrt{2 \times 2} \times \sqrt{3}$$

$$2 \times 2 \times \sqrt{3}$$

$$4 \sqrt{3}$$

# Entire to Mixed

continued

## Reducing Radicals

Prime Factorization

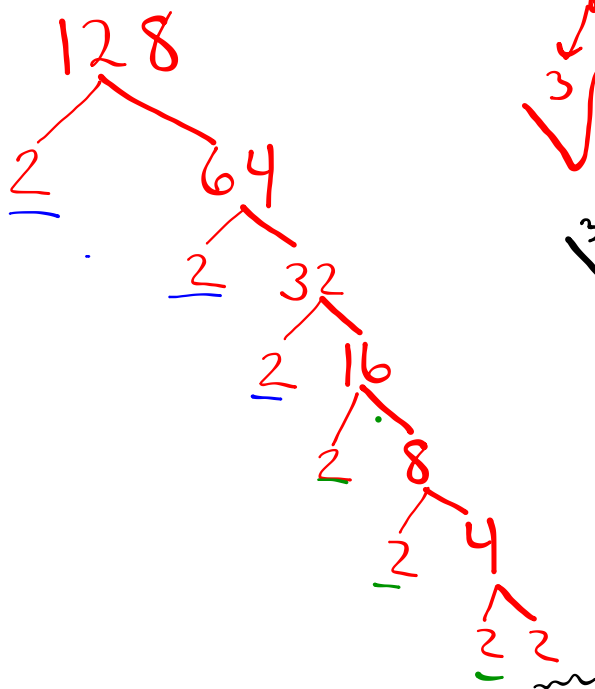
To reduce  $\sqrt{125}$   
 you must find the **largest** square number  
 that will divide into 125 evenly!

$$\begin{aligned}
 \text{a) } & \sqrt[3]{128} \\
 &= \sqrt[3]{64 \times 2} \\
 &= \sqrt[3]{64} \times \sqrt[3]{2} \\
 & \quad 4 \sqrt[3]{2}
 \end{aligned}$$

Entire to Mixed

$$\begin{aligned}
 \text{b) } & \sqrt{54} \\
 &= \sqrt{9 \times 6} \\
 &= \sqrt{9} \times \sqrt{6} \\
 & \quad 3 \sqrt{6}
 \end{aligned}$$

Prime Factorization



group in 3's

$$\begin{aligned}
 & \sqrt[3]{(2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 2} \\
 & \sqrt[3]{(2 \times 2 \times 2)} \times \sqrt[3]{2 \times 2 \times 2} \times \sqrt[3]{2} \\
 & 2 \times 2 \times \sqrt[3]{2} \\
 & \quad 4 \sqrt[3]{2}
 \end{aligned}$$



We can also use prime factorization to simplify a radical.

### Example 1 Simplifying Radicals Using Prime Factorization

Simplify each radical.

a)  $\sqrt{80}$       b)  $\sqrt[3]{144}$       c)  $\sqrt[4]{162}$



**SOLUTION**

Same questions using largest perfect nth factors

$$\begin{aligned} \text{a) } \sqrt{80} &= \sqrt{16 \cdot 5} = \sqrt{16} \times \sqrt{5} \\ &= 4\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{b) } \sqrt[3]{144} &= \sqrt[3]{8 \cdot 18} = \sqrt[3]{8} \cdot \sqrt[3]{18} \\ &= 2\sqrt[3]{18} \end{aligned}$$

4.3 Mixed and Entire Radicals



CHECK YOUR UNDERSTANDING

$$\begin{aligned} \text{c) } \sqrt[4]{162} &= \sqrt[4]{81 \cdot 2} = \sqrt[4]{81} \sqrt[4]{2} \\ &= 3\sqrt[4]{2} \end{aligned}$$

# Homework

Entire  $\rightarrow$  Mixed

$\sqrt{\#}$   $\rightarrow$  coeff  $\sqrt{\text{radical}}$   
 $\sqrt{12}$   $\rightarrow \sqrt{4 \cdot 3} \Rightarrow 2\sqrt{3}$

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#4, #7a, #8a, #9, #10, #11

a)  $c^2 = a^2 + b^2$

$$c = \sqrt{a^2 + b^2}$$

$$\sqrt{3^2 + 6^2}$$

$$\sqrt{9 + 36}$$

$$\sqrt{45}$$

not  
multip  
it  
is  
add

4a)  $\sqrt{8}$

$$\sqrt{4 \times 2}$$

$$\sqrt{4} \times \sqrt{2}$$

$$2\sqrt{2}$$

b)  $\sqrt{45} = \sqrt{9 \cdot 5}$

$$= \sqrt{9} \sqrt{5}$$

$$3\sqrt{5}$$