$$8\log_2 \sqrt[4]{x} - \frac{5}{3}\left[9\log_2 x^5 + 12\left(\log_2 \sqrt[5]{x^3} - 3\log_2 x^{-2}\right)\right]$$

$$8\log_2 x^{1/4} - \frac{5}{3}\left[9\log_2 x^5 + 12\log_2 x^{3/5} - 36\log_2 x^{-2}\right]$$

$$8\log_2 x^{1/4} - 15\log_2 x^5 - 20\log_2 x^{3/5} + 60\log_2 x^{-2}$$

$$\log_2 x^2 - \log_2 x^{75} - \log_2 x^{12} + \log_2 x^{-120}$$

$$\log_2\left[\frac{x^2 \cdot x^{-120}}{x^{75}\, x^{12}}\right]$$

$$\log_2\left[\frac{x^{-118}}{x^{87}}\right]$$

$$\log_2 x^{-205}$$

2. Given that $\log_r x = -6$, $\log_r y = -2$, and $\log_r z = 9$, evaluate the expression $\log_r\left(\dfrac{\sqrt[3]{x^2 z^5}}{r^{-3} y^4}\right)$.

$\log_r x^{2/3} + \log_r z^{5/3} - \log_r r^{-3} - \log_r y^4$

$\frac{2}{3}\log_r x + \frac{5}{3}\log_r z + 3\log_r r - 4\log_r y$

$\frac{2}{3}(-6) + \frac{5}{3}(9) + 3 - 4(-2)$

$= -4 + 15 + 3 + 8$

$= \underline{22}$

$$\log_2\left[\log_x\left(\log_2 2\sqrt{8}\right)\right] = -2$$

$$\log_2\left[\log_x\left(\log_2 2^1 \cdot 2^{3/2}\right)\right] = -2$$

$$\log_2\left[\log_x \frac{5}{2}\right] = -2$$

$$2^{-2} = \log_x \frac{5}{2} \longrightarrow \left(x^{\frac{1}{4}}\right)^4 = \left(\frac{5}{2}\right)^4$$

$$\cancel{\left(x^{-\frac{1}{4}}\right)^{-4} = \left(\frac{5}{2}\right)^{-4}}$$

$$x = \cancel{\left(\frac{2}{5}\right)^4 = \frac{16}{625}} \qquad x = \frac{625}{16}$$

Exponential
Equations solved
using logs

Exponential Equations with Like Bases

- In an Exponential Equation, the variable is in the exponent. There may be one exponential term or more than one, like…

$$3^{2x+1} - 5 = 4 \quad \text{or} \quad 3^{x+1} = 9^{x-2}$$

$3^{5+1} = 9^{5-2}$

$3^6 = 9^3$

$729 = 729.$

- If you can isolate terms so that the equation can be written as two expressions with the same base, as in the equations above, then the solution is simple.

$3^{x+1} = 9^{x-2}$

$3^{x+1} = (3^2)^{x-2}$

$3^{x+1} = 3^{2x-4}$

$x+1 = 2x-4$

$1+4 = 2x-x$

$5 = x$

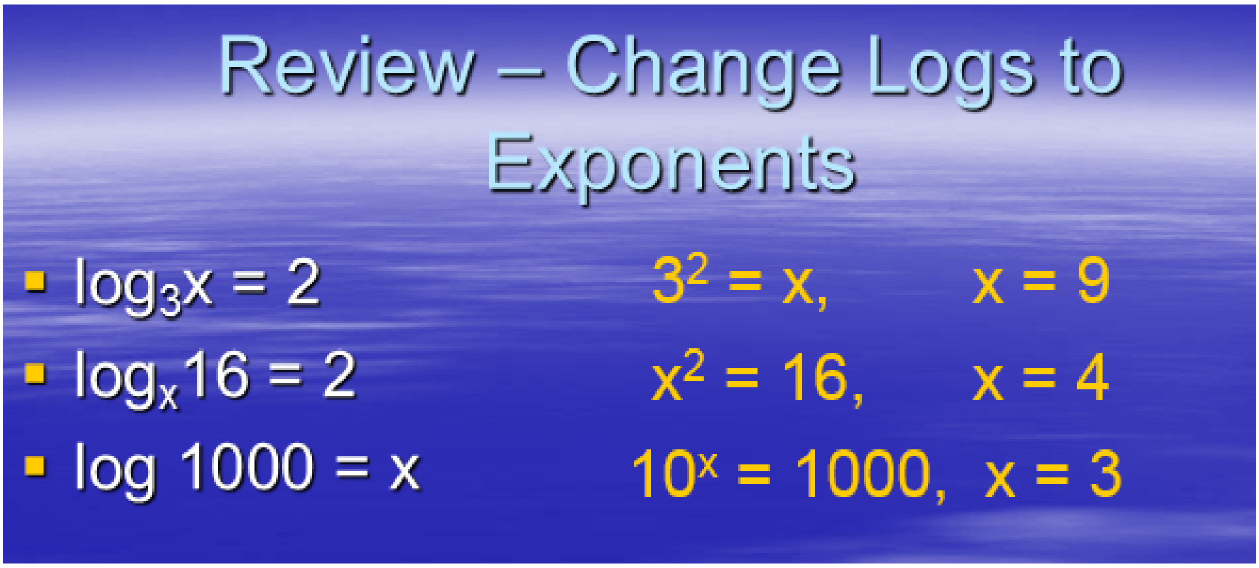
Exponential Equations with Like Bases

⌘ Example #1 - One exponential expression.

