

Log Equations

Solving Logarithmic Equations

STEPS... (1) Write left side & right side as a single logarithm

NOTE: $\log_a a = 1$

(2) Set arguments equal & solve the equation

(3) Check for extraneous roots

EXAMPLE #1: $\log_3 x - \log_3 4 = \log_3 12$

log law for subtraction

~~$\log_3 \frac{x}{4} = \log_3 12$~~

$\frac{x}{4} = 12$

$x = 48$

** note all base 3.*

** bring each side together so there is one log.*

EXAMPLE #2:

Solve the following equation... $\log_{10}(x+2) + \log_{10}(x-1) = 1$

$\log_{10}(x+2)(x-1) = 1$

$\log_{10}(x+2) + \log_{10}(x-1) = 1$

$\log_{10}(x+2)(x-1) = 1$ OK

$\log_{10}(x+2) + \log_{10}(x-1) = 1$

$\log_{10}(x+2)(x-1) = 1$

~~$\log_{10}(x+2)(x-1) = \log_{10} 10$~~

$(x+2)(x-1) = 10^1$

$(x+2)(x-1) = 10$

$x^2 + x - 2 = 10$

$x^2 + x - 2 = 10$

$x^2 + x - 12 = 0$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$(x+4)(x-3) = 0$

$x = -4$ (3)

$x = -4$ (3)

Issue: The argument of a log-function cannot be negative. -4 is extraneous

EXAMPLE #3:

Solve: $\log_2(x+1) + \log_2(x-1) = 3$ WRONG $x=3$

$\log_2(x+1)(x-1) = 3$ OK

$(x+1)(x-1) = 2^3$

$\log_2(x+1)(x-1) = \log_2 8$

$x^2 - 1 = 8$

$x^2 - 9 = 0$

$(x-3)(x+3) = 0$

$x = 3$ (3) ← extraneous

↓ watch the negative

$$\log_5(4x+2) - \log_5(x-6) = 2$$

$$\log_5 \frac{4x+2}{x-6} = 2$$

$$\frac{4x+2}{x-6} = 5^2$$

$$\frac{4x+2}{x-6} = 25$$

$$4x+2 = 25(x-6)$$

$$4x+2 = 25x-150$$

$$-21x = -152$$

$$x = \frac{-152}{-21}$$

$$x = 152/21 \quad \checkmark$$

$$\log_2(3x-1) + \log_2(x+1) = 5$$

$$\log_2(3x-1)(x+1) = 5$$

$$(3x-1)(x+1) = 2^5$$

$$(3x-1)(x+1) = 32$$

$$3x^2 + 2x - 1 = 32$$

$$3x^2 + 2x - 33 = 0$$

$$\rightarrow 3x^2 + 11x - 9x - 33 = 0$$

$$x(3x+11) - 3(3x+11) = 0$$

$$(3x+11)(x-3) = 0$$

$$x = -11/3 \quad \text{or} \quad 3$$

extraneous

$$\log_2(5x-2)+\log_2(3x+2)=6$$

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(a) $\log_5(3x+1) + \log_5(x-7) = 2$

(b) $\log_2(5x-6) + \log_2(x+6) = 7$

(c) $\log_9(4x+5) - \log_9(x-5) = 1$

(c) $\log_9(4x+5) - \log_9(x-5) = 1$
 $\log_9 \frac{4x+5}{x-5} = 1$
 $\frac{4x+5}{x-5} = 9^1$
 $4x+5 = 9(x-5)$
 $4x+5 = 9x-45$
 $4x-9x = -45-5$
 $-5x = -50$
 $x = 10$

(d) $\log_6(5x-3) + \log_6(5x+3) = 3$

$\log_2(5x-6) + \log_2(x+6) = 7$

(b) $(5x-6)(x+6) = 2^7$

$5x^2 + 30x - 6x - 36 = 128$

$5x^2 + 24x - 36 - 128 = 0$

$5x^2 + 24x - 164 = 0$

$\frac{-24 \pm \sqrt{3856}}{10}$

-3.81
 ~~-8.61~~
 extraneous

(d) $\log_6(5x-3) + \log_6(5x+3) = 3$

$\log_6(5x-3)(5x+3) = 3$

$(5x-3)(5x+3) = 6^3$

$25x^2 - 9 = 216$

$25x^2 - 225 = 0$

$25(x^2 - 9) = 0$

$25(x-3)(x+3)$

$x = 3$
 ~~$x = -3$~~
 extraneous

(e) $\log_{12}(4-x) + \log_{12}(1-3x) = 2$

$\log_{12}(4-x)(1-3x) = 2$

$(4-x)(1-3x) = 12^2$

$4 - 12x - 1x + 3x^2 = 144$

$3x^2 - 13x - 140 = 0$

$\frac{13 \pm \sqrt{1849}}{6}$
 $\frac{13 - 43}{6} = -5$ ✓
 $\frac{13 + 43}{6} = 9.3$
 extraneous

#8. (e) $\log_2 \sqrt{x^2 + 4x} = \frac{5}{2}$

$$\left(2^{\frac{5}{2}}\right)^2 = \left(\sqrt{x^2 + 4x}\right)^2$$

$$2^5 = x^2 + 4x$$

$$32 = x^2 + 4x$$

$$0 = x^2 + 4x - 32$$

$$0 = (x+8)(x-4)$$

$$x = -8, +4$$

* Solve a log equation:

① change to exp. form

OR

write each term as a log