

Test 2013 Practice test 2014.doc

MC : Practice

- | | | |
|------|-------|-------|
| 1. B | 8. C | 15. B |
| 2. A | 9. C | 16. A |
| 3. B | 10. B | |
| 4. C | 11. C | |
| 5. C | 12. A | |
| 6. D | 13. B | |
| 7. A | 14. C | |

1,2

3. a) $\log_2 32^{20}$ (b) $\log_6 15 + \log_6 18 - \log_6 10 + 2\log_6 4 - \log_6 2$

$$20 \log_2 32$$

$$20(5)$$

$$100$$

$$\log_6 \frac{15(18)(4)^2}{10(2)}$$

$$\log_6 216$$

$$= 3$$

4. (a) $3(4)^{-2x+1} - 1 = 23$

$$3(4)^{-2x+1} = 24$$

$$4^{-2x+1} = 8$$

change to base 2

$$(2^2)^{-2x+1} = 2^3$$

$$2^{-4x+2} = 2^3$$

$$-4x+2 = 3$$

$$-4x = 1$$

$$x = -\frac{1}{4}$$

or log:

$$\log 4^{-2x+1} = \log 8$$

$$(-2x+1) \log 4 = \log 8$$

$$-2x+1 = \frac{\log 8}{\log 4}$$

$$-2x+1 = 1.5$$

$$-2x = 0.5$$

$$x = -0.25$$

(b) $\log_9 x = -\frac{3}{2}$

$$9^{-\frac{3}{2}} = x$$

$$\frac{1}{9^{\frac{3}{2}}} = x$$

$$\frac{1}{27} = x$$

(c) $\log_5 (9x-2) + \log_5 (x+2) = 3$

$$\log_5 (9x-2)(x+2) = 3$$

$$(9x-2)(x+2) = 5^3$$

$$9x^2 + 18x - 2x - 4 = 125$$

$$9x^2 + 16x - 129 = 0$$

$$\frac{-16 \pm \sqrt{4900}}{18}$$

extraneous

$$\frac{-16 \pm 70}{18}$$

3

(d) $\log_{2x} 256 = 4$

$$(2x)^4 = 256$$

$$16x^4 = 256$$

$$x^4 = 16$$

$$x = \sqrt[4]{16} \text{ or } 16^{\frac{1}{4}}$$

$$x = 2$$

$$\begin{aligned}
 5. \quad & 3 \log_b x + 9 \log_b x + 3 \log_b x - 12 \log_b x - \frac{1}{2} \log_b x^4 \\
 & \log_b x^3 + \log_b x^9 + \log_b x^3 - \log_b x^{12} - \log_b x^2 \\
 & \log_b \frac{x^3 \cdot x^9 \cdot x^3}{x^{12} \cdot x^2} \\
 & \log_b \frac{x^{15}}{x^{14}} \\
 & \log_b x
 \end{aligned}$$

$ \begin{aligned} 6.(a) \quad & V = 200(2)^{t/25} \\ & 2000 = 200(2)^{t/25} \\ & 10 = 2^{t/25} \\ & \log 10 = \frac{t}{25} \log 2 \\ & \frac{\log 10}{\log 2} = \frac{t}{25} \\ & 3.3219 = \frac{t}{25} \\ & t = 83.05 \text{ days} \end{aligned} $	$ \begin{aligned} (b) \quad & T = 37.6(0.987)^{5t} \\ & 30 = 37.6(0.987)^{5t} \\ & \frac{30}{37.6} = 0.987^{5t} \\ & 0.7978723 = 0.987^{5t} \\ & \log 0.7978723 = 5t \log 0.987 \\ & \frac{\log 0.7978723}{\log 0.987} = 5t \\ & 17.256598 = 5t \\ & 3.4513 = t \\ & \text{hrs} \end{aligned} $
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$$\begin{aligned}
 7. \quad & D = 500(0.68)^t \\
 a) \quad & 32\% \\
 b) \quad & D = 500(0.68)^2 \\
 & = 231.2 \text{ mg} \\
 c) \quad & 100 = 500(0.68)^t \\
 & 0.2 = 0.68^t \\
 & \log 0.2 = t \log 0.68 \\
 & \frac{\log 0.2}{\log 0.68} = t \\
 & t = 4.17 \text{ hrs}
 \end{aligned}$$

Practice test logs and exponents.doc

Pre-Calculus 12A

Practice Unit Test: Exponential and Logarithmic Functions

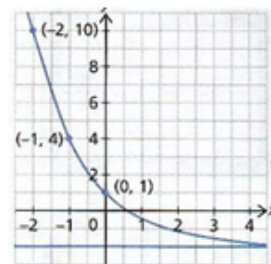
I. Multiple Choice: Place the letter corresponding to the correct solution on the scantron sheet provided.