$$3^{2x+1} - 5 = 4$$
$$3^{2x+1} = 9$$
$$3^{2x+1} = 3^2$$
$$2x + 1 = 2$$
$$2x = 1$$
$$x = \frac{1}{2}$$

1. Isolate the exponential expression and rewrite the constant in terms of the same base.

2. Set the exponents equal to each other (drop the bases) and solve the resulting equation.


Review – Change Logs to Exponents
$\log_3 x = 2$
$3^2 = x$, $x = 9$
$\log_x 16 = 2$
$x^2 = 16$, $x = 4$
$\log 1000 = x$
$10^x = 1000$, $x = 3$

Exponential Equations with *Different* Bases

- The Exponential Equations below contain exponential expressions whose bases cannot be rewritten as the same rational number.

$$3^{2x+1} - 5 = 11 \quad \text{or} \quad 3^{x+1} = 4^{x-2}$$

$$3^{2x+1} = 16$$

- The solutions are irrational numbers, we will need to use a log function to evaluate them.

Ex: $3^x = 50$

cannot get to the same base:

① take the log of both sides

② using log. laws rewrite the equation so the variable is not an exponent

$\log 3^x = \log 50$

$x \log 3 = \log 50$

$x = \dfrac{\log 50}{\log 3}$

$x = 3.56\ldots$

OR: $3^x = 50$

$x = \log_3 50$

$x = \dfrac{\log 50}{\log 3} = 3.56\ldots$

$\cancel{2}^x = 543$

$x = \log_2 543$

$x = \dfrac{\log 543}{\log 2}$

$x = 9.08\ldots$

$4^{\sqrt{x}} = 88$

$\sqrt{x} = \log_4 88$

$x = (\log_4 88)^2$

$x = 10.43$

$3^{2x-5} = 900$

$2x-5 = \log_3 900$

$2x = \log_3 900 + 5$

$x = \frac{\log_3 900 + 5}{2}$

$x = 5.596$

$\Rightarrow 2x-5 = \frac{\log 900}{\log 3}$

$2x-5 = 6.191806549$

$2x = 11.191506549$

$x = \frac{11.191506549}{2}$

$\log 3^{2x-5} = \log 900$

$(2x-5)\log 3 = \log 900$

$5^{2x} = 100^{x+1}$

① $2x = \log_5 100^{x+1}$

$2x = (x+1)\log_5 100$

$\frac{2x}{x+1} = \log_5 100$

$\frac{2x}{x+1} = 2.86135$

$2x = 2.86135(x+1)$

$2x = 2.86135x + 2.86135$

$2x - 2.86135x = 2.86135$

$-0.86135x = 2.86135$

$x = \frac{2.86135}{-0.86135}$

$x = -3.32$

OR ② $\log 5^{2x} = \log 100^{x+1}$

$2x \log 5 = (x+1)\log 100$

$\frac{2x}{x+1} = \frac{\log 100}{\log 5}$

$\frac{2x}{x+1} = 2.86135$

$$4^{2x-3} = 12^{x+1}$$

$2x-3 = \log_4 12^{x+1}$ oR $\log 4^{2x-3} = \log 12^{x+1}$

$(2x-3)\log 4 = (x+1)\log 12$

$2x-3 = (x+1)\log_4 12$

$\frac{2x-3}{x+1} = \frac{\log 12}{\log 4}$

$\frac{2x-3}{x+1} = \log_4 12$

$\frac{2x-3}{x+1} = 1.79248$

$2x-3 = 1.79248(x+1)$

$2x-3 = 1.79248x + 1.79248$

$2x - 1.79248x = 1.79248 + 3$

$0.20752x = 4.79248$

$x = \frac{4.79248}{0.20752}$

$x = 23.09$

$2x-3 = \log_4 12(x+1)$

$2x-3 = (\log_4 12)x + \log_4 12$

$2x - \log_4 12x = \log_4 12 + 3$

$x(2 - \log_4 12) = \log_4 12 + 3$

$x = \frac{\log_4 12 + 3}{2 - \log_4 12}$

$$19^{3x+2} = 4^{6x}$$

$$3x+2 = \log_{19} 4^{6x}$$

$$3x+2 = 6x \log_{19} 4.$$

$$\frac{3x+2}{6x} = \log_{19} 4$$

$$\frac{3x+2}{6x} = 0.470818$$

$$3x+2 = 2.824908x$$

$$3x - 2.824908x = -2$$

$$0.175092x = -2.$$

$$x = \frac{-2}{0.175092}$$

$$x =$$

exponential equations solved using logs.doc

Attachments

exponential equations solved using logs.doc