Practice test 2013.doc

1. $t_8 = \frac{13}{3}$ $t_2 = \frac{1}{3}$

(a)

① $\frac{13}{3} = a + 7d$

② $\frac{1}{3} = a + 1d$

$\frac{12}{3} = 6d$

$4 = 6d$

$\frac{4}{6} = d$

$d = \frac{2}{3}$

$\frac{1}{3} = a + \frac{4}{6}$

$\frac{1}{3} = a + \frac{2}{3}$

$\frac{1}{3} - \frac{2}{3} = a$

$-\frac{1}{3} = a$

(b) $t_n = a + (n-1)d$

$t_n = -\frac{1}{3} + (n-1)\frac{2}{3}$

$t_n = -\frac{1}{3} + \frac{2}{3}n - \frac{2}{3}$

$t_n = \frac{2}{3}n - \frac{3}{3}$

OR

$t_n = \frac{2}{3}n - 1$

(c) (i) $t_{12} = a + (n-1)d$

$= -\frac{1}{3} + (12-1)\frac{2}{3}$

$= -\frac{1}{3} + 11(\frac{2}{3})$

$= -\frac{1}{3} + \frac{22}{3}$

$= \frac{21}{3} = 7$

(ii) $t_{26} = -\frac{1}{3} + (26-1)\frac{2}{3}$

$= -\frac{1}{3} + 25(\frac{2}{3})$

$= -\frac{1}{3} + \frac{50}{3}$

$= \frac{49}{3}$

(d) (i) $t_n = a + (n-1)d$

$9 = -\frac{1}{3} + (n-1)\frac{2}{3}$

$9 = -\frac{1}{3} + \frac{2}{3}n - \frac{2}{3}$

$9 = -\frac{3}{3} + \frac{2}{3}n$

$9 = -1 + \frac{2}{3}n$

$10 = \frac{2}{3}n$

$30 = 2n$

$15 = n$

(ii) $t_n = a + (n-1)d$

$\frac{97}{3} = -\frac{1}{3} + (n-1)\frac{2}{3}$

$\frac{97}{3} = -\frac{1}{3} + \frac{2}{3}n - \frac{2}{3}$

$\frac{97}{3} = -\frac{3}{3} + \frac{2}{3}n$

$\frac{97}{3} + \frac{3}{3} = \frac{2}{3}n$

$\frac{100}{3} = \frac{2}{3}n$

$\frac{300}{3} = 2n$

$100 = 2n$

$50 = n$

(e) $S_{200} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{200}{2}[2(-\frac{1}{3}) + (200-1)\frac{2}{3}]$

$100[-\frac{2}{3} + 199(\frac{2}{3})]$

$100[-\frac{2}{3} + \frac{398}{3}]$

$100[\frac{396}{3}]$

$100[132]$

$13200$

2.(a) $t_n = ar^{n-1}$

$162 = ar^{5-1}$

$162 = ar^4$

$13122 = ar^{9-1}$

$13122 = ar^8$

① $13122 = ar^8$

② $162 = ar^4$

① ÷ ② $81 = r^4$

$\sqrt[4]{81} = r$

$3 = r$

② $162 = a(3)^4$

$162 = a(81)$

$2 = a$

(b) $t_n = ar^{n-1}$

$t_n = 2(3)^{n-1}$

(c) (i) $t_3 = ar^{n-1}$

$= 2(3)^{3-1}$

$= 2(3)^2$

$= 18$

(ii) $t_{10} = 2(3)^{10-1}$

$= 2(3)^9$

$= 39366$

(d) (i) $t_n = ar^{n-1}$

$9565938 = 2(3)^{n-1}$

$\frac{9565938}{2} = 3^{n-1}$

$4782969 = 3^{n-1}$

$\log_3 4782969 = n-1$

$14 = n-1$

$15 = n$

(ii) $1458 = 2(3)^{n-1}$

$729 = 3^{n-1}$

$\log_3 729 = n-1$

$6 = n-1$

$7 = n$

(e) $S_{10} = \frac{a[r^n - 1]}{[r-1]}$

$= \frac{2[3^{10} - 1]}{[3-1]}$

$= 59048$

3. $t_n = 15-4n+2n^2$

$t_1 = 15-4(1)+2(1)^2 = 13$

$t_2 = 15-14(2)+2(2)^2 = 15$

$t_3 = 21$

$t_4 = 31$

$t_5 = 45$

4. Need to find *n* first:

(a) $12582912 + 6291456 + 3145728 + 1572864 + \ldots + 3$

geometric

$a = 12582912$

$r = \frac{1}{2}$

$t_n = ar^{n-1}$

$3 = 12582912\left(\frac{1}{2}\right)^{n-1}$

$\frac{3}{12582912} = \left(\frac{1}{2}\right)^{n-1}$

$\log_{(\frac{1}{2})}\left(\frac{3}{12582912}\right) = n-1$

$22 = n-1$

$23 = n$

now find

$S_{23} = \frac{a[r^n - 1]}{[r-1]}$

$= \frac{12582912\left[\left(\frac{1}{2}\right)^{23} - 1\right]}{\left[\frac{1}{2} - 1\right]} = 25165821$