- Express the following in exponential notation: $\log_5 M = C$
 [A] $b^M = C$ [B] $M^b = C$ [C] $M^C = b$ [D] $b^C = M$
- Solve for x : $5^{x-12} = 25^{3x}$
 [A] $-\frac{12}{5}$ [B] -3 [C] -6 [D] $-\frac{6}{7}$
- The value of $\log_2 56 - \log_2 7 + \log_2 2$ is
 [A] $\log_2 51$ [B] 1 [C] 2 [D] 4
- If $\log x^2 - \log 2x = 2$, then x equals...
 [A] 4 [B] 50 [C] 100 [D] 200
- Find all real solutions for the following equation: $\log_2 x + \log_2 (x-2) = 3$
 [A] $x = 3, -1$ [B] $x = 4$ [C] $x = 4, -2$ [D] $\frac{5}{2}$
- Evaluate: $10^{(\log 5 + \log 2 - \log 100)}$
 [A] -1 [B] 10^{-1} [C] 1 [D] 10
- Express as a single power of 2: $\frac{8^{3n+2} \cdot 4^{n-1}}{16^2} = ?$
 [A] 2^{11n-4} [B] 2^{11n+12} [C] 2^{11n-7} [D] 2^{11n+9}

8. Which of the following equations would define the following graph?

[A] $f(x) = 2^x - 2$ [B] $f(x) = \left(\frac{1}{2}\right)^x - 2$

[C] $f(x) = 3\left(\frac{1}{2}\right)^x - 2$ [D] $f(x) = 3(2)^x - 2$



9. The logarithmic expression $\log\left(\frac{a^3}{\sqrt{bc}}\right)$ equals... $\log \frac{a^3}{b^{1/2} c^{1/2}}$

[A] $\frac{1}{3} \log a - 2 \log b + \log c$

[B] $\frac{1}{3} \log a - 2 \log b - \log c$

[C] $3 \log a - \frac{1}{2}(\log b - \log c)$

[D] $3 \log a - \frac{1}{2} \log b - \frac{1}{2} \log c$

10. If $f(x) = 5^{-2(x-2)}$, then find the value of $f(1)$.

[A] 25

[B] 2

[C] $\frac{1}{125}$

[D] $\frac{1}{25}$

11. Jason has been collecting die-cast cars for several years and realizes that the value of each car increases exponentially with time. One car in his collection was purchased for \$1.80 in 1970 and its value has doubled every 6 years since then. Which of the following equations could be used to model the value of the car (V) in terms of the number of years (t) since it was purchased?

[A] $V = 1.80(2)^{6t}$

[B] $V = 1.80(2)^{\frac{t}{6}}$

[C] $V = 2(1.80)^{6t}$

[D] $V = 2(1.80)^{\frac{t}{6}}$

12. Determine the value of x : $\log_x 16 = -\frac{2}{3}$

[A] -64

[B] $\frac{1}{64}$

[C] 64

[D] $-\frac{1}{64}$

13. Solve for x : $3(4)^{-2x+1} - 1 = 23$

[A] $-\frac{1}{2}$

[B] $-\frac{1}{4}$

[C] $-\frac{7}{2}$

[D] $\frac{1}{2}$

14. Cesium-137 is an exceptionally dangerous radioactive isotope with a half-life of 30 years. How long would it take for a sample of 800 mg of Cesium-137 to decay to 100 mg?

[A] 3 years

[B] 120 years

[C] 90 years

[D] 60 years

15. If $5[\log_2(\log 10)] = x$, then x is equal to ...

[A] 0

[B] 1

[C] 5

[D] 2

$$5(\log_2(\log 10)) = x$$

$$5(\log_2 1) = x$$

$$5(0) = x$$

$$0 = x$$

II. Open Response: All work for each of the following must be shown in the space provided. (37 Marks)

1. Given the exponential function: $y = 3(2)^{3x+6} + 7$

- a) describe the transformations of the function when compared to the function $y = (2)^x$
b) state the domain, the range, the y-intercept and the equation of the asymptote

Handwritten student work for the problem. The student shows the original function $y = 3(2)^{3x+6} + 7$ and rewrites it as $y = 3(2)^{3(x+2)} + 7$. For part (a), they list: Vertical stretch = 3, Horizontal stretch = 1/3, Left 2, and Up 7. For part (b), they list: Domain: $x \in \mathbb{R}$, Range: $y > 7$, and Horizontal Asymptote (HA): $y = 7$.

