1.(a) $t_2 = \frac{1}{3}$; $t_8 = \frac{13}{3}$

* Given 2 terms
Set up 2 eq'n to solve for unknowns

$t_n = a + (n-1)d$

$\frac{1}{3} = a + (2-1)d$ $\quad\quad$ $\frac{13}{3} = a + (8-1)d$

$\frac{1}{3} = a + d$ $\quad\quad$ $\frac{13}{3} = a + 7d$

① $\frac{13}{3} = a + 7d$

② $\frac{1}{3} = a + d$

① − ② $\quad \frac{12}{3} = 6d$

$4 = 6d$

$\frac{4}{6} = d$

$= \frac{2}{3}$

$d = \frac{2}{3}$ $\quad\quad$ $\frac{1}{3} = a + \frac{2}{3}$

$\frac{1}{3} - \frac{2}{3} = a$

$-\frac{1}{3} = a$

b) General term; t_n

$t = a + (n-1)d$

$t_n = -\frac{1}{3} + (n-1)\frac{2}{3}$

$t_n = -\frac{1}{3} + \frac{2}{3}n - \frac{2}{3}$

$t_n = \frac{2}{3}n - \frac{3}{3}$

$\boxed{t_n = \frac{2}{3}n - 1}$

(c) $t_{12} = \frac{2}{3}n - 1$

$= \frac{2}{3}(12) - 1$

$= \frac{24}{3} - 1$

$= 8 - 1$

$= 7$

$t_{26} = \frac{2}{3}(26) - 1$

$\frac{52}{3} - 1$

$\frac{52}{3} - \frac{3}{3}$

$\frac{49}{3}$

d) term number is "n"

$t_n = a + (n-1)d$

$9 = -\frac{1}{3} + (n-1)\frac{2}{3}$

$9 = -\frac{1}{3} + \frac{2}{3}n - \frac{2}{3}$

$9 = -\frac{3}{3} + \frac{2}{3}n$

$9 = -1 + \frac{2}{3}n$

$10 = \frac{2}{3}n$

$30 = 2n$

$n = 15$

$t_n = \frac{2}{3}n - 1$

$9 = \frac{2}{3}n - 1$

$10 = \frac{2}{3}n$

$30 = 2n$

$15 = n$

$\frac{97}{3} = \frac{2}{3}n - 1$

$\frac{97}{3} + 1 = \frac{2}{3}n$

$\frac{97}{3} + \frac{3}{3} = \frac{2}{3}n$

$3 \times \frac{100}{3} = \frac{2}{3}n \times 3$

$100 = 2n$

$50 = n$

*n has to be a whole #.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{200} = \frac{200}{2}\left[2\left(-\frac{1}{3}\right) + (200-1)\frac{2}{3}\right]$