$2187 + 2916 + 3888 + \ldots$

$16382$

$a = 2187$

$r = \frac{4}{3}$

$n = ?$

$t_n = ar^{n-1}$

$16382 = 2187\left(\frac{4}{3}\right)^{n-1}$

$\frac{16382}{2187} = \left(\frac{4}{3}\right)^{n-1}$

$\log_{(\frac{4}{3})}\left(\frac{16382}{2187}\right) = n-1$

4. (b) $24+30+36+42+\ldots+12030$ arithmetic
$a=24$
$d=6$

find n first

$t_n = a+(n-1)d$

$12030 = 24+(n-1)6$

$12030 = 24+6n-6$

$12030 = 18+6n$

$12012 = 6n$

$2002 = n$

Now find

$S_{2002} = \frac{n}{2}[2a+(n-1)d]$

$= \frac{2002}{2}[2(24)+(2002-1)6]$

$= 12066054$

5. (a) $t_{35}=-411$ $t_{20}=-231$

$-411 = a+(35-1)d$ $-231 = a+(20-1)d$

① $-411 = a+34d$ ② $-231 = a+19d$

① $-411 = a+34d$
② $-231 = a+19d$
①-② $-180 = 15d$
$-12 = d$

Sub $d=-12$

$-231 = a+19(-12)$

$-231 = a-228$

$-3 = a$

$S_{300} = \frac{300}{2}[2(-3)+(300-1)(-12)]$

$= 150[-6+299(-12)]$

$= \cancel{-90600}$ $-539100$

5(b) $t_n = ar^{n-1}$ :

$1835008 = ar^{10-1}$ $112 = ar^{3-1}$

$1835008 = ar^9$ $112 = ar^2$

$1835008 = ar^9$
$\div$ $112 = ar^2$
$16384 = r^7$

$\sqrt[7]{16384} = r$

$4 = r$

$112 = a(4)^2$

$112 = 16a$

$\frac{112}{16} = a$

$7 = a$

$S_{14} = \frac{7[4^{14}-1]}{[4-1]} = \boxed{626349395}$

6. $5+15+45+135+$

$$S_n = \frac{a[r^n - 1]}{[r-1]}$$

$$\underset{n\to\infty}{} = \frac{5[3^n - 1]}{3-1}$$

Diverges
(gets bigger)

term 1, $t_2$, $t_3$

7. $5000, 5125, 5253.125, \ldots$

geometric $r = 1.025$

$t_n = ar^{n-1}$

$t_n = 5000(1.025)^{n-1}$

(b) $t_1 = $ yr 2000 $= 5000$

$t_2 = $ yr 2001 $= 5125$

$t_3 = $ yr 2002 $= 5253.125$

$t_4 = $ yr 2003 $= 5384.45\ldots$

(c) 2008 is term 9

$t_9 = 5000(1.025)^{9-1}$

$= \$6092.01$

(d) $t_n = ar^{n-1}$

$11866 = 5000(1.025)^{n-1}$

$2.3732 = (1.025)^{n-1}$

$\log_{1.025} 2.3732 = n-1$

$35 = n-1$

$36 = n$

In the yr 2036

8. $S_8 = -3280 \quad r = -3$

$$-3280 = \frac{a[(-3)^8 - 1]}{[-3-1]}$$

$$-3280 = \frac{a[6560]}{[-4]}$$

$$2 = a$$

$\therefore t_1 = 2$

9. (a) $-3/4$ conv to $-3/4$

(b) $\lim_{x \to \infty} \frac{14x^2 + 29x - 15}{10x^2 - 6x - 4}$

$= \frac{14}{10} = 7/5$ conv to $7/5$

(c) $\frac{0}{6}$ conv to 0

(d) conv to 0

(e) Diverge

(f) conv. to 0

10. (a) $\sum_{K=1}^{122} 4k^3$

$$4\left[\frac{n(n+1)}{2}\right]^2$$

$$4\left[\frac{122(123)}{2}\right]^2$$

$$= 225180036$$

(b) $\sum_{K=1}^{1500} 12$

$$= 12n$$

$$= 12(1500)$$

$$= 18000$$

(c) $\sum_{K=1}^{75}\left(5K^2-10K+2\right)$

$$5\left[\frac{n(n+1)(2n+1)}{6}\right]-10\,\frac{n(n+1)}{2}+2n$$

$$5\left[\frac{75(76)(151)}{6}\right]-10\left[\frac{75(76)}{2}\right]+2(75)$$

$$= 688900$$

(d) $\sum_{K=50}^{150}\left(K^3-3\right)$

$$\sum_{K=1}^{150}\left(K^3-3\right) - \sum_{K=1}^{49}\left(K^3-3\right)$$

$$\left[\left[\frac{n(n+1)}{2}\right]^2-3n\right]_{K=150} - \left(\left[\frac{n(n+1)}{2}\right]^2-3n\right)_{K=49}$$

$$126562050 - 1500478 = \boxed{125061572}$$

## Multiple Choice

1. C
2. $t_n=6n+9$
3. D
4. B
5. B
6. D
7. B
8. C
9. A
10. A
11. no 11 on sheet
12. C
13. D
14. D
15. A

$t_{26}=82000(1.016)^{26-1}$

16) 82000, 82... ($\times 1.016$)

2000, 2001, 2002, ....2025

n=1 n=2 n=3 n=26

# Attachments

Practice test 2013.doc