1. $y = 3(2)^{3x+6} + 7$
 $y = 3(2)^{3(x+2)} + 7$

(a) Vert. stretch = 3
Horizontal stretch = $\frac{1}{3}$
Left 2
Up 7

(b) Domain: $x \in \mathbb{R}$
Range: $y > 7$
HA: $y = 7$

2. For the function $y = -4\left(\frac{1}{6}\right)^{x+5} - 8$,

- a) describe the transformations of the function when compared to the function $y = \left(\frac{1}{6}\right)^x$
- b) state the domain, the range, the y-intercept and the equation of the asymptote

2. $y = -4\left(\frac{1}{6}\right)^{x+5} - 8$

(a) Vert. Stretch 4
Horizontal Stretch -
Left 5
Down 8
reflected x-axis

(b) Domain $x \in \mathbb{R}$
Range $y < -8$
HA: $y = -8$ (because it is reflected x-axis)

3. For the function $y = 2\log_4(x-3)$,

- a) describe the transformations of the function when compared to the function $y = \log_4(x)$
b) state the domain, the range, the y-intercept and the equation of the asymptote

$$3. y = 2\log_4(x-3)$$

(a) Vert. stretch 2
Horizontal stretch -
Right 3

(b) Domain $x > 3$
Range $y \in \mathbb{R}$
VA $x = 3$

4. Sketch $y = 2(3)^{x+5} - 1$ show clearly the y-intercept and horizontal asymptote

4. $y = 2(3)^{x+5} - 1$

y-int: $y = 2(3)^{0+5} - 1$
 $y = 487$

HA: $y = -1$
 growth

Find x-int:
 $0 = 2(3)^{x+5} - 1$
 $1 = 2(3)^{x+5}$
 $\frac{1}{2} = 3^{x+5}$
 $\log_3 0.5 = x+5$
 $-0.63 = x+5$
 $-5.63 = x$