$100\left[-\frac{2}{3} + 199\left(\frac{2}{3}\right)\right]$

$100\left[-\frac{2}{3} + \frac{398}{3}\right]$

$100\left[\frac{396}{3}\right]$

$100[132] = 13200$

2. $t_n = ar^{n-1}$

$t_5 = 162$ $\quad$ $t_9 = 13122$

$162 = ar^{5-1}$ $\quad$ $13122 = ar^{9-1}$

$162 = ar^4$ $\quad$ $13122 = ar^8$

① $13122 = \cancel{a}r^8$

② $162 = \cancel{a}r^4$

①÷②

$81 = r^4$

$\sqrt[4]{81} = r$

$3 = r$

sub r = 3

$162 = a(3)^4$

$162 = a(81)$

$\frac{162}{81} = a$

$2 = a$

(b) general term

$t_n = ar^{n-1}$

$\boxed{t_n = 2(3)^{n-1}}$

* cannot multiply a & r

(c) $t_3 = ar^{n-1}$

$= 2(3)^{3-1}$

$= 2(3)^2$

$= 2(9)$

$= 18$

$t_{10} = ar^{n-1}$

$= 2(3)^{10-1}$

$2(3)^9$

$= 2(19683)$

$= 39366$

(d) term # ⇒ n

$t_n = ar^{n-1}$

$\frac{9565938}{2} = \frac{\cancel{2}(3)^{n-1}}{\cancel{2}}$

$4782969 = 3^{n-1}$

$\log_3 4782969 = n-1$

$14 = n-1$

$14+1 = n$

$15 = n$

$\frac{\log(4782969)}{\log(3)}$

$\frac{1458}{2} = \frac{\cancel{2}(3)^{n-1}}{\cancel{2}}$

$729 = 3^{n-1}$

$\log_3 729 = n-1$

$6 = n-1$

$7 = n$

$\frac{\log(729)}{\log(3)}$

$S_n = \frac{a[r^n - 1]}{[r-1]}$

$S_{10} = \frac{2[3^{10} - 1]}{[3-1]}$

$= \frac{2[59048]}{2}$

$= 59048$

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1. $t_8 = \frac{13}{3}$ $t_2 = \frac{1}{3}$

(a)

① $\frac{13}{3} = a + 7d$

② $\frac{1}{3} = a + 1d$

$\frac{12}{3} = 6d$

$4 = 6d$

$\frac{4}{6} = d$

$d = \frac{2}{3}$

$\frac{1}{3} = a + \frac{4}{6}$

$\frac{1}{3} = a + \frac{2}{3}$

$\frac{1}{3} - \frac{2}{3} = a$

$-\frac{1}{3} = a$

(b) $t_n = a + (n-1)d$

$t_n = -\frac{1}{3} + (n-1)\frac{2}{3}$

$t_n = -\frac{1}{3} + \frac{2}{3}n - \frac{2}{3}$

$t_n = \frac{2}{3}n - \frac{3}{3}$

OR

$t_n = \frac{2}{3}n - 1$

(c) (i) $t_{12} = a + (n-1)d$

$= -\frac{1}{3} + (12-1)\frac{2}{3}$

$= -\frac{1}{3} + 11\left(\frac{2}{3}\right)$

$= -\frac{1}{3} + \frac{22}{3}$

$= \frac{21}{3} = 7$

(ii) $t_{26} = -\frac{1}{3} + (26-1)\frac{2}{3}$

$= -\frac{1}{3} + 25\left(\frac{2}{3}\right)$

$= -\frac{1}{3} + \frac{50}{3}$

$= \frac{49}{3}$

(d) (i) $t_n = a + (n-1)d$

$9 = -\frac{1}{3} + (n-1)\frac{2}{3}$

$9 = -\frac{1}{3} + \frac{2}{3}n - \frac{2}{3}$

$9 = -\frac{3}{3} + \frac{2}{3}n$

$9 = -1 + \frac{2}{3}n$

$10 = \frac{2}{3}n$

$30 = 2n$

$15 = n$

(ii) $t_n = a + (n-1)d$

$\frac{97}{3} = -\frac{1}{3} + (n-1)\frac{2}{3}$

$\frac{97}{3} = -\frac{1}{3} + \frac{2}{3}n - \frac{2}{3}$

$\frac{97}{3} = -\frac{3}{3} + \frac{2}{3}n$

$\frac{97}{3} + \frac{3}{3} = \frac{2}{3}n$

$\frac{100}{3} = \frac{2}{3}n$

$\frac{300}{3} = 2n$

$100 = 2n$

$50 = n$

(e) $S_{200} = \frac{n}{2}[2a + (n-1)d]$

$= \frac{200}{2}\left[2\left(-\frac{1}{3}\right) + (200-1)\frac{2}{3}\right]$

$100\left[-\frac{2}{3} + 199\left(\frac{2}{3}\right)\right]$

$100\left[-\frac{2}{3} + \frac{398}{3}\right]$

$100\left[\frac{396}{3}\right]$

$100[132]$

13200

2.(a) $t_n = ar^{n-1}$

$162 = ar^{5-1}$ $\quad$ $13122 = ar^{9-1}$

$162 = ar^4$ $\quad$ $13122 = ar^8$

① $13122 = ar^8$
② $162 = ar^4$

① ÷ ② $\quad 81 = r^4$

$\sqrt[4]{81} = r$

$3 = r$

② $162 = a(3)^4$

$162 = a(81)$

$2 = a$

(b) $t_n = ar^{n-1}$

$t_n = 2(3)^{n-1}$

(c) (i) $t_3 = ar^{n-1}$

$= 2(3)^{3-1}$

$= 2(3)^2$

$= 18$

(ii) $t_{10} = 2(3)^{10-1}$

$= 2(3)^9$

$= 39366$

(d) (i) $t_n = ar^{n-1}$

$9565938 = 2(3)^{n-1}$

$\frac{9565938}{2} = 3^{n-1}$

$4782969 = 3^{n-1}$

$\log_3 4782969 = n-1$

$14 = n-1$

$15 = n$

(ii) $1458 = 2(3)^{n-1}$

$729 = 3^{n-1}$

$\log_3 729 = n-1$

$6 = n-1$

$7 = n$

(e) $S_{10} = \frac{a[r^n - 1]}{[r-1]}$

$= \frac{2[3^{10} - 1]}{[3-1]}$

$= 59048$

3. $t_n = 15 - 4n + 2n^2$

$t_1 = 15 - 4(1) + 2(1)^2 = 13$

$t_2 = 15 - 14(2) + 2(2)^2 = 15$

$t_3 = 21$

$t_4 = 31$

$t_5 = 45$

4. Need to find n first:

(a) $12582912 + 6291456 + 3145728 + 1572864 + \ldots + 3$

geometric

$a = 12582912$

$r = 1/2$

$t_n = ar^{n-1}$

$3 = 12582912\left(\frac{1}{2}\right)^{n-1}$

$\frac{3}{12582912} = \left(\frac{1}{2}\right)^{n-1}$

$\log_{(1/2)}\left(\frac{3}{12582912}\right) = n-1$

$22 = n-1$

$23 = n$

now find

$S_{23} = \frac{a[r^n - 1]}{[r-1]}$

$= \frac{12582912\left[\left(\frac{1}{2}\right)^{23} - 1\right]}{[1/2 - 1]} = 25165821$

$\times 2$

$3 + \ldots\ldots\ldots + 12582912$

$t_n = ar^{n-1}$

$\frac{12582912}{3} = \frac{3(2)^{n-1}}{3}$

$4194304 = 2^{n-1}$

$\log_2 4194304 = n-1$

$2^{\square} = 2^{n-1}$

$3(4/3)$

4. (b) $24+30+36+42+\ldots+12030$ arithmetic
$a=24$
$d=6$

find n first

$t_n = a+(n-1)d$
$12030 = 24+(n-1)6$
$12030 = 24+6n-6$
$12030 = 18+6n$
$12012 = 6n$
$2002 = n$

Now find

$S_{2002} = \frac{n}{2}[2a+(n-1)d]$
$= \frac{2002}{2}[2(24)+(2002-1)6]$
$= 12066054$

* Remember n has to be a whole #.

5. (a) $t_{35}=-411$ $\quad t_{20}=-231$

$-411 = a+(35-1)d$ $\quad -231 = a+(20-1)d$
① $-411 = a+34d$ $\quad$ ② $-231 = a+19d$

① $-411 = a+34d$
② $-231 = a+19d$
①-② $-180 = 15d$
$-12 = d$

Sub $d=-12$
$-231 = a+19(-12)$
$-231 = a-228$
$-3 = a$

$S_{300} = \frac{300}{2}[2(-3)+(300-1)(-12)]$
$= 150[-6+299(-12)]$
$= -539100$

5(b) $t_n = ar^{n-1}$:

$1835008 = ar^{10-1}$ $\quad 112 = ar^{3-1}$
$1835008 = ar^9$ $\quad 112 = ar^2$

$\div$ $\frac{1835008 = ar^9}{112 = ar^2}$
$16384 = r^7$
$\sqrt[7]{16384} = r$
$4 = r$

$112 = a(4)^2$
$112 = 16a$
$\frac{112}{16} = a$
$7 = a$

$r^2 = 64$ | $r^3 = 729$
$r = \sqrt{64}$ | $r = \sqrt[3]{729}$

$r^7 = 16384$
$r = \sqrt[7]{16384}$
$(16384)^{1/7}$

$S_{14} = \frac{7[4^{14}-1]}{[4-1]} = \boxed{626349395}$

6. $5+15+45+135+$

$$S_n = \frac{a[r^n-1]}{[r-1]}$$

$n \to \infty$ $$= \frac{5[3^n-1]}{3-1}$$

Diverges
(gets bigger)

term 1 t_2 t_3

7. 5000, 5125, 5253.125, ...

geometric $r = 1.025$

$t_n = ar^{n-1}$

$t_n = 5000(1.025)^{n-1}$

(b) $t_1 = $ yr 2000 $= 5000$

$t_2 = $ yr 2001 $= 5125$

$t_3 = $ yr 2002 $= 5253.125$

$t_4 = $ yr 2003 $= 5384.45...$

(c) 2008 is term 9

$t_9 = 5000(1.025)^{9-1}$

$= \$6092.01$

(d) $t_n = ar^{n-1}$

$11866 = 5000(1.025)^{n-1}$

$2.3732 = (1.025)^{n-1}$

$\log_{1.025} 2.3732 = n-1$

$35 = n-1$

$36 = n$

In the yr 2036

8. $S_8 = -3280 \quad r = -3$

$$-3280 = \frac{a[(-3)^8 - 1]}{[-3-1]}$$

$$-3280 = \frac{a[6560]}{[-4]}$$

$2 = a$

$\therefore t_1 = 2$

9. (a) $-3/4$ conv to $-3/4$

(b) $\lim\limits_{x \to \infty} \frac{14x^2 + 29x - 15}{10x^2 - 6x - 4}$

$= \frac{14}{10} = 7/5$ conv to $7/5$

$$\frac{(2x+5)(7x-3)}{(5x+2)(2x-2)}$$

(c) $\frac{0}{10}$ conv to 0

(d) conv to 0

(e) Diverge

(f) conv. to 0

10. (a) $\sum_{K=1}^{122} 4k^3$

$4\left[\frac{n(n+1)}{2}\right]^2$

$4\left[\frac{122(123)}{2}\right]^2$

$= 225180036$

(b) $\sum_{K=1}^{1500} 12$

$= 12n$

$= 12(1500)$

$= 18000$

(c) $\sum_{K=1}^{75}(5K^2-10K+2)$

$5\left[\frac{n(n+1)(2n+1)}{6}\right]-10\frac{n(n+1)}{2}+2n$

$5\left[\frac{75(76)(151)}{6}\right]-10\left[\frac{75(76)}{2}\right]+2(75)$

$= 688900$

(d) $\sum_{K=50}^{150}(K^3-3)$

$\sum_{K=1}^{150}(K^3-3) - \sum_{K=1}^{49}(K^3-3)$

$\left[\left[\frac{n(n+1)}{2}\right]^2-3n\right]_{K=150} - \left[\left[\frac{n(n+1)}{2}\right]^2-3n\right]_{K=49}$

$128255175 - 1500478$

$= 126754697$

Multiple Choice

1. C
2. $t_n=6n+9$
3. D
4. B
5. B
6. D
7. B
8. C
9. A
10. A
11. C
12. No 12
13. D
14. C
15. D
16. D
17. A

14. $11+\frac{11}{3}+\frac{11}{9}+\frac{11}{27}$

$a=11$

$r=\frac{1}{3}$

$S_n=\frac{11\left[\left(\frac{1}{3}\right)^n-1\right]}{\left[\frac{1}{3}-1\right]}$

$n\to\infty$

$\frac{11(0-1)}{\left(\frac{1}{3}-1\right)}=\frac{11(-1)}{-\frac{2}{3}}$

$=\frac{-11}{-\frac{2}{3}}=\frac{33}{2}$

16. $S_n=\frac{2\left[\left(\frac{1}{5}\right)^n-1\right]}{\left(\frac{1}{5}-1\right)}$

$n\to\infty$

$=\frac{2(0-1)}{\left(\frac{1}{5}-1\right)}$

$=\frac{-2}{-\frac{4}{5}}=\frac{10}{4}=\frac{5}{2}$

$82000(1.016)^{25}$

$\boxed{2000-2025}$ *

$15+15\left(\frac{8}{9}\right)+15\left(\frac{8}{9}\right)^2+15\left(\frac{8}{9}\right)^3+\ldots$

$S_n=\frac{15\left[\left(\frac{8}{9}\right)^n-1\right]}{\left[\frac{8}{9}-1\right]}$

$S_\infty=\frac{15\left[\left(\frac{8}{9}\right)^\infty-1\right]}{\left[\frac{8}{9}-1\right]}\to\frac{15[0-1]}{-\frac{1}{9}}$

$=135$

0.05 0.07

$5, 7, 9, \ldots$

+2 +2

24^{th} day

$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$S_{24}=\frac{24}{2}\left[2(0.05)+(24-1)(0.02)\right]$

Attachments

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