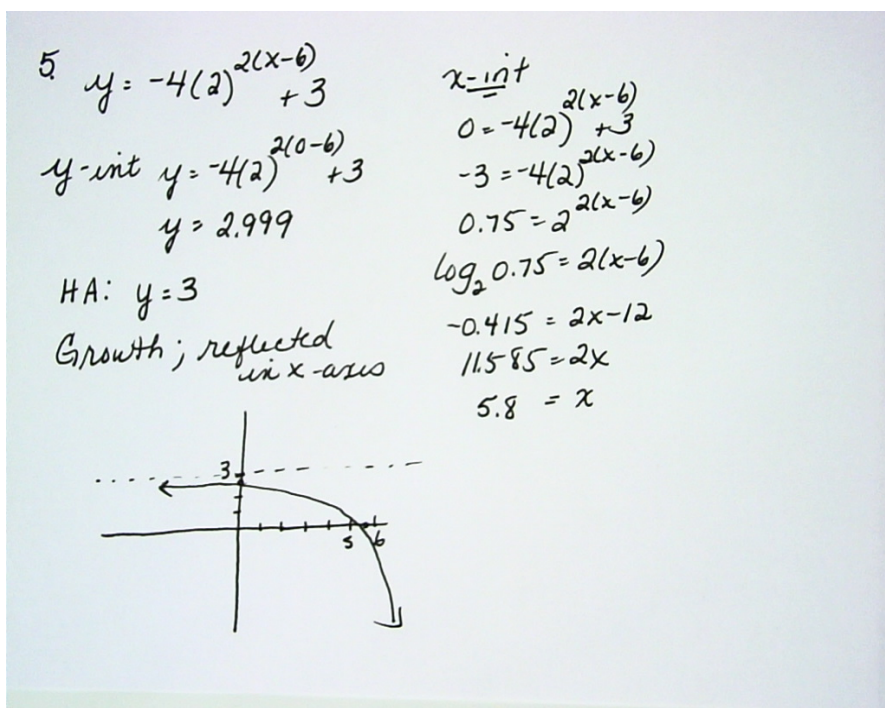
cannot be -1

$\log_b x$

$y = b^x$

can be -1

5. Sketch $y = -4(2)^{2(x-6)} + 3$, show clearly the y-intercept and horizontal asymptote



6. A cup of coffee contains approximately 100 mg of caffeine. When you drink coffee, the caffeine is absorbed into the bloodstream and eventually metabolized by the body. Every three hours the amount of caffeine in the bloodstream is reduced by 42%.
- Write an equation which express the amount of caffeine, C (in mg), in the bloodstream as an exponential function of the elapsed time, t (in hours), since drinking one cup of coffee.
 - Find the amount of caffeine in the bloodstream after 8 hours.
 - How many hours (**accurate to the nearest hundredth**) does it take for the amount of caffeine to be reduced to 10 mg?

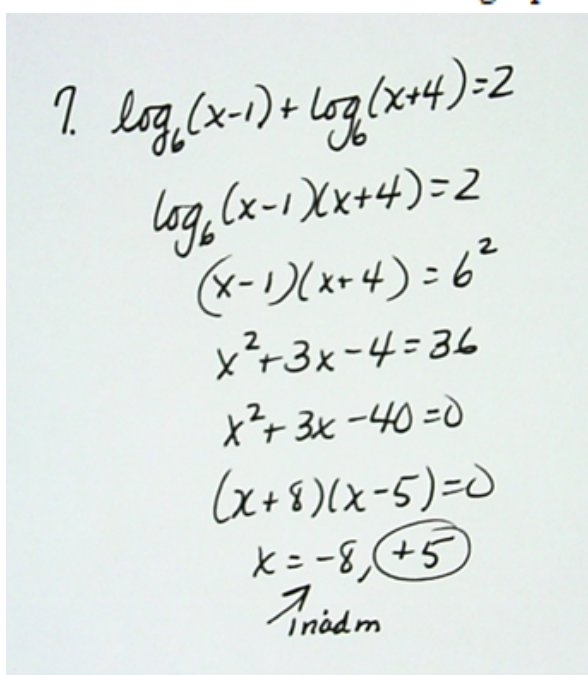
6.

(a) $C = 100(0.58)^{t/3}$

(b) $C = 100(0.58)^{8/3}$
 $= 23.4 \text{ mg}$

(c) $10 = 100(0.58)^{t/3}$
 $0.1 = 0.58^{t/3}$
 $\log_{0.58} 0.1 = \frac{t}{3}$
 $4.227 = \frac{t}{3}$
 $t = 12.68 \text{ hrs}$

7. Solve for x in each of the following equations: $\log_6(x-1) + \log_6(x+4) = 2$



Handwritten solution for the equation $\log_6(x-1) + \log_6(x+4) = 2$:

$$\begin{aligned} 7. \log_6(x-1) + \log_6(x+4) &= 2 \\ \log_6(x-1)(x+4) &= 2 \\ (x-1)(x+4) &= 6^2 \\ x^2 + 3x - 4 &= 36 \\ x^2 + 3x - 40 &= 0 \\ (x+8)(x-5) &= 0 \\ x &= -8, \text{ } \boxed{+5} \\ &\uparrow \\ &\text{indm} \end{aligned}$$

8. a) Given that $\log_r x = -2$, $\log_r y = 5$, and $\log_r z = 4$, evaluate the expression $\log_r \left(\frac{x^3 z^2}{r \sqrt[5]{y}} \right)$

b) Express the following as a **single logarithm in simplest form**: $5 \log_2 x - \frac{3}{4} [4 \log_2 x^3 - 12 \log_2 x^2]$

8.

(a) $\log_r \frac{x^3 z^2}{r y^{1/5}}$

$$3 \log_r x + 2 \log_r z - \log_r r - \frac{1}{5} \log_r y \quad \checkmark$$

$$3(-2) + 2(4) - 1 - \frac{1}{5}(5)$$

$$-6 + 8 - 1 - 1$$

$$= 0$$

(b) $5 \log_2 x - \frac{3}{4} [4 \log_2 x^3 - 12 \log_2 x^2]$

$$5 \log_2 x - 3 \log_2 x^3 + 9 \log_2 x^2$$

$$\log_2 \left[\frac{x^5 x^{18}}{x^9} \right]$$

$$\log_2 x^{14}$$

Attachments